High Speed Aerodynamics

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Lecture No. # 23

Multi-dimensional Flow Problems

So until now, we have we are discussing flow in one dimension, and that flow in one dimension as helped us to understand mini gas dynamical problem and in particular, we have seen that, we have been able to use that one dimensional analysis to compute the lift and wave drag experienced by an air fall; even that would have been if easy to extend to a wing problem using the one dimensional analysis. So that supersonic flow problem in particular, this one dimensional analysis takes us to a long way in understanding various aero dynamical and gas dynamical problem. However, for subsonic and in particular for transonic flow problems, this one dimensional analysis does not help much and also in general, most of our flow problems are essentially multi dimensional.

So now, we will consider multi dimensional flow problems and first of all, we will see the governing equations for flow in multi multiple dimension. The basic flow equations as before remain the conservation laws, that is the conservation of mass, conservation of momentum and conservation of energy, and this gives us the most important governing equations for fluid flow problem. These equations are derived in earlier process in aerodynamic. So, we will now go for a formal derivation here, but we will try to revisit these equations, and rewrite the equations and try to see what do they convey.

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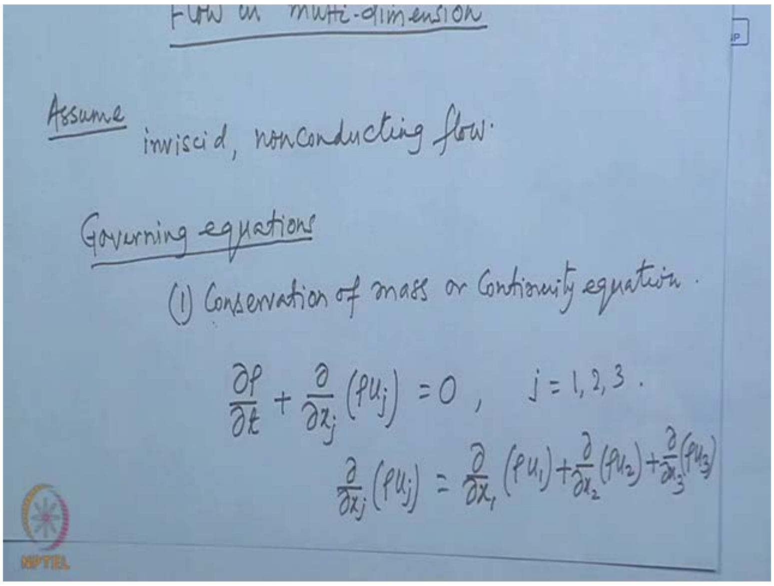

So, the… We will of course, restrict to inviscid non conducting fluid. So, we will consider flow in multi dimension and we will consider inviscid non conducting flow. So, the effect of viscosity and effect of heat conduction will not be considered in these equations that we are going to discuss. The governing equation, the first of the governing equations is the mass conservation or commonly called as the continuity equation conservation of mass or continuity equation. It simply states that, the mass within a control volume remain conserved; that is the rate of change of mass within the control volume is balanced by the mass that is coming into the control volume, and going out of the control volume or either, the rate of change of mass flow in the control volume is balanced by the mass fluxes from the boundary of the control volume. This then can be written as the rate of change of mass flow and in fluid dynamical problem usually the equations are written in unit volume bases.

So the mass per unit volume that is the density. So the rate of change density within the control volume is balanced by the fluxes must fluxes through the boundary of the control surface. What we have essentially used is known as the cartesian tensor notation or a repertory index in a term implies summation; that is this term represents d dx 1 rho u 1 plus d dx 2 rho u 2 plus d dx 3 rho u 3, where u 1, u 2, u 3 are the three component of the velocity and x 1, x 2 x 3 are the three reactions; that is we are considering three dimensional space in which j equal to 1, 2, 3; j can take the value of 1, 2, 3.

