High Speed Aerodynamics

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Lecture No. # 24

Multi-Dimensional Flow Problems (Contd.)

So, we will continue our discussion on the equations of fluid motion in multiple dimensions, and as we have seen that for an in viscid non conducting fluid, if the flow is adiabatic and the pressure filled is time independent then the total enthalpy and total entropy they remain constant along a straight line. And in such situation we had mentioned that you can express the equation in streamline, and normal to the streamline direction and can be used to our advantage.

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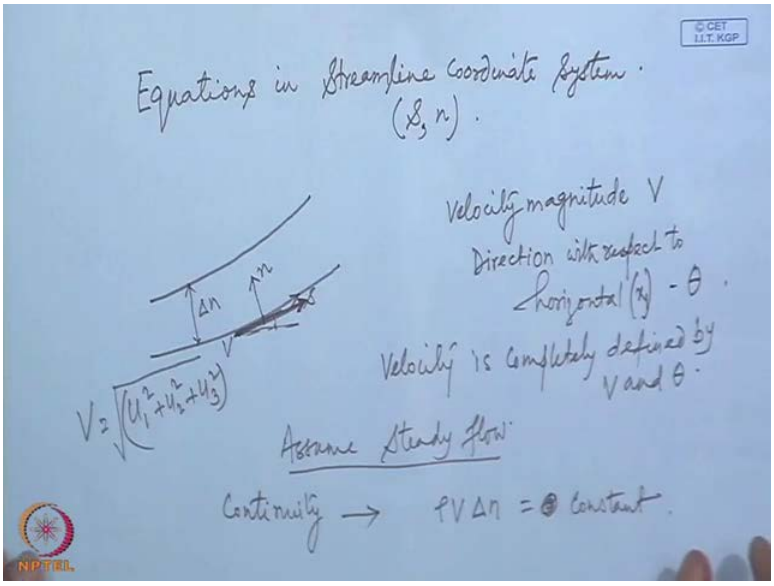

Now, we will see what are the form of the equations in this streamline coordinate system; however, denoting the direction streamline direction as S and normal to the streamline as n. Let us consider that two such streamline. So, this is the direction of S and n is the normal to it, as you know that at any point on the streamline the velocity is…velocity vector is tangential to these. So, at this point let us say this is at direction of the velocity vector, and let us denote the velocity magnitude by V the velocity magnitude by V and its direction with respect to the horizontal is theta; and direction with respect to horizontal let us say that is a x axis is theta.

So, the velocity is expressed by… the velocity is completely defined by V and theta. (No audio 03:02 to 03:38) Now, let us assume first simplicity as study flow. Now, the continuity or mass flow conservation equation can be very easily written using the 1 dimensional analysis, or we have seen that density, into velocity, into the cross sectional area remain constant. Now, we can consider the channel formed by 2 streamlines and within unit depth then, the mass flow rate from 1 dimensional analysis will simply be density into, velocity into the width between the 2 streamlines, and unity that will be the area.

So, the continuity equation simply becomes sorry not 0 constant, mass flow rate is constant.

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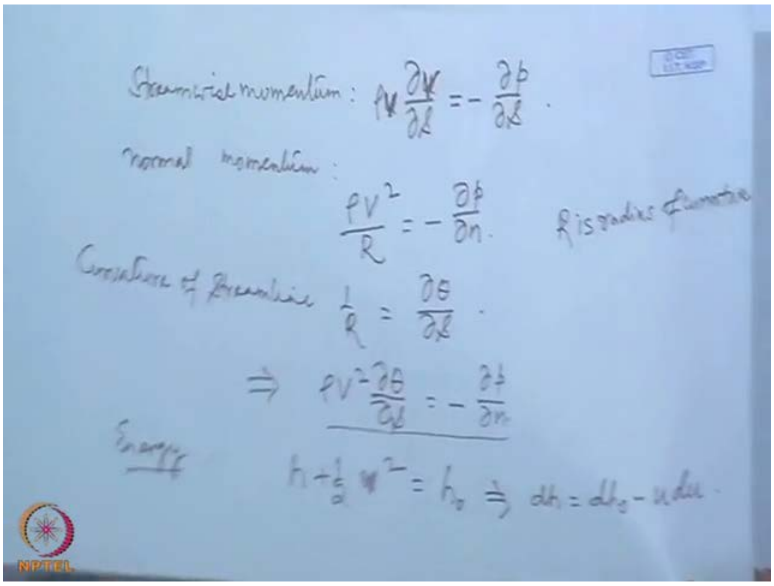

