High Speed Aerodynamics

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Lecture No. # 28

Linearized Flow Problems (Contd.)

From our solution of supersonic flow past a wave, we have extended that solution to flow over an air foil and we have seen that pressure coefficient can very easily be obtained from the local surface inclination. So, supersonic linearized problem can be solved quite easily and this can also be extended to a three dimensional wing, where in case of a three dimensional wing.

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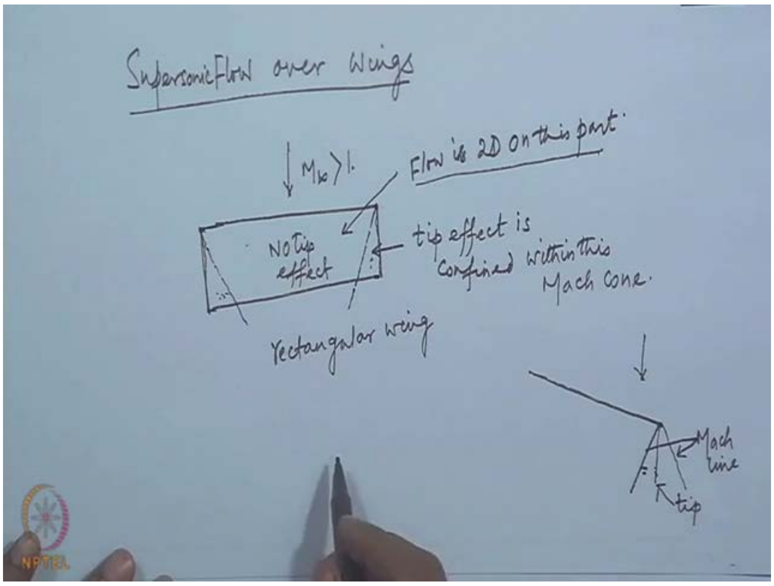

See, if we have a rectangular wing as you know from our (No audio from 0:55 to 1:13) supersonic flow over wings (No audio from 1:15 to 1:24). We can see that for supersonic flow over a wing, if considering a straight rectangular wing a rectangular wing, we know from our study on incompressible flow that, because of that tip effect, the lift reduces (Refer Slide Time: 1:24); however, in case of a supersonic flow, we know that there is a limited upstream influence, hence if we have the mach line from the tip and hence in a three dimensional sense, it is a mach cone, that the tip effect will not be felt outside this mach cone.

So, this part of the wing (No audio from 02:22 to 02:42), so tip effect is here only(No audio from 02:46 to 03:01) within this mach cone (No audio from 03:03 to 03:14) as a consequence see this, over this part, the flow is two dimensional over this part (No audio from 03:24 to 03:41) (Refer Slide Time: 02:21). And the pressure coefficient at any point and hence the difference between low surface and upper surface pressure which is usually called the loading, on this part of the wing will remain as in case of a two dimensional flow.

So, the loss in lift is only due to this part of the wing and (No audio from 04:05 to 04:19) this part is purely two dimensional. For a soft wing, the situation may be a little different and there might be two particular cases where both the mach lines from this tip leading edge are lying on the wing or one of them is lying on the wing, the other outside the wing as in this case (No audio 04:54 to 05:58) (Refer Slide Time: 04:00).

So, in this case the situation is almost similar to this and only the effect, tip effect is felt here and the loss loss of lift is only because of this part; however, another situation may arise with a supersonic wing (No audio 06:27 to 07:12) (Refer Slide Time: 06:00).