So, the velocity vector u velocity vector u is written as u j and the three coordinates direction written as x j, and the term j being the index which is repeated within this term implies a summation given by this. So this is, what is the continuity equation or conservation of mass equation. One more things would be recalled that the velocity u or the velocity field u as a function of x 1, x 2, x 3 and t represents the Eulerian velocity; that is it is velocity at a point fixed in the space, but not the velocity of any fluid particle; counter u to the Lagrangian definition of velocity which is used in all rigid body mechanics.

However, the velocity represents the velocity of rigid particle or a body. In fluid mechanics, most often the Eulerian velocity or the velocity field is used. However, the velocity refers to a fixed point in the space; that is u is the velocity vector at a point fixed in the space, and any fluid material or fluid particle that is passing through that point at that instant we will have the velocity u, but it is not the velocity of the fluid.

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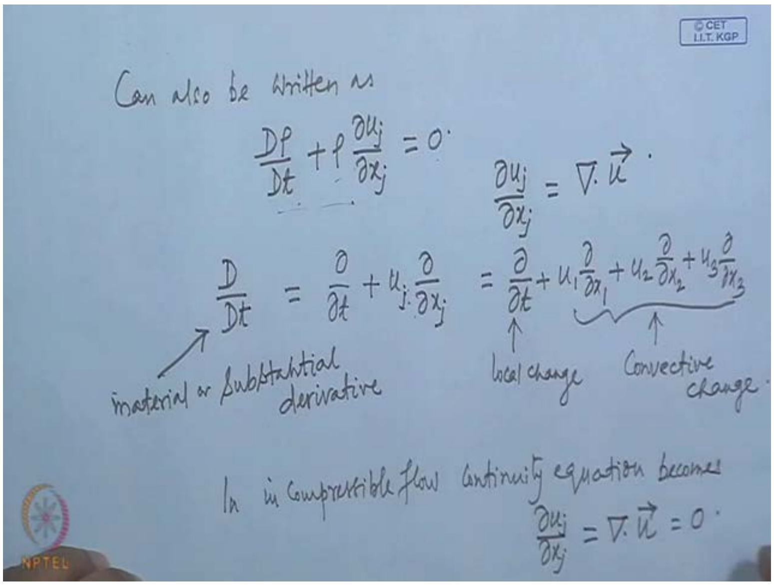

Now, this equation can also be written in the form; even also be written as written as D rho Dt plus rho du j dx j equal to 0, and this rho du j dx j or other du j dx j is our conventional diversion separator, and this D Dt which is called a material or substantial derivative which is know as material or substantial derivative is D Dt (No audio 10:16 to 11:03) where this is so called local or unsteady derivative local change, and this is this represents the convective change. In derivation of this continuity equation; that is this or the earlier form it is assumed that, there is no mass sources within the control volume.

However, if there are mass sources within the control volume that right hand side 0 will be replaced by that mass source pyramid volume. About this repertory indices this repertory indices, they are also called the dummy indices, and they can be change at will; that is we can write them either as i or k or l anything; it hardly it does not matter. Only thing must remember that particular index is not used in that equation for other purposes. Of course, this and we can see that in an in incompressible flow this changes to the well known equation the diversions of the velocity field is 0 or the velocity field is diversions free. So, we can write that in in in incompressible flow, continuity equation becomes… The equation flow can further be written as in this form that

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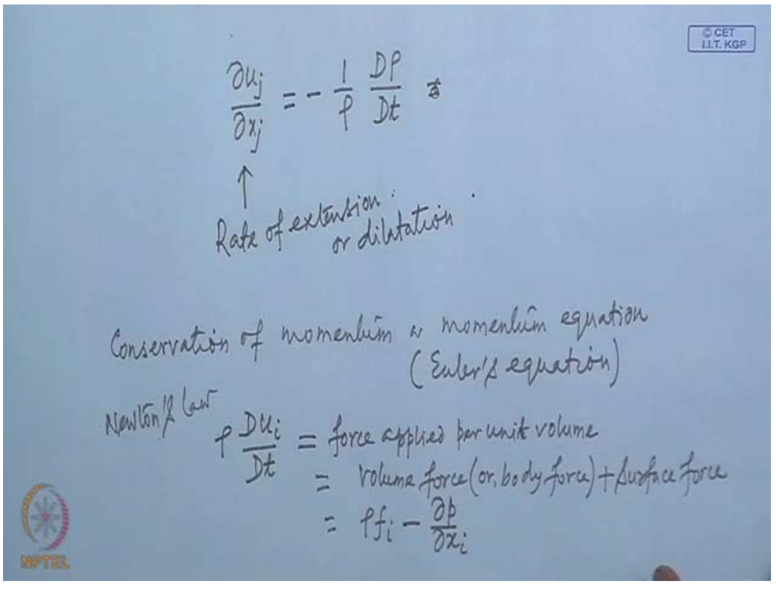