Similarly, the stream wise momentum equation can be simply written as stream wise momentum equation can be written as say rho, u, d, u, d, s; equal to… again we can see that this is identical to the 1 dimensional (( )) equation; however, in this case you will also have a normal component of the momentum equation, or normal component of momentum equation, which gives that pressure gradient in the normal reaction will be balanced by a centripetal force sorry rho we should write rho V rho V square by r equal to minus d p by d n and or r is a radius of curvature, r is radius of curvature and the curvature of streamline 1 by r is d theta d s.

So, substituting this the equation becomes, rho V square d theta d s, energy equation and simply be written as that h plus half V square equal to h 0 which of course, gives d h is equal to d h naught minus u d u in differential form.

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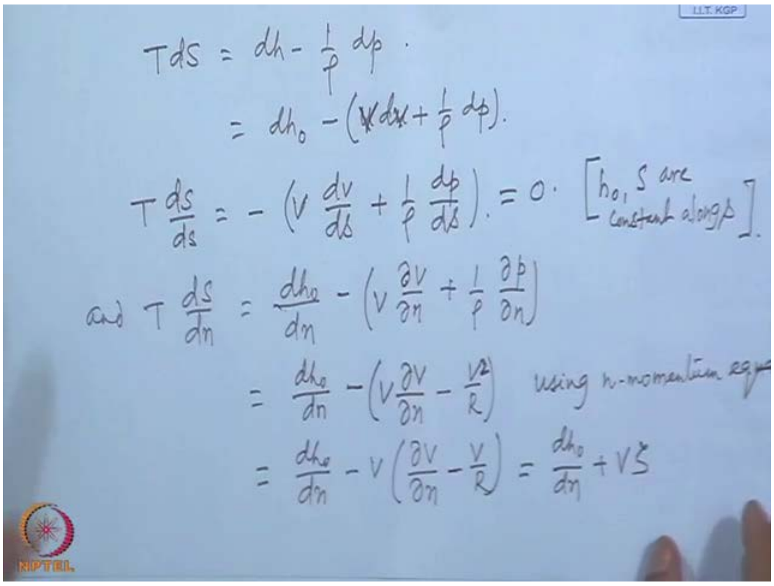

Bringing the definition of entropy bringing in the definition of entropy T d S is d h minus 1 by rho d p and where, d h is d h 0 minus . So, if you know differentiate this along the stream wise direction, what in a sense is that is V V S d S, the total enthalpy remain constant along the streamline direction. So, we have sorry V, d V, d S and this is of course, non 0 as in total enthalpy changes from 1 streamline to other normal reaction and (No audio 10:34 to 11:06) also since, entropy is constant along stream wise direction, this is 0, h 0 and S are constant along S.

Now, using the definition of the radius of curvature, or the normal momentum equation this d p, d n can be replaced which gives (No audio 11:58 to 12:28) using n momentum, n momentum equation n momentum equation. So, this is square. So, plus V zeta; zeta is the velocity which, as we know is defined mathematically by the curve of the velocity vector. So, in the streamline coordinate system this becomes the velocity.