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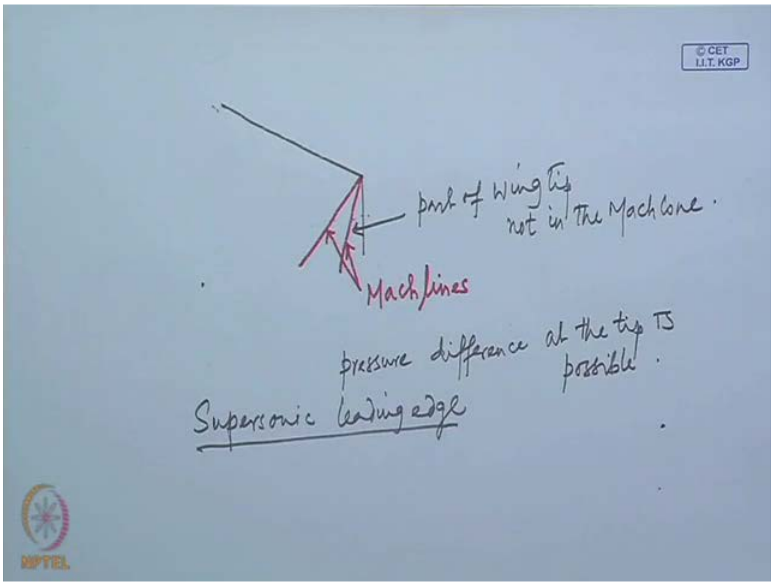

In this type of situation where both the mach lines are over the wing and there is a part of wing which is outside this mach cone (No audio from 07:25 to 07:32), part of wing tip, not in the mach cone (No audio from 07:42 to 07:50) in this type of situation the lower part and lower surface and upper surface of the wing will not felt the effect of the other and it is possible to have a pressure difference at the tip.

So, pressure difference at the tip is possible in this case (No audio from 08:17 to 08:33). So, in this type of wing, there will be decrease in pressure over this part. However, that will again be gained by this part and if the flow configuration is like this, then it is called a supersonic leading edge (No audio from 08:54 to 09:07).

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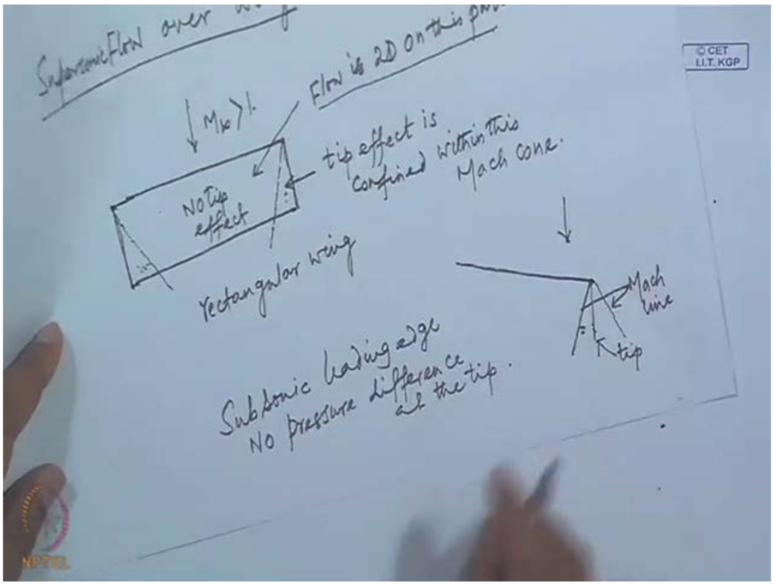

However, in the earlier two cases that we discussed that is for the rectangular wing and a soft wing in this configuration, these are called subsonic leading edge (No audio from 09:18 to 09:26) and in this case, no pressure difference at the tip (No audio from 09:32 to 09:48). So, in case of a supersonic leading edge, tip loading is possible, however we will not go for any quantitative discussion of these at this stage.

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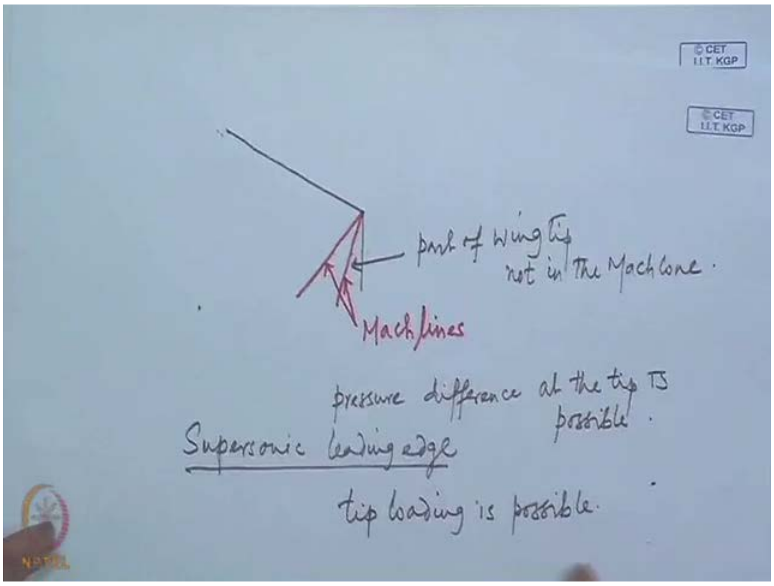