du j dx j (No audio 14:11 to 14:55) and this is gives the rate of extension or dilatation. So it says that, the rate of extension or dilatation of a material fluid element or the rate of volume change is equal to the rate fractional rate of density change. Now, let us come to the conservation of momentum and the resulting equations in for inviscid flow or inviscid non conducting flow are also known as Euler's equations. The corresponding equation for viscous conducting flow are called the Navier-Stokes equations.

Now, this momentum conservation equation is essentially the application of Newton's law which states, the rate of change of momentum of a particle is equal to the force applied on it, and rate of change of momentum is written as mass time acceleration. So, when you write it for fluid dynamical problem usually, we write it for unit volume. So, the mass actually becomes the density. So, the mass into acceleration for unit volume becomes density into acceleration, but the acceleration of a fluid particle. So, the acceleration of fluid particle which is written as rho Du i Dt, this equal the force that applied on the surface of the fluid element.

Now, the forces are usually of two types. That is of course, per unit volume and the forces are usually of two type; the volume force or body force volume force or body force plus surface force and the volume force that usually act in the fluid are may be due to gravity or may be electromagnetic force and similar rather, and the surface forces are usually the pressure or the viscous stresses. Since, we are not considering viscous flow here the surface force only comes from the pressure. So, this volume force if we that is f i is the volume force or body force per unit mass; f i is the body force per unit mass in the i-th direction plus surface force which is

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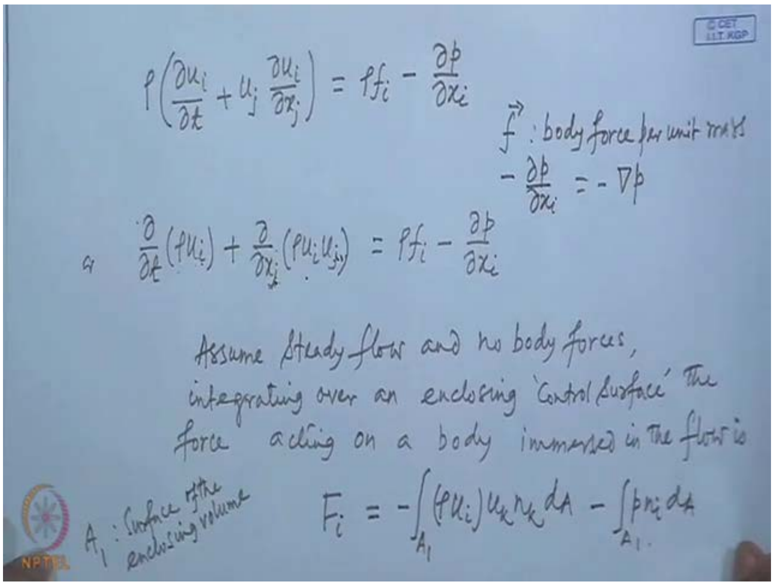

the equation can be written by a expanding that substantial derivative (No audio 20:14 to 20:53) f body force per unit mass. So in, if the gravitational force is the only body force present then, the shape becomes the gravitational acceleration g and as you can see that minus dp dx i is minus gradient of p; this can also be written as (No audio 21:33 to 22:13) about the repeated indices that we are mentioning earlier that j in this case is the repeated indice and or domain indice indices index and that can be changed whatever we want, we can make it k, l, m, n anything. However, in this case this i is being used for a separate purpose; this j should not be or cannot be make i; j cannot be written as i otherwise, it can be replaced by any other index.