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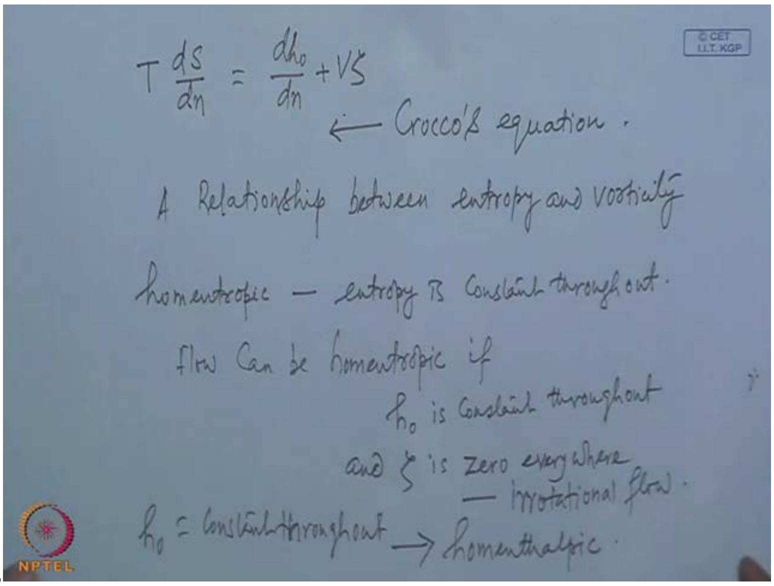
,

So, this relation, this equation which will write once again T, d S, d n is it gives an explicit relationship between entropy and velocity; and known as the Crocco's equation, or Crocco's theorem. So, this is called Crocco's equation in streamline coordinate system. A relationship between entropy, and velocity, which clearly show that these 2 quantities are interconnected in this context, you may recall again that system that we have encountered in 1 dimensional motion in particular shock interactions.

In 1 dimensional flow of course, there is no velocity; however, I have seen that about this slipstream, there is a difference in tangential velocity, or jump in tangential velocity. Tangential velocity is discontinuous about that system and of course, the entropy on the 2 sides are also different.

So, you have seen there that the differential entropy created jump in tangential velocity, or discontinuity in tangential velocity, and we also know that velocity are seat vertex also creates a difference in tangential velocity and. So, there is we notice that similarity between that sleep stream and vertex (( )). And coming to this multidimensional problem we now have a explicit relationship between entropy and velocity. A flow is called hometropic if. So, when entropy is constant everywhere, homentropic entropy is constant throughout while as we have mentioned earlier isentropic is entropy, is constant along a streamline.

So, homentropic imply that in all streamlines, the entropy is constant, and clearly that will happen, if entropy does not change in normal direction as well and that can only happen, if 0 is constant; and zeta is 0 . So, flow can be homentropic throughout and the velocity zeta is 0 everywhere, and we know this is what is called irrotational similarly, where h 0 is constant throughout that is called homenthalpic, throughout it is called homenthalpic.

So, the flow will be homentropic, if the flow is homenthalpic as well as irrotational and in viscid non conducting flow with steady pressure field will be homentropic, if it is homenthalpic and irrotational.

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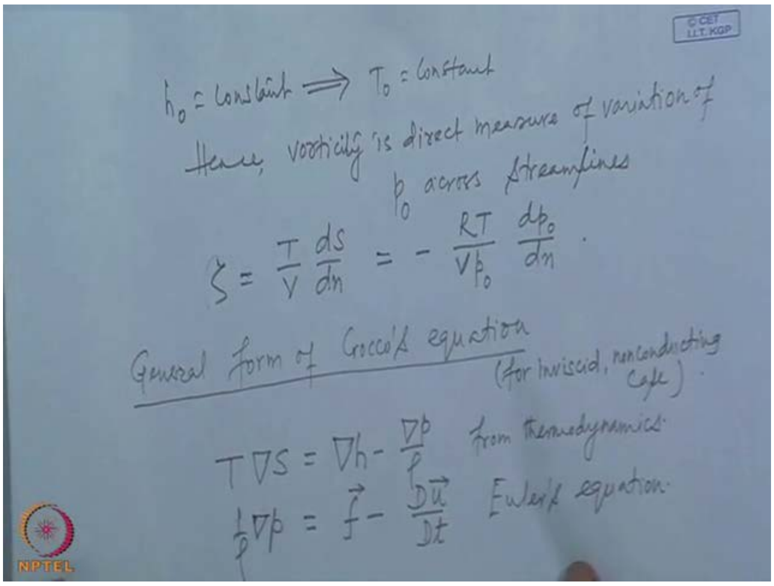

