High Speed Aerodynamics

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Lecture No. # 21
Isothermal Flow in Ducts with Friction

So, when the duct is very long there is enough area to heat transfer take place from and to the flow stream and consequently the flow is not adiabatic. However, the flow often reaches an approximate isothermal state, and this flow can be analyzed as isothermal flow in a duct and the most common example of such type of flow is transporting of gas over a long distance through pipes. Now, question usually comes that a transportation of gas over a long distance through pipe is essentially takes place at quite a low speed, then should the flow be treated as compressible or it can be treated as incompressible.

However, the flow mach number is usually quite low for gas transportation through a pipe however, over a long distance a substantial change in pressure takes place and which necessitates the for more appropriate analysis the flow should be treated as compressible. Now, essentially the analysis is similar to the adiabatic flow analysis that we have already discussed, with some minor verification. First of all since the flow is not adiabatic the total temperature T 0 is not constant, it is changing from point to point remember that T 0 is the total temperature that the flow would have reached, if it were brought to rest adiabatically, so at any point corresponding to a particular T there is a T 0 and in this case this T 0 changes throughout along the flow.

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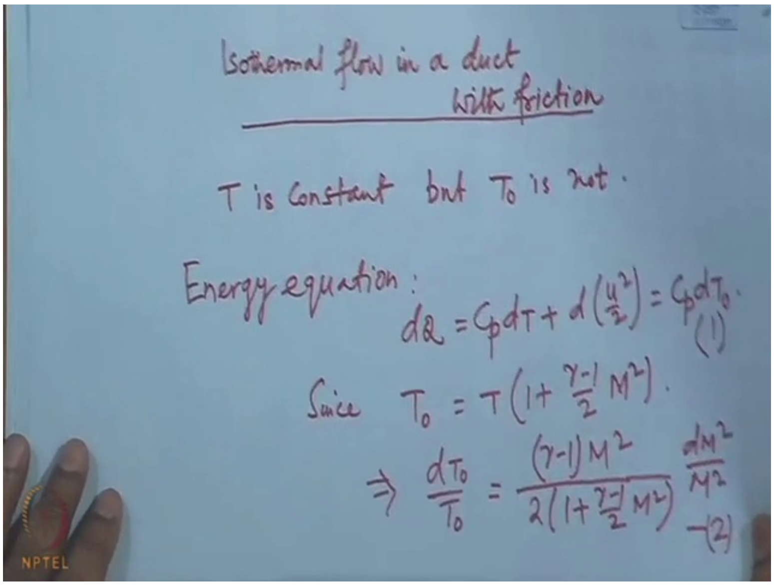

And so if we use these in a duct with friction the duct is of course, of uniform cross sectional area so it really does not appear in the equations, so T is constant in this case, but t 0 is not now in this case the energy equation takes the form energy equation takes the form dQ equal to C p dT plus d u square by 2 that equal to c p dT 0, since by definition T 0 is T into 1 plus gamma minus 1 to M square this implies dT 0 by T 0 equal to gamma minus 1 M square by 2 into 1 plus gamma minus 1 by 1 by 2 M square into dM square by M square. We will denote this as equation number two and this as equation number 1.

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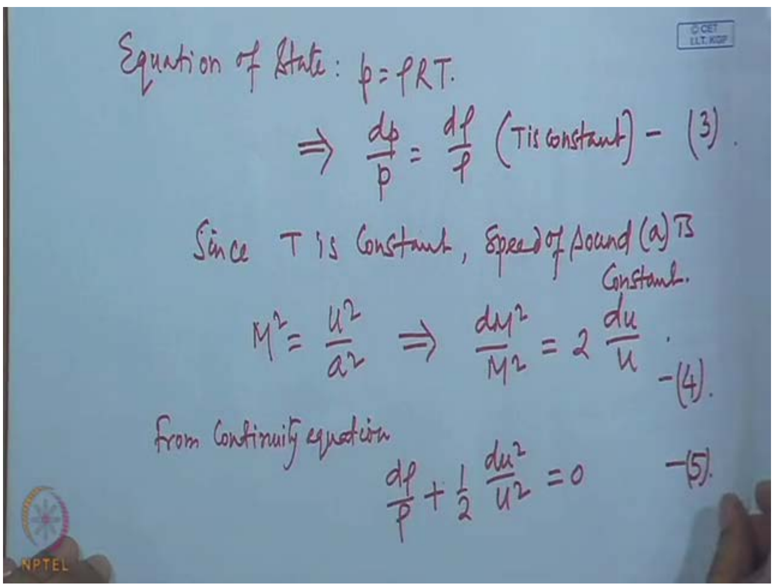