We will now consider some other application of linearized flow problems, see this linearized flow problems are, can be categorised mainly a two dimensional flow which we have discussed earlier, a planar flow, of course a two dimensional cases are special case of this planar flow problem. And a very important application of this linearized flow theory or small perturbation theory is a cylinder body, or elongated body, or body is of (()) body is of the revolution (No audio from 11:05 to 11:14).

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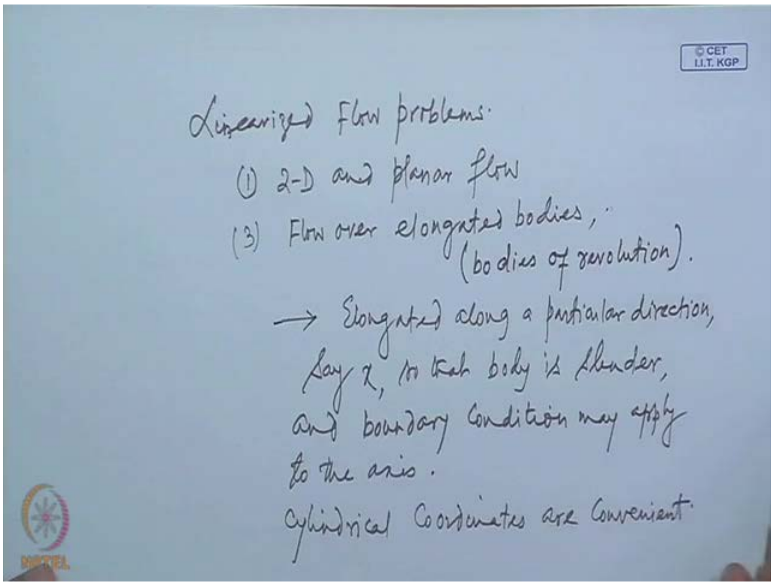

So, linearized flow problems are broadly of 2 D and planar flow and (No audio from 11:32 to 11:55) usually these are bodies of revolution. Now, in this case consistent with small perturbation theory, the basic assumptions lies that the slope of the surface at everywhere is small and in particular flow over elongated body that is (No audio from 12:29 to 12:35) elongated along a particular direction (No audio from 12:44 to 12:52) say x, so that body is highly slender.

That is, in one direction it is, its length is so large compared to its length in the other direction, slender and boundary condition may apply to the axis, as in case of a two dimensional and a planar flow, we have satisfied the boundary condition on say y equal to 0, that is normal velocity on the surface is taken as the normal velocity at y equal to 0.

So, for elongated bodies also are similar type of approximation can be used. Now, for elongated bodies, cylindrical coordinates are more convenient and we will try to derive this potential equation in cylindrical coordinate.

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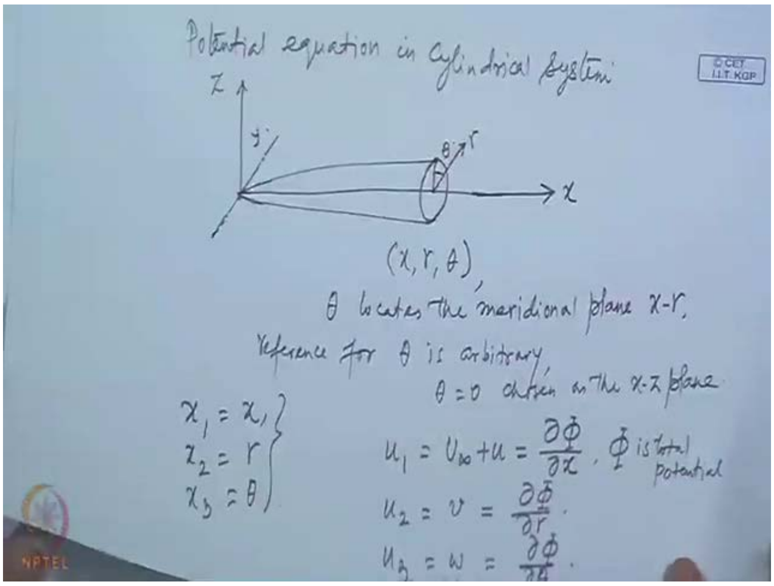