Now, h 0 equal to constant that implies T 0 is constant. So, in this case velocity is direct measure of variation of p 0 across streamlines, that is from this equation you can see that when h 0 is constant; the change in entropy occurs due to velocity, and as we have seen earlier that entropy changes may be caused by change in total temperature and total pressure, and when h 0 is constant total temperature is also constant. So, the change in entropy in that case will be only due to change in total pressure. And hence, this can be written as that when h 0 is constant zeta is T by V d S d n and that becomes…

So, variation of stagnation pressure is directly related to velocity. Now we will try to see the form of this crocco's equation in cartesian system for in viscid non conducting case. Using the second law combine first; and second law, rather T grad S into grad h minus grad p by rho from thermodynamics. Now from Euler's equation we have from euler's equation we have delta p or 1 by rho grad p 1 by rho grad pal to which is the euler's equation.

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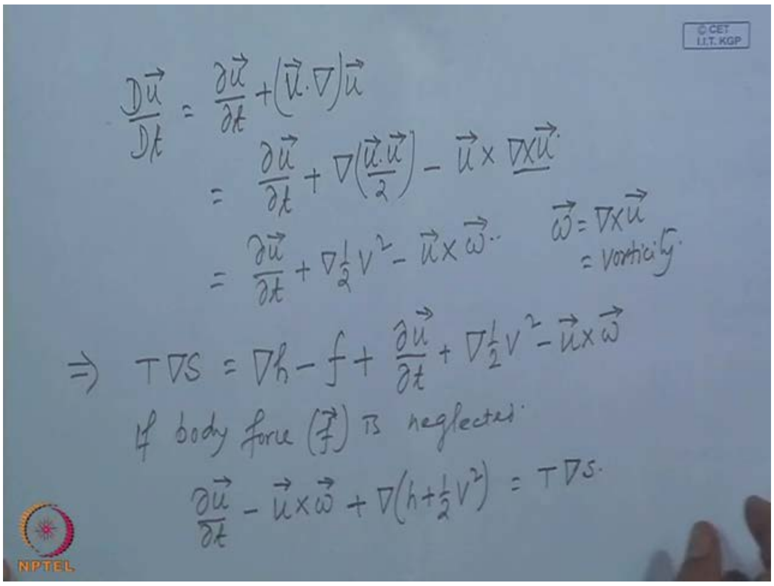

Now, the acceleration of the fluid particle that can be written as the convective velocity, and which using vector identity becomes… (No audio 25:47 to 26:34) this is the quantity denoted as the velocity. Then substituting these we get T grad S you will grad h minus f plus . So, if you neglect body force these we combined this.

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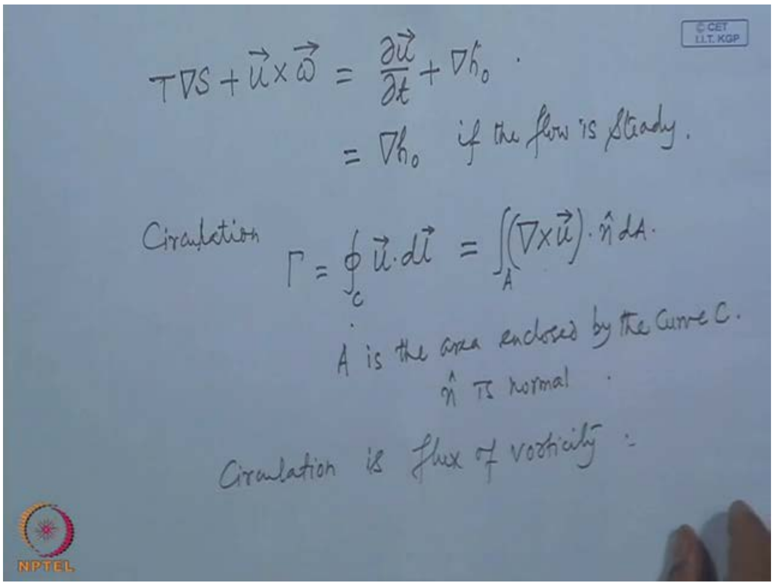

Now, can be written as that T grad S plus here. So, we get and identical relation for the general equation.