So, the equation of state for a perfect gas equation of state that is p equal to rho RT for a perfect gas and this implies dp by P equal to d rho by rho t is constant and we denote this by equation three and since t is constant speed of sound a is constant and M square equal to U square by a square this gives dM square by M square equal to 2 du by u, and the continuity momentum and energy equation sorry continuity and momentum equations remain same, and consequently from continuity what we have obtained in the case of adiabatic flow equation, we get the same relation d rho by rho plus half of du square by U square equal to 0 and this we call equation number 5.

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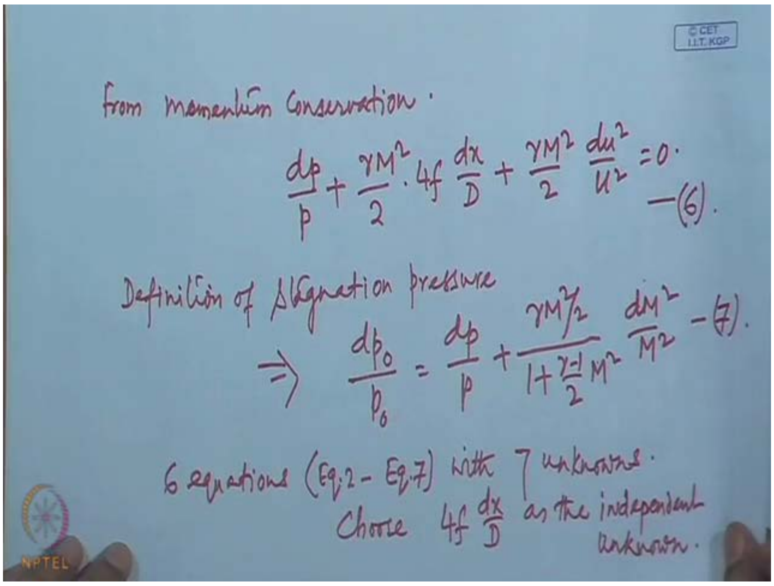

Similarly, from momentum conservation from momentum conservation, we get the same equation that we had in case of adiabatic flow with friction in a uniform area duct namely dp by p by gamma M square by 2 into 4f d x by D plus gamma M square by 2 du square by U square equal to 0, and from the definition of stagnation pressure this will be our equation six this gives us as before dp 0 by p 0 dp by p plus gamma M square by 2 divided by 1 plus gamma minus 1 by 2 M square into dM square by M square.

We can write again another equation for impulse function however the impulse function for this problem is not very important in this case, and consequently we are not going to use the impulse function, so hence we have here once again six equation that is from equation two, three, four, five, six, and seven with seven unknown six equations with seven unknown, and once again we choose that six equations equation two to equation seven with seven unknowns, and we once again choose 4f d x by D as a independent one.