Now, let us (No audio 14:49 to 15:20) potential equation potential equation in cylindrical system (No audio 15:37 to 16:19), the cylindrical coordinates are as convenience x, r, theta and theta locates the meridian plane x r and let us say we have, we choose theta equal to 0 which represents x z plane (Refer Slide Time: 14:49).

So, of course, the choice is arbitrary, theta locates the meridional plane, meridional plane x-r, reference for theta is arbitrary (No audio from 17:18 to 17:35). However, as we mentioned this is arbitrary, we can choose any. So, in our this coordinate system, we have our x 1 equal to x, x 2 equal to r, and x 3 is theta, and the velocity components u 1, as before we will consider as u infinity plus u that equal to d phi d a x, phi in this case is total potential (No audio from 18:21 to 18:34), u 2 will choose as (Refer Slide Time: 18:35) and similarly, u 3 can be chosen as w which is d phi d theta. So, the velocity components w or u 3 is along the theta direction, so basically this is a tangential or circumferential velocity.

Now, in the derivation of the potential equation, we have seen earlier that we started with the continuity equation and in which we replaced density using momentum and energy equation, introducing speed of sound. And then speed of sound is further removed by, replaced by velocity components using the energy equation, (()) speed of sound and velocity components using the energy equation. And then a complete equation was obtained in terms of speed of sound at infinity and the velocity component, or in terms of free stream Mach number and velocity components which was then replaced by the perturbation potential. And then further, we assumed a small perturbation approximation to simplify the equations which resulted in the that is cartesian equation.

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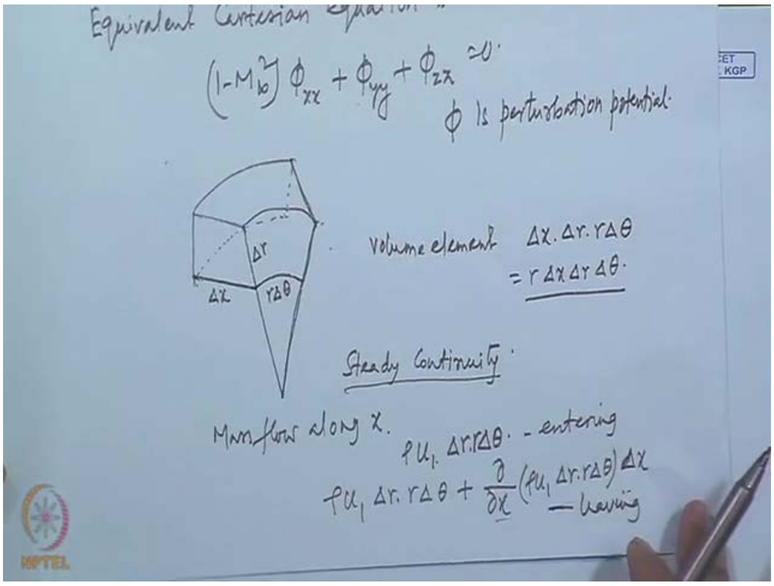

So, equivalent Cartesian equations, if you remember equivalent Cartesian equation (No audio from 20:54 to 21:12) where phi x x, phi y y, phi z z are second derivative of phi with respect to x y of z in the Cartesian system and phi is perturbation potential (Refer Slide Time: 20:54). So, we will try to obtain an equation corresponding to this in the cylindrical system. So, first of all, let us consider the continuity equation in the Cartesian system sorry in the cylindrical system, let us consider a small unit volume, a small volume element (No audio 22:10 to 23:02), so for this small volume element (No audio from 23:05 to 23:29), delta x into delta r into r delta theta that equal to r delta x delta r delta theta that is the volume of this infinitesimal element.