Now, we know that another very important fluid dynamical quantity is circulation which is also related to velocity. So, very important quantity in fluid dynamics or aerodynamics is circulation and as we know that which, is directly related to the lift produced by an fall or wing. So, circulation which is defined as integration over a closed curve. And this by application of stokes theorem which changes are control integral to a area integral is a 1 by a is the area enclosed by the curve c and n is normal . So, see the circulation is simply a flux of velocity, and hence the link between circulation and entropy is quite evident.

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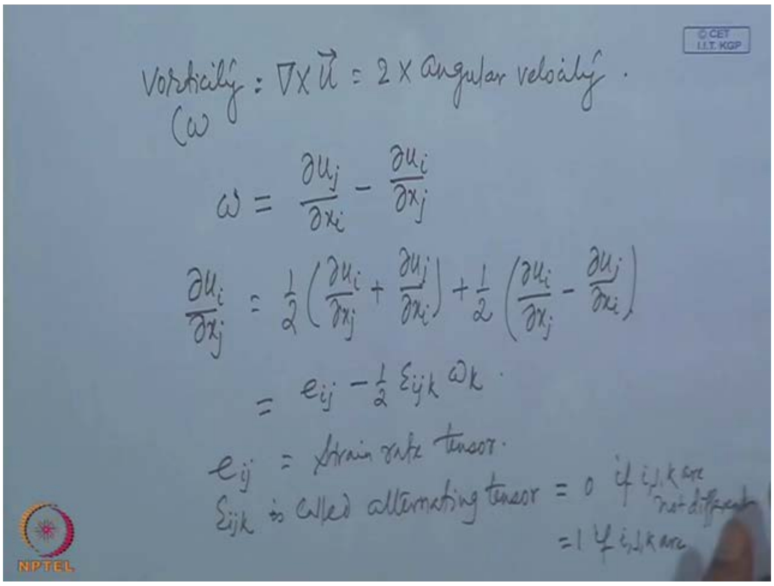

Velocity is also defined as 2 times the angular velocity, 2 into angular velocity of the fluid particle, and omega is defined as d u j, d x i minus. Now, these velocity gradient are essentially tensor. So, it will appear that velocity is also a tensor; however, this is a purely anti symmetric tensor having only non-zero diagonal elements; all other elements are 0. Consequently this is velocity is not exactly a tensor rather it is a pseudo tensor or pseudo vector and most often we can write the velocity gradient at a point d u i, d x j as half of d u I, d x j plus d u j d x i.

Where this matrix is written as sum of 2 matrices; 1 is symmetric; the other is anti symmetric. This symmetric matrix or the symmetric tensor is known as the strain rate tensor epsilon i j k is called the alternating tensor; It is 0, if all 3 are not different. 0, if i k j are not different, that is, if 2 of these indices are same; then this becomes 0. And it is 1, if i j k are cyclic that is when i j k are all different and they are cyclic, this epsilon i j k takes the value 1 and it takes the value minus 1, if not cyclic that is either they may be one, 2, 3 ; 2, 3 one, or 3 , one, 2. Assuming that i j k can take only the value of one, 2, 3 . That is we are considering only 3 dimensional space then, i j k can take the value one, 2, 3 and it will be 0, if any 2 of them are same when all 3 are different, and they are arranged in a cyclic order that is either one, 2, 3 ; 2, 3 , one, or 3 1 2 it takes the value 1, but if it is not cyclic then it is minus 1.