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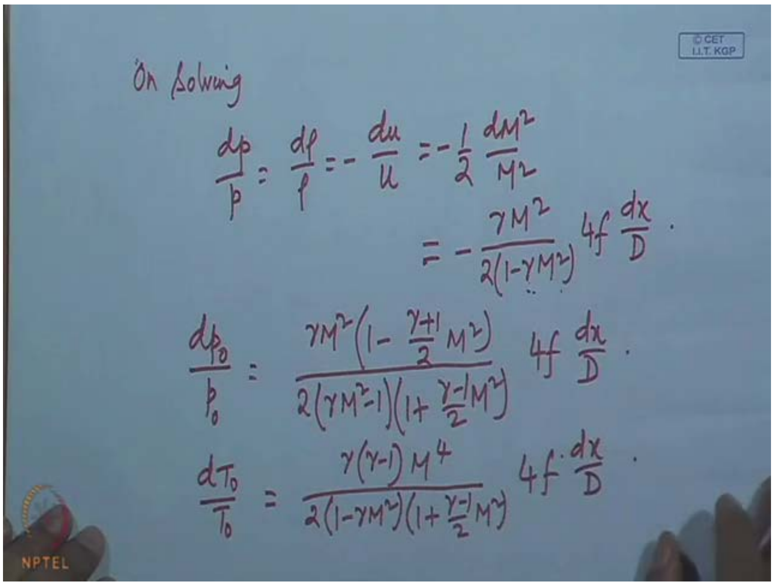

Now, solving these six algebraic equations, that is equation two to equation six what we get is dp by p is drho by rho into minus du by U that is minus half dM square by M square, this is equal to minus gamma M square by 2 into 1 minus gamma M square into 4f d x by D. We also have dp 0 by p 0 equal to gamma M square into 1 minus gamma plus 1 by 2 M square by 2 into gamma M square minus 1 into 1 plus gamma minus 1 by 2 M square into 4f dx by D and dT 0 by T 0 equal to gamma into gamma minus 1 M to the power 4 by 2 into 1 minus gamma M square into 1 plus gamma minus1 by 2 M square 4f dx by D.

Now, you see that in this case the sign of these terms depends not M square alone but, on gamma M square or the direction of change depends on gamma M square but, not M square alone.

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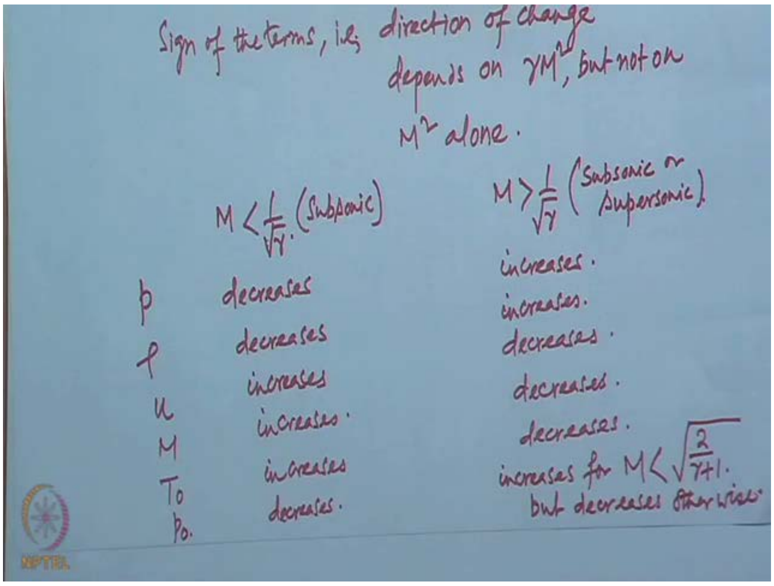

So, this shows sign of the terms that is direction of change depends on gamma M square but, not on M square alone now four f d x by d is positive always, hence we can see that the pressure will decrease if M square is less than one minus gamma sorry if M square is less than one by gamma however, it will increase if M square is greater than one by gamma.

Similarly, the other parameters will also the sign of these other parameters that is density change, velocity change in mach number, change in total pressure and total temperature will depend on whether M square is greater than or less than one by gamma. So, we can summarize this change in this that if M is less than one by root gamma this is of course, the flow is subsonic and the other case is M greater than one by root gamma in this case it might be subsonic or supersonic both. What you see that if M is less than one by root gamma or M square is than one by gamma pressure in this case decreases, while in this case pressure increases.