Now, this form of this equation we could have obtained straight away if we had started with conservation of momentum in the conventional sense; that is considering a control volume and then rate of change of momentum within the control volume plus the momentum flux through the boundary of the control surface which is being balanced by the applied forces. This is the form that would have obtained straight away. However, this form and this form they are basically the same equation that as can be seen that, if we apply continuity equation the additional term that are contained here, they become 0 due to the application of continuity equation. So, this and this are same form of this equation.

Now, this equation can be integrated to over obtain the forces that are acting on a body immerged in the flow. So, if we assume steady flow and no body forces, integrating over a enclosing control surface. The force acting on a body immerged in the flow is minus rho u i u k n k d a integrated over the enclosing control surface (No audio 26:05 to 26:50) that is it is integration is carried out over the enclosing control surface. So, this gives the force acting on a body that is immerged in this inviscid non conducting flow.

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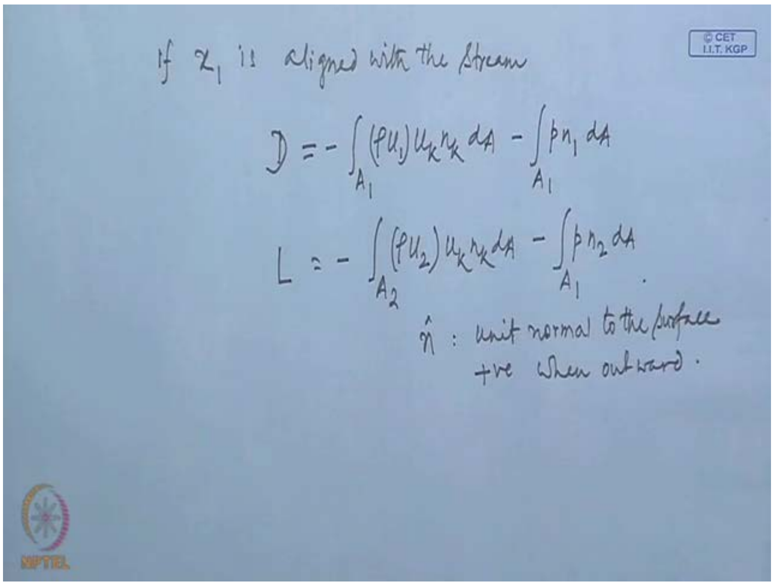

In particular, if we aligned our x 1 axis along the flow and x 2 axis along the normal to the flow, if x 1 is aligned with the stream then, the force along x 1 is the conventional drag force and then, this drag becomes and n is the normal to the surface and taken positive outward (No audio 28:50 to 29:30).

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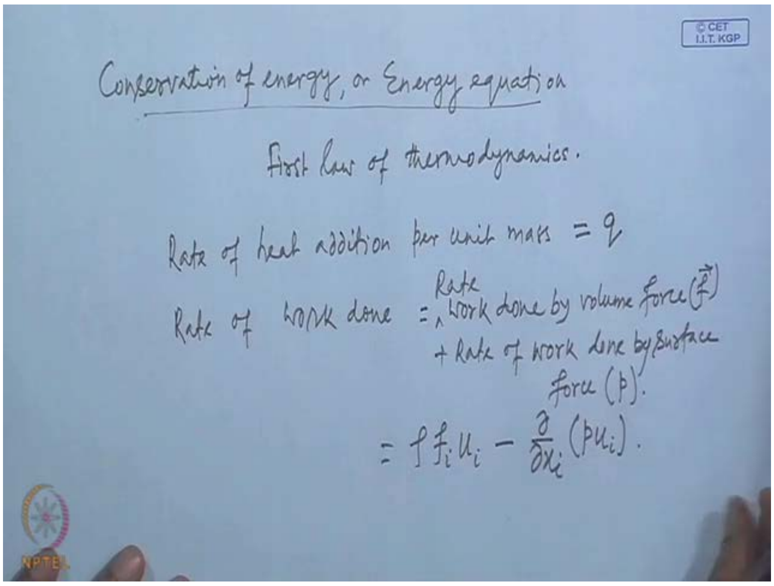