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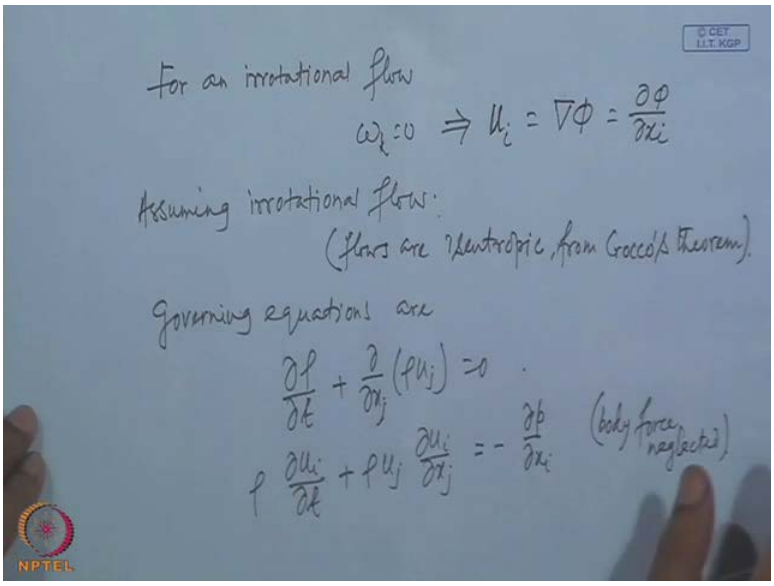

Now, when the velocity is 0 everywhere, we have all the flow to be irrotational and under such condition, the velocity vector can be defined simply as a gradient of a scalar potential or gradient of a scalar function. So, for an irrotational flow for an irrotational flow 0, and this implies u i is gradient of a scalar potential for an irrotational flow, the velocity field can be expressed simply as a gradient of a scalar potential.

Now, if we consider an irrotational flow, and already we have seen from crocco's theorem that when the flow is irrotational there, also a isentropic. The governing equations now become the continuity of course, does not change. The momentum equation is body force neglected which is of course, quite customary in aerodynamics that body in most of the only body force is gravitational, and the effect of gravity on the flow, aero dynamical flow is not considerable because, what we are interested is mostly in the difference between pressure and other properties, and over a small region gravity does not play a major role.

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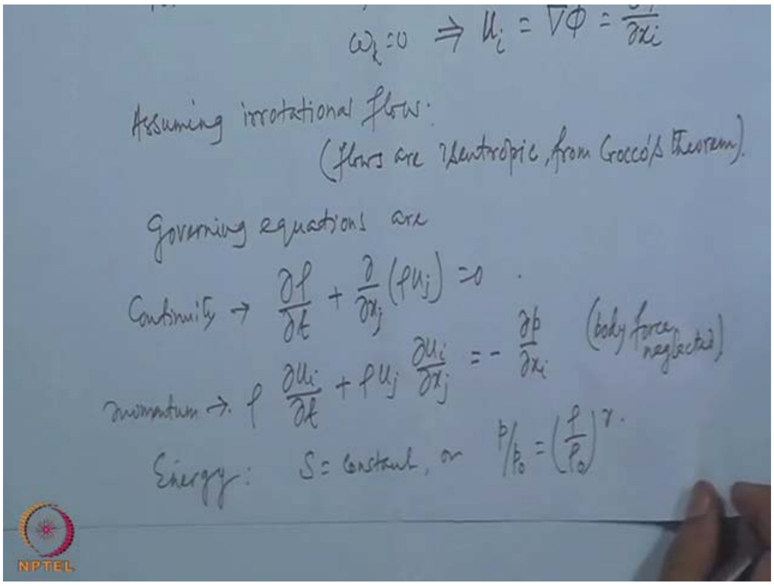

So, we can mark this is the continuity momentum. The energy equation can take a very simple form which we can write as or .