The density too decreases and increases in this case u increases, in this case it is decreases, M increases up to one by root gamma, and in this case decreases up to root one by gamma, T 0 increases,, in this case it is decreases, p 0 decreases however, coming to p 0, we see another situation that when M square is greater than one by gamma that is when this is positive, there is there is a possibility that this term can still be negative giving this trying to be negative that is p 0 may decrease and however there is also the possibility that this term may be positive, when M square is greater than one by gamma numerator is of course, positive the denominator can also be positive for certain value of m giving dp 0 by p 0 as positive meaning that the stagnation pressure may increase for some specific value of mach number which, we can see that it is increases for M less than square root of 2 by gamma plus 1 but, decreases otherwise.

That is for mach number greater than one by root gamma, but less than square root of two by gamma plus 1 the stagnation pressure increases however, when mach number is greater than square root of 2 by gamma plus 1 stagnation pressure decreases.

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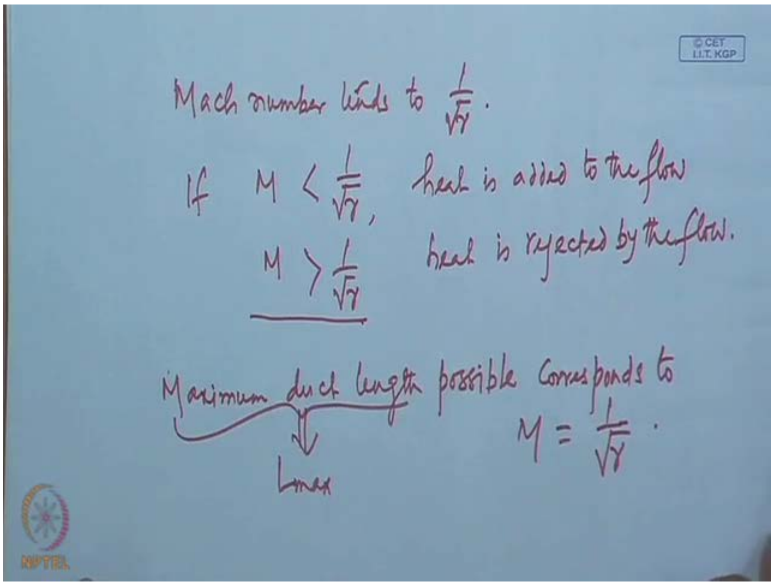

What you find here that the mach number always tends towards infinity sorry 1 by root gamma mach number tends to 1 by root gamma however, 1 by root gamma is essentially subsonic, so implies in this isothermal flow the subsonic flow will always remain subsonic however, the supersonic flow may come down to a subsonic flow with minimum mach number of one by root gamma.

Also, we see that when mach number is less than one by, if mach number is less than one by root gamma heat is added to the flow stream if M is greater than one by root gamma heat is rejected by the stream. Since, the subsonic flow accelerates to the maximum value of one by root gamma mach number maximum value of mach number one by root gamma and subsonic flow decelerates to the minimum value of one by root gamma, so you can see that that can be the maximum length of the duct possible where flow mach number reaches one by root gamma.

So, maximum duct length possible corresponds to M, and again this maximum duct length if we denote it by l max, then once again we can integrate the relation from 0 to l max and at 0

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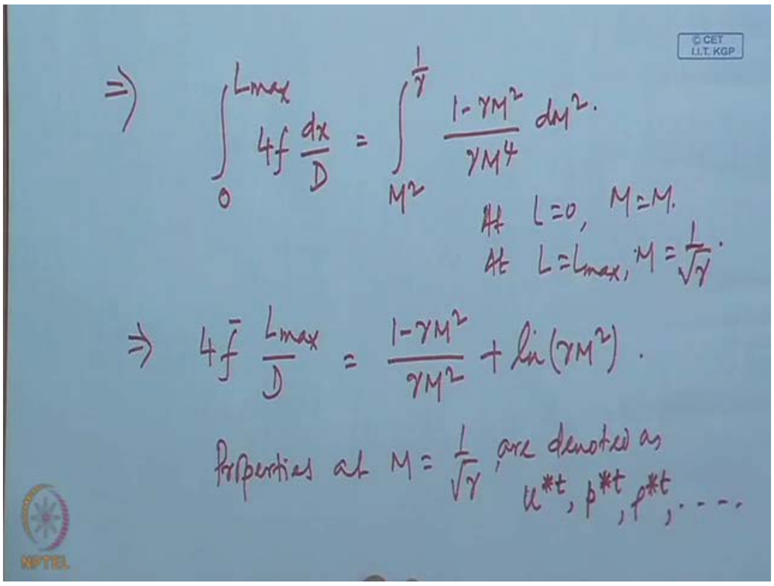