Now, for the continuity equation in steady flow, we know that sum total of mass fluxes entering minus the sum total of mass fluxes leaving equals to 0, so for steady flow continuity equation (No audio from 23:12 to 24:33). Now, let us say consider flow velocity along x or mass flow along x, so the mass flow along x (No audio from 24:45 to 25:01), let us say mass is coming from this side and leaving from this side. So, through this face mass flow entering is rho u 1 into delta r into delta r delta theta rho u 1 into delta r into r delta theta and mass flow that is leaving is through this face, again rho u 1 delta r into r delta theta plus its rate of change along x d d x of rho u 1 delta r into r delta theta into delta x, this is what is the mass flow leaving. So, subtracting this from this, this is what is the net mass flow rate along x and similarly, we can find the net mass flow rate along the two other directions (Refer Slide Time: 26:39).

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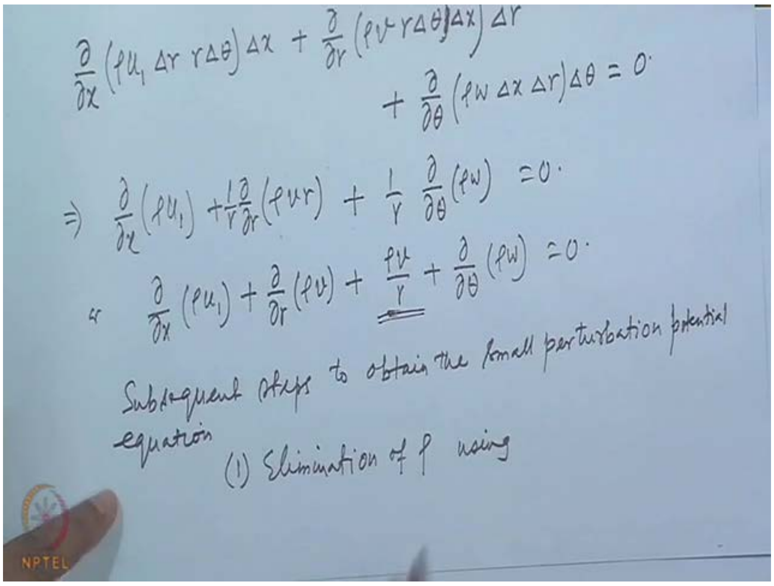

And consequently, this becomes d d x of rho u 1 delta r r delta theta into delta x plus d d r of what we have considered v, rho v r delta theta sorry r delta theta delta x into delta r plus d d theta of rho w delta x delta r into delta theta that equal to 0 (Refer Slide Time: 26:58). So, net accumulation within the control volume is 0 and on an unit volume basis, for an unit volume this becomes d d x of rho u 1 plus 1 by r d d r of rho v r plus 1 by r d d theta of rho w equal to 0.

So, this is the continuity equation in the cylindrical coordinate system and we can see here this becomes d d x of rho u 1 plus (No audio 29:14 to 29:44) (Refer Slide Time: 29:06). So, if we compare this with the Cartesian system continuity equation, we see an additional term which is present in this case and of course, this will give additional term in the small perturbation potential equation also, the subsequent steps are (No audio from 30:16 to 30:29) to obtain (No audio from 30:31 to 30:59).

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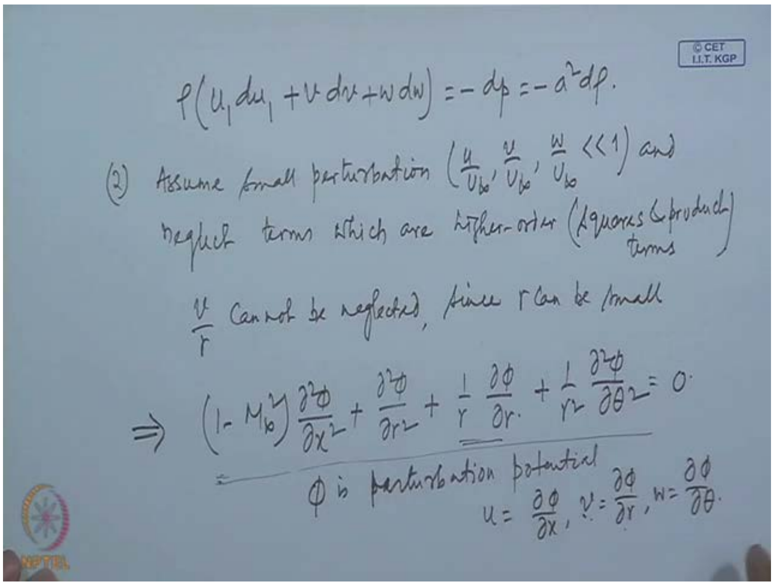