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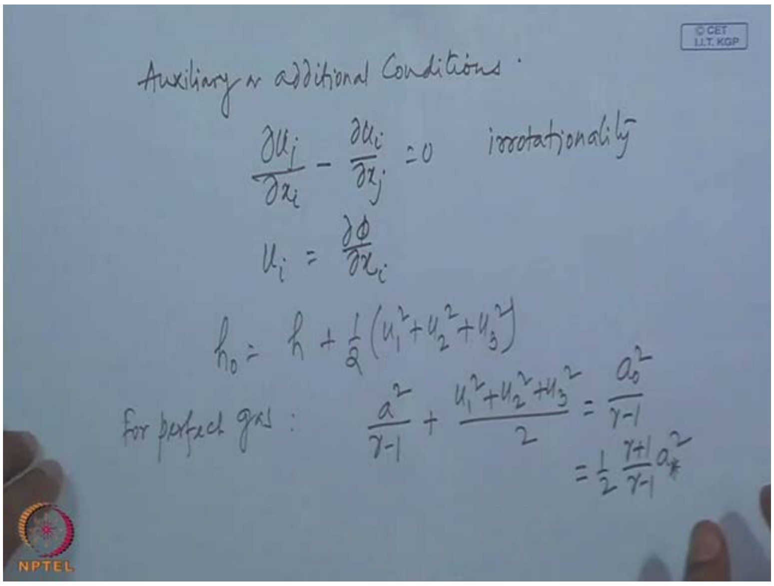

In addition we also have some auxiliary conditions that is the velocity, some auxiliary conditions, irrotationality d u j d x i minus also, we have this of course, a great simplicity because, in that equation instead of 3 velocity component we now have only a single unknown it is scalar potential phi. And the equation can often be expressed in terms of this scalar potential phi, also we can use the relation h 0 equal to h plus for perfect gas; this can be written as this can be written as… this is when we use the sonic condition as the reference look at this.

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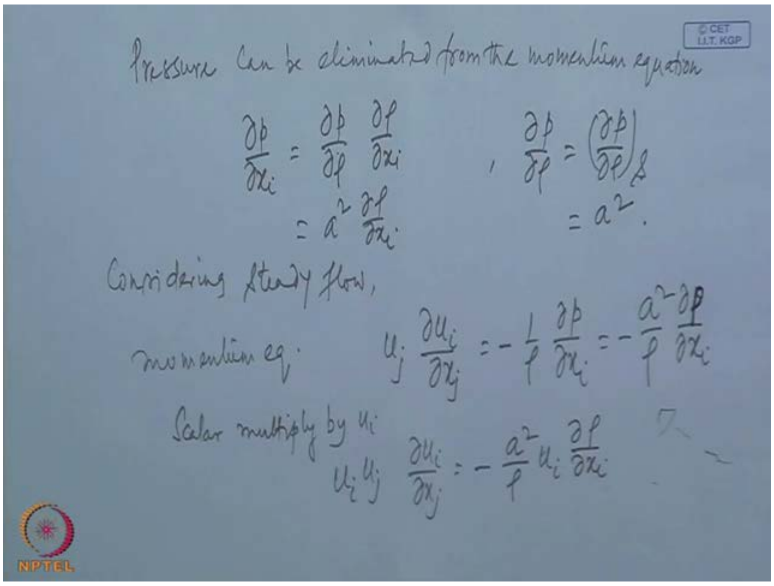

Now, we see the pressure is present in the momentum equation, from the momentum equations using the simple relation that d p d x i is d p d rho into d rho d x i and since, this flow is isentropic since this flow is isentropic. So, d p d rho is d p d rho S, in this case the flow is isentropic. So, this derivative is isentropic derivative which is equal to the speed of sound. So, this equation becomes. So, considering steady flow the momentum equation is the momentum equation can be written as u j d u i d x j equal to minus 1 by rho d p d x i which is sorry and a square d rho d x i

Now, scalar multiply this with u i which gives this relation u i u j d u i d x j minus a square by rho u i d rho d x i.