Next, we will consider the energy equation. The conservation of energy is basically, the first law of thermodynamics applied to a flow. You may recall that the first law of thermodynamics states that, rate of work done or the work done on a fluid plus heat added to a fluid is equal to the change in its internal energy. Now in case of a flow other than internal energy, there is kinetic energy. In most of the aero dynamical problem, we neglect potential energy; so otherwise, that also should be taken into account. So, the change in energy in this case must be the change in total energy, and since in fluid dynamics the equations are usually written as rate equations per unit volume. So, this first law of thermodynamics must be applied to win that in the rate from that is rate of feed added to the flow plus rate of work done by the flow rate of work done on the flow is equal to the rate of change of energy of the flow.

Now considering a control volume, the change in energy it in that control volume will be given by the local change plus the convective change and the heat addition. If we consider, we will add a rate of heat addition par unit mass t q. So, rate of work done… Now this work done, we have in this case two forces; that is the volume force or the body force. So the rate of work done is work done by the volume force plus work done by the surface force. So work done by the volume force of course rate plus rate of work done by surface force which happens to be the pressure in this case. So, this can be written as rho f i u i that is the so called scalar product of two vector minus d dx i of pu i. So the rate of heat addition per unit mass plus q plus rho f i u i minus a force per unit volume.

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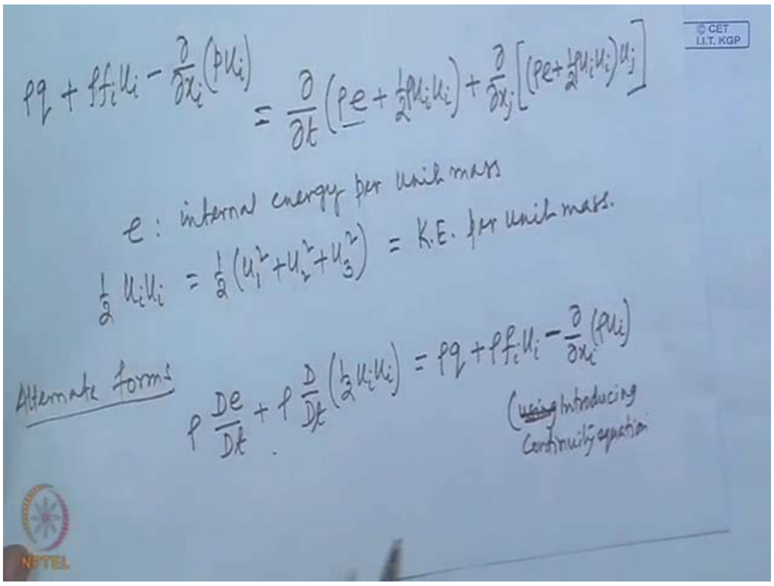

And this can be written as that rho q plus rho f i u i (No audio 35:15 to 36:24) u i u i is the sorry and one more rho is also required here.

So in, this is internal energy per unit mass that multiplied by the mass per unit volume gives it energy internal energy per unit volume rho e, and half u i u i which do to this cartesian tension notation becomes half of u 1 square plus u 2 square plus u 3 square kinetic energy per unit mass. Now, this equation can be written in many other form particularly, it can be manipulated and the continuity equation or Euler's equation can be introduced here, and the equation then will be written in different form, and some alternate form alternate forms; one is (No audio 38:20 to 39:09) that is only the this right hand side; now here is written in the on the left hand and this equation is manipulated and continuity equation is introduced in the manipulation. So this is obtained from this equation using or introducing rather introducing continuity. This equation can again be manipulated