we consider (No Audio 24:27 to 25:43) again defining an average friction factor, this gives us four f power l max by D to 1 minus gamma M square by gamma M square plus log of gamma M square.

Now, let us denote the properties, properties at M equal to 1 by root gamma are denoted U star, U star t, p star t that is t two represent an isothermal that, this is u star isothermal case and p star isothermal case, and rho star t and so on.

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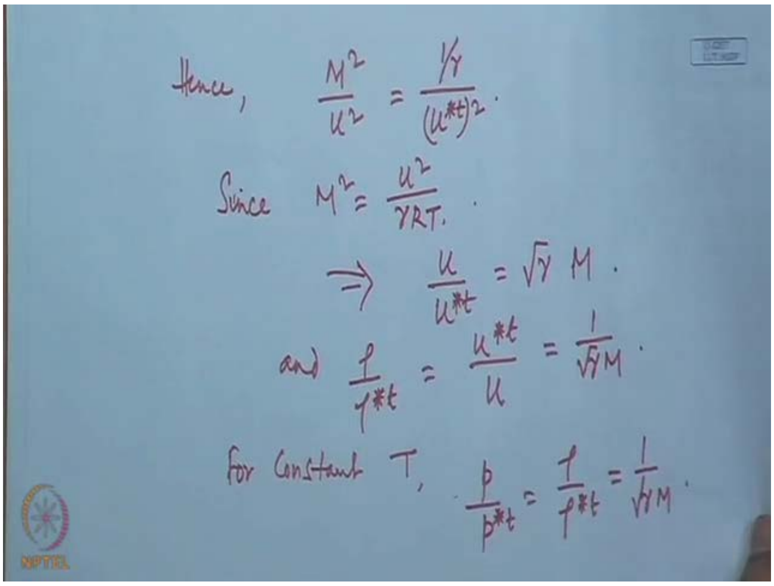

Then hence the parameter M square by U square can be written as 1 by gamma and this is U star t square, since we have M square equal to u square by gamma RT, and T is constant this gives us U by U star t equal to root gamma into M.

And rho by rho star t is in continuity in a uniform duct is 1 by root gamma M, now using perfect gas relationship for constant T we have p by p star t is same as rho by rho star t which is also 1 by root gamma M.

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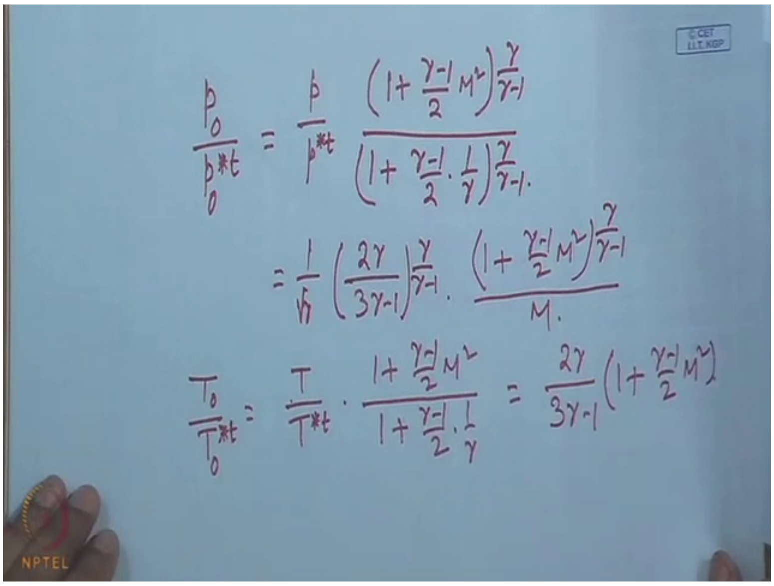