The small perturbation potential equation is first, elimination of rho rho using u 1 d 1 plus v d v plus w d w equal to minus d p that equal to minus a square d rho, once rho is eliminated (No audio 31:57 to 33:04) which are higher order that is all squares and crossed cross product terms (No audio from 33:10 to 33:27). However, is that v by r cannot be neglected, since r can be small and as a consequence, when all these are applied, the equation that finally we get are (No audio from 34:18 to 34:42), as can we see this is simply v by r plus 1 by r square where phi is now perturbation potential u is d phi d x v is d phi d r and w is d phi d theta (Refer Slide Time: 34:12). You can see that the perturbation, since there was no undisturbed stream in r and theta direction, the perturbation potential, gradient of perturbation potential and gradient of total potential in r and theta direction are the same, giving rise to the perturbation velocities in v and w. So, this is the equation that holds for potential flow over potential flow in x r theta coordinate or cylindrical coordinate system.

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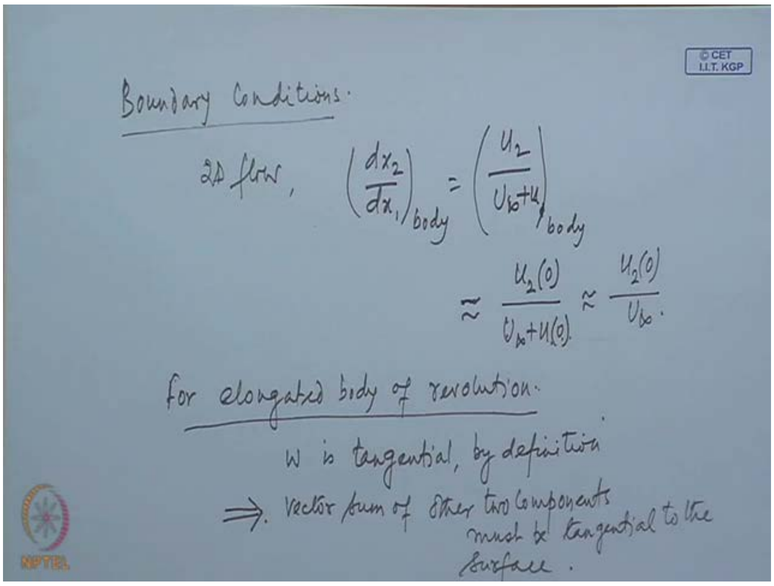

Now, let us come to the boundary conditions (No audio from 36:28 to 36:37), we have seen in two dimensional case for 2 D flow and planar flow, we have seen that d x 2 d x 1 on the body is u 2 by u infinity plus u body, and which was approximated to be u 2 at 0 by u infinity plus u 0 that is, we approximated that the u 2 component of the velocity and the perturbation x 1 component of the perturbation velocity on the body surface can be replaced by their respective values at x p equal to 0, since the body is very thin.

And so, the difference in velocity from y equal to 0 from x 2 equal to 0 to x 2 over the body is negligible and this approximation holds, and this also further we approximated that, this is u 2 at 0 y u infinity. Since, this is much larger than this, we made this approximation. So, this is what we use for two dimensional flow, now for for elongated body of revolution (No audio from 38:46 to 38:55) body of revolution, again the physical boundary condition that the mass flow rate through the body surface must be 0 which again imply that the velocity must be tangential to the body surface, or the normal component of the velocity is 0.

Now, from this physical boundary condition, we can see that w is tangential by definition (No audio from 39:36 to 39:46), so the w components satisfy the boundary condition on its own. So, what is required is that (No audio from 39:55 to 40:16) vector sum of other two components must be tangential (No audio from 40:31 to 40:42) must be tangential to the body surface, now other two component definitely referred to the meridional plane.