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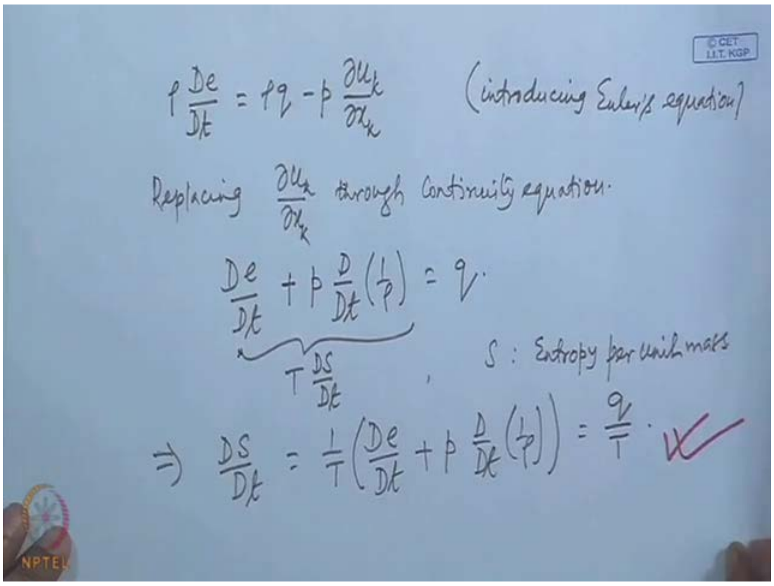

and the momentum equation can be introduced into it and (No audio 40:10 to 41:07) and again, if we introduce continuity equation in this term to replace du k dx k using continuity (No audio 41:15 to 41:46) can write this equation to be De Dt plus p D Dt of 1 by rho. Further, we can see here that this is, what is the definition of entropy? From the definition of entropy this gives (No audio 42:35 to 43:11). So, we have again this equation can be written as DS Dt which is 1 by T into De Dt plus p D Dt of 1 by rho will be q by T.

So, this is also another form of this energy equation for inviscid non conducting flow. In this case, I think we should remember that this q which is only heat added externally that is externally added heat; it is not by heat contained at the latent heat of the flow; it when it does not include by heat transfer through conduction. So, it is only heat that is coming from outside to the flow. So, our radiation is included.

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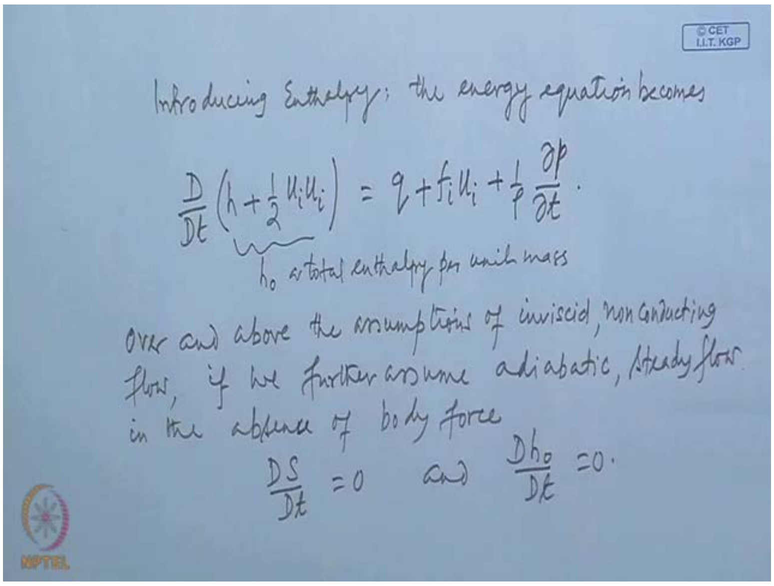