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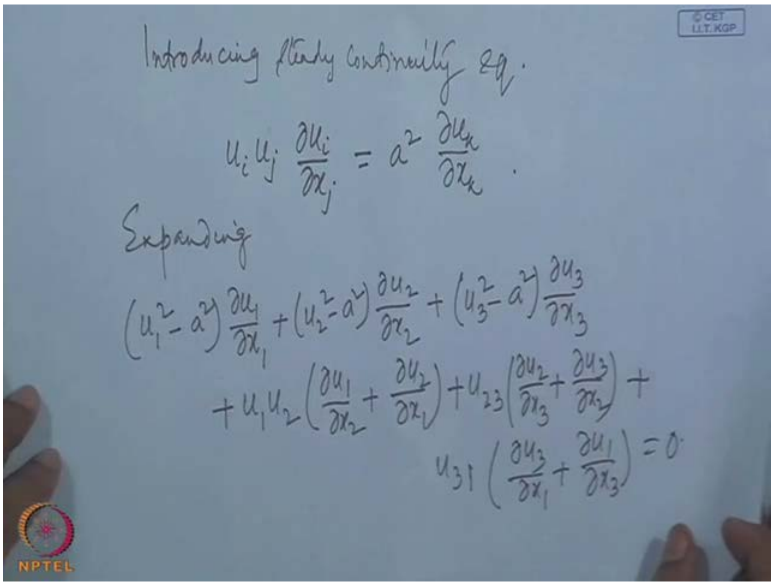

Now, from steady flow continuity equation that, if you use steady flow continuity equation steady continuity equation, what we get is, then u i u j d u i d x j a square d u k d x k and on expanding we have u 1 square minus a square (No audio 51:48 to 52:31) plus 1 2 2 3 . So, this is the equation that we obtain combining all these equations. So, see that for an isentropic irrotational flow for an irrotational flow, we can combine all the equations to obtain 1 single equation.

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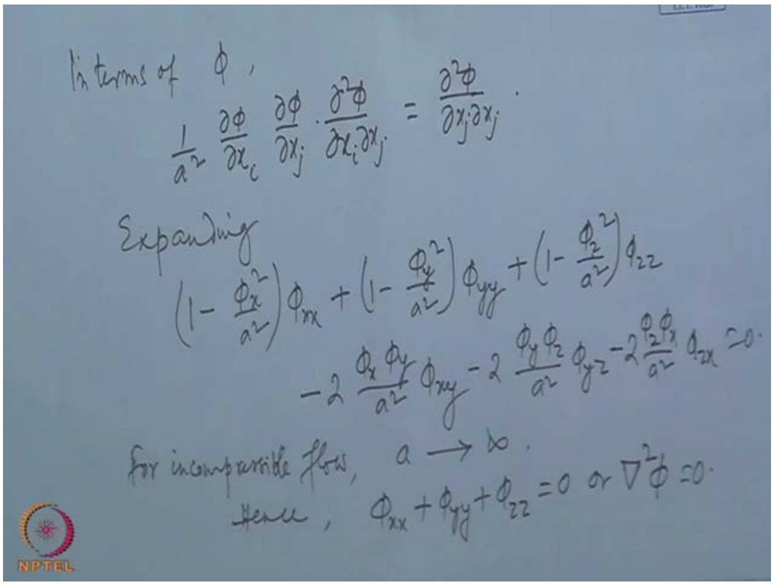

If, you introduce velocity potential, the equation can be written as in terms of phi, the equation becomes 1 by a square d phi d x i d phi d x j into phi d x i d x j. The right hand side term is simply the laplacian of phi, and this we can expan, we get 1 minus phi x square by a square into phi x x plus 1 minus phi y square by a square into phi y, y minus. (No audio 55:12 to 56:02)

So, this is the potential equation for irrotational compressible flow, and we can see that for incompressible flow this becomes… we know for incompressible flow the speed of sound approaches infinity and consequently this equation becomes, this term vanishes, this vanishes, and this vanishes. And all these terms vanishes leaving only phi x x plus phi y y plus phi z z equal to 0, or laplacian of phi equal to 0, which is quite well known and we have used extensively earlier. So, what we obtain in terms is a general equation for the potential flow problem.

So, to summaries we have derived crocco's equation, and seen what is the relationship between entropy and velocity? And we have seen under what conditions the flow will then become isentropic or homentropic, and consequently we have simplified the equation for irrotational flow. Where the flow velocity can be expressed in terms of a scalar potential, and we have derived the appropriate potential equation for high speed compressible flow, and also see that this takes the form of simple laplacian equal to 0 the form for incompressible flow.