Now, using the definition of stagnation pressure we also have p 0 by p 0 star t into p by p star t 1 plus gamma minus 1 by 2 M square to the power gamma by gamma minus 1 divided by 1 plus gamma minus 1 by 2 into 1 by gamma to the power gamma by gamma minus 1, and substituting the value for t by p star t this becomes 1 by root gamma into 2 gamma by 3 gamma minus 1 to the power gamma by gamma minus 1 into 1 plus gamma minus 1 by 2 M square to the power gamma by gamma minus 1 by M.

The stagnation temperature accordingly can also be written as T 0 by T equal to T 0 by T 0 star t is t by t star t into one plus gamma minus one by two M square divided by one plus gamma minus one by two into one by gamma which is two gamma by three gamma minus one into one plus gamma minus one by two m square.

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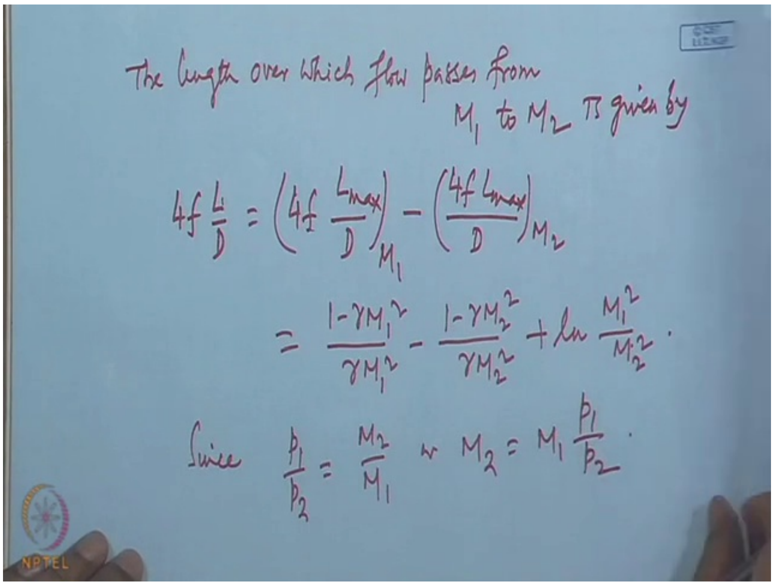

The length over which the length over which flow passes from M 1 to M 2 is given by 4 f L by D where L is this required length is 4 f sorry M 1 minus M 2, and using the relation that have been obtained for 4f L max by L max by D we get one minus gamma M one square by gamma M one square minus one minus gamma M two square by gamma M two square plus we have got M one square by M two square.

We have already seen that p 1 by p 2 is same as M 2 by M 1 or M 2 equal to M 1 one into p 1 by p 2 and substituting this relationship here.

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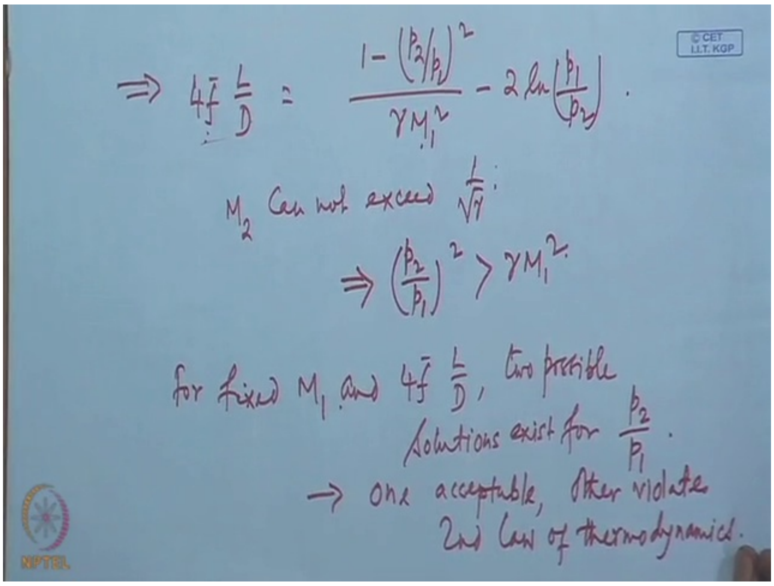