Similarly, if we introduce enthalpy, introducing enthalpy the energy equation becomes (No audio 45:01 to 46:03). Now, we see that these last two equation the t p q is 0 and the flow is steady. Of course, we have already assumed that the flow is inviscid and non conducting. In addition, if we further assume that no heat is added that is the flow is adiabatic, and it is steady then, we can see that the entropy and the total enthalpy this can also be written as the total enthalpy. So, over and above the assumptions of inviscid non conducting flow if we further assume the flow to be adiabatic that is q is 0, and it is steady and no body force is present then, both entropy and total enthalpy remain constant. However in this case, the material derivative is 0. So, over and above the assumptions of inviscid non conducting flow if we further assume adiabatic steady flow. So that this term becomes 0 sorry; this is 1 by rho dp dt. In the absence of body force then, both Ds Dt equal to 0, and that is the material derivative of S and h 0 are 0.

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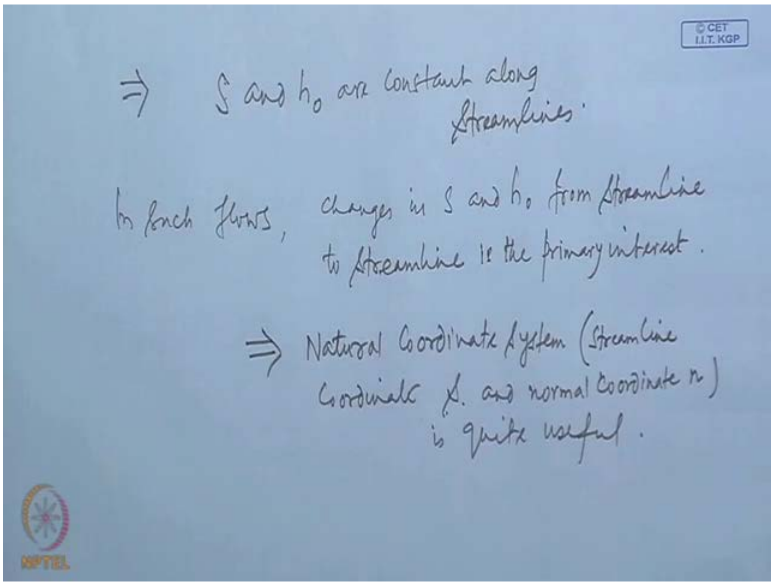

And this implies that S and h 0 remain constant along stream lines (No audio 49:44 to 50:15).

So once again, we come back to what we saw earlier that adiabatic frictionless flow of a non conducting fluid is essentially isentropic. However, here we have seen find that it is to be isoenergetic; this d pressure must be steady; the pressure field must be steady. In a non steady pressure field, if even if the flow is adiabatic, non conducting and inviscid, there can still be change in total enthalpy if the pressure field is unsteady. So, in such flows in such flows changes in S and h 0 from stream line from one stream line to a different stream line, and in this case this implies that so called natural coordinate system (No audio 52:13 to 53:07).

Since, in this isentropic an isenthalpic enthalpy flow, it is a importance to find the changes from one stream line to the other stream line because, entropy and enthalpy total enthalpy they remain constant on a particular stream line . So, how the changes from one streamline to the other streamline is of more important and since, this can be investigated more naturally using the natural coordinate system. However, the two coordinate directions are the streamline coordinate direction s and normal to the streamline coordinate direction n and to separate or to distinguish between the stream line coordinate s and the entropy S will be using small s for stream line coordinate and capital S for entropy.

So, we have today discussed or revisited the governing equations which tends from the conversation laws. We have not gone for a formal derivation, but discussed how they are derived, and we have written down the equations since, they are derived in earlier courses in aerodynamics, the full derivation was not required not thought required, and only the equation. Important features of the equations are mentioned particularly the energy equation are written in energy equation is written in various form also the cartesian tension notation is introduced here to write the equations for convenience and here also, we have found that a flow which is inviscid and non conducting will become isentropic if no heat is added to it; that is if the flow is adiabatic and in addition, the flow will also be total enthalpy will remain constant if the pressure field is steady and no body force is present. And for isentropic flow, we have also mentioned that or we can that natural coordinate system or stream line normal coordinate system will be more useful. We will continue this discussion in our next lecture.