You get 4f bar L by D into one minus p 2 by p 1 square by gamma M one square minus two log p 1 by p 2, now since M 2 cannot exceed one by root gamma M 2 cannot exceed one by root gamma, and this gives p 2 by p 1 square is less than gamma M 1 square. Now, if you have a fixed value of M 1 and also of fixed friction coefficient then, we have two solutions for p 2 by p 1, this relation clearly shows that if we have a fixed value of M 1 and a fixed value of the friction 4f L by D then we have two possible solution for p 2 by p 1 however, only one of them is acceptable the other is not because that violates the second law of thermodynamics.

So, for fixed M 1 and 4f L by D two possible solutions exist for p 2 by p 1 however, one is one acceptable, other the other violates second law of thermodynamics, if we consider a very small part of small pressure drop a small in percentage, and then we can expand this relation in power series of this fractional pressure drop.

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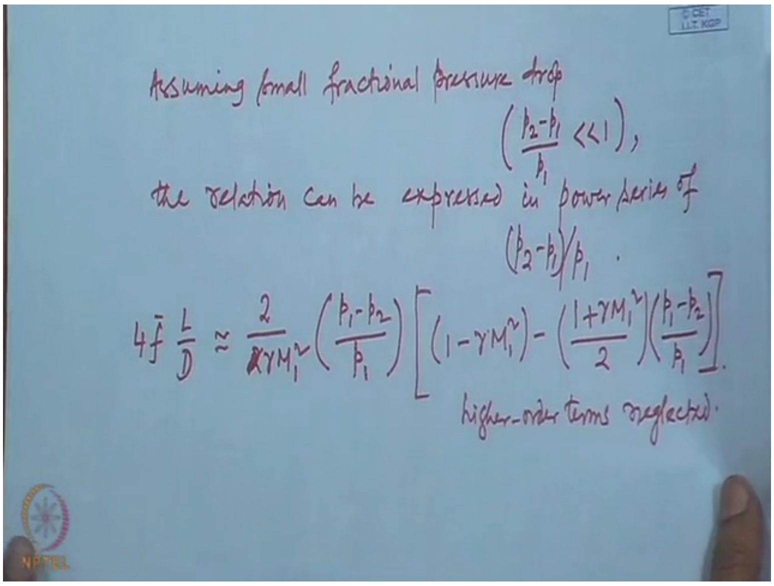

Assuming small fractional pressure drop that is p 2 minus p 1 by p 1 the relation in power series of p 2 minus p 1 by p 1 and this gives 4f bar L by D is two by gamma sorry two by gamma M 1 square into p 1 minus p 2 by into one minus gamma M 1 square minus one plus gamma M 1 square by two into p 1 minus p 2 by p 1, where higher order terms are neglected, and we can see that; this formula is similar to the conventional pressure drop formula in incompressible flow where of course, these term within the square bracket is just unity. Now, solving this solving this quadratic equation.

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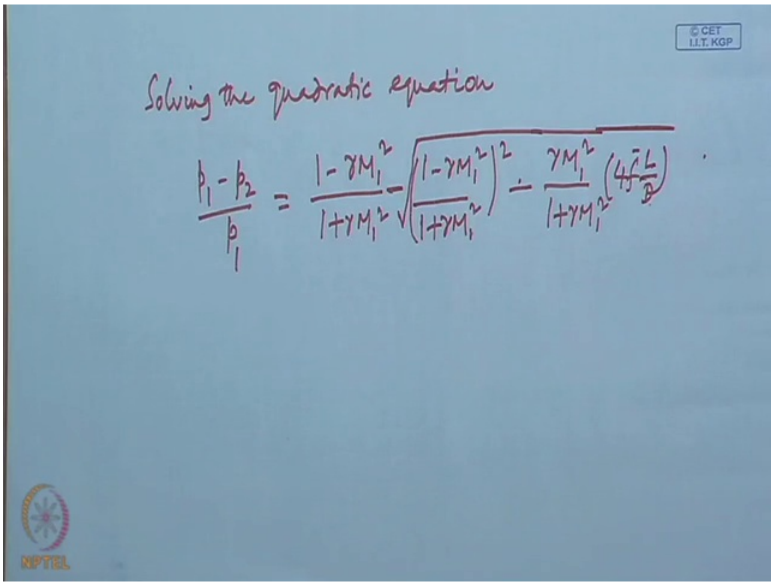

Solving the quadratic equation one minus (No Audio 44:07 to 45:01), we see that in this solution the plus term is dropped, because that is not physically expectable solution as it violates the second law of thermodynamics, so this gives the pressure drop over a portion of, or over a length of the duct given by l, or the average friction factor is f bar and this also a measure of the power required to drive this flow through a duct of uniform area over a length l.

We have also seen that for a given value of M 1 there is a maximum length for continuous isothermal flow, and at the end of that length there is the mach number reaches is one by root gamma. Hence, it follows that of choking effect may occur in similar fashion to those for adiabatic flow however, one thing should be noticed here very clearly that as mach number is very close to one by root gamma the changes in all the parameters are very rapid that is looking back to the relations for dp by p or any of these that all these relations suggest that the change in the parameters is very fast or in the mach number is very close to one by root gamma and consequently in that rapid change the assumptions of approximately isothermal flow will may not hold that is when the flow reaches very close to M equal to one by root gamma, it is most likely that the flow will be there nearly adiabatic instead of nearly isothermal.

However, looking from our simplified analysis of one dimensional isothermal flow, what we have seen that for a long duct, over a long duct with a uniform cross sectional area where friction is present the flow reaches or tends to a mach number one by root gamma and we have as you have seen here that, if the flow is subsonic that is the maximum mach number then can reach one by root gamma that is the flow still remains subsonic in this case however, in the supersonic case the flow mach number can come down to one by root gamma; which is subsonic that means, a supersonic flow when flows isothermally in a long duct with uniform area and friction can become subsonic.

We have seen that the parameters, pressure of density decreases in the duct for mach number less than one by root gamma however, they increases if the mach number is less than one by root gamma similarly, the parameter flow velocity, flow mach number, total temperature increases for flow with mach number less than one by root gamma however, they decreases if the flow mach number remains so lesser than one by root gamma.

The stagnation pressure decreases for mach number one by root gamma however, for the mach number root more than one by root gamma case, there is a reason or there is a range of mach number which lies between one by root gamma and square root of two by gamma plus one in that range the stagnation pressure increases however, if the mach number is outside this range, that is the mach number is greater than square root of two by gamma plus one then stagnation pressure decreases.

We have also seen that, when mach number is less than one by root gamma because of the friction heat is added to the flow, and heat is rejected by the flow when mach number is one by root gamma we have also seen what will be the length of a duct required to change the mach number from M 1 to M 2 and we have also seen that what will be the pressure drop over a length L over which mach number changes from M 1 to M 2 and pressure decreases changes from p 1 to p 2.

So, that concludes our discussion on isothermal flow in a long duct with friction, next we will consider third problem belonging to these category of flow problems, that is flow in a duct which is flow in a duct with heat heating or cooling

See, in a duct the flow can changes because of change in area, because of friction or because of heating or cooling, we have already considered change in area only change in area also you have considered change only in the friction, presence of friction or no friction but, similarly, now we will consider when there's simply heat is added.

However, a combination of all these or any two of these three is practical and quiet possible and they are more perhaps more realistic however, they cannot be solved analytically even in one dimensional framework.

So, anyway our next problem of discussion will be flow in ducts with heating or cooling with that we.