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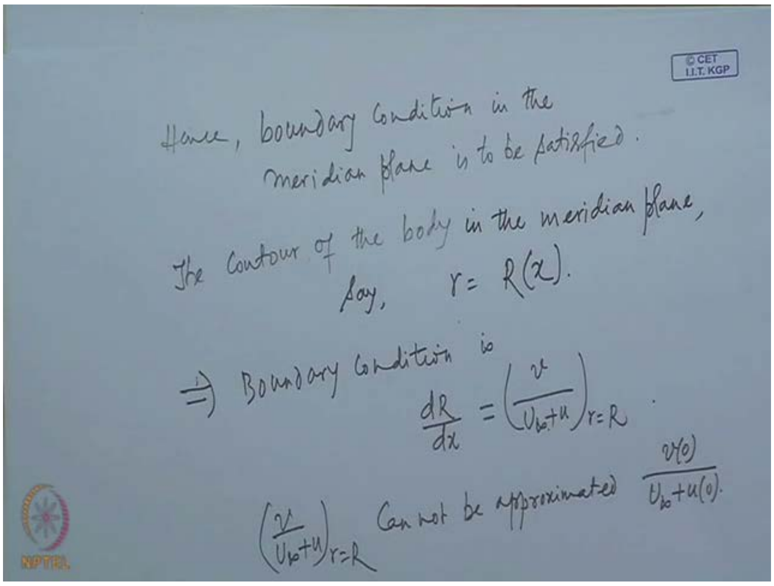

So, what we need is in the meridian plane (No audio from 41:01 to 41:11), hence boundary condition in the meridian plane is (No audio from 41:31 to 41:49). Now, in the meridian plane, let us say the contour of the body in the meridian plane, say r equal to R x (No audio from 42:31 to 42:40). Hence, the boundary condition, (No audio from 42:45 to 42:58) d R d x is v by u infinity plus u evaluated at r plus R. However, in this case, this we cannot cannot be approximated cannot be approximated, we cannot make this substitution here and further simplify.

Now, we will see why this approximation is not possible why this approximation is not possible in a sense, so if in case of a two dimensional flow, we made this approximation that the velocity on the surface of a two dimensional body is same or nearly the same as the velocity it would be on the axis so that we simply transfer the surface to the axis. This is not possible in case of bodies of revolution, or when we are considering the cylindrical coordinate system, the reason is the presence of the term rho v y r in the continuity equation or in the mass flow equation.

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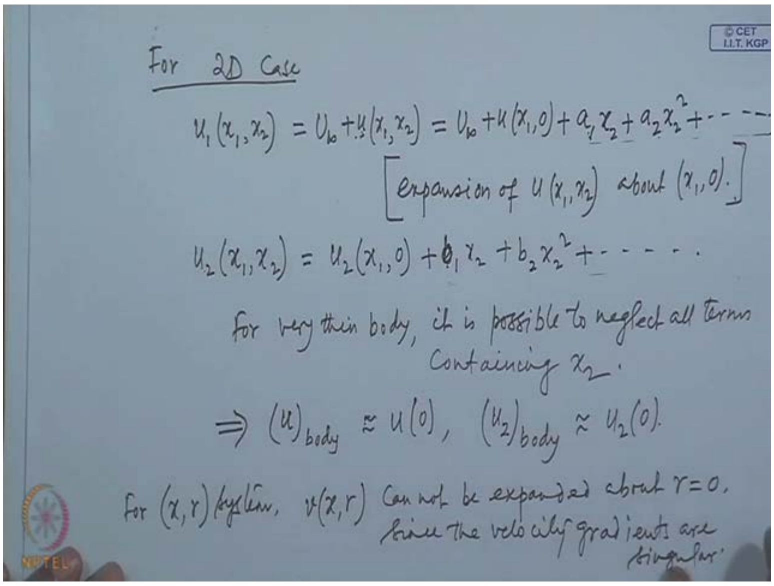

So, you can see that (No audio from 45:19 to 45:30) for two dimensional case, what we have done is that this u 1 is a function of say x 1 and x 2 is simply U infinity plus u x 1 x 2 and if we make a Taylor series expansion of this, this becomes U infinity plus u x 1 0 plus a 1 x 2 plus a 2 x 2 square plus, this is what is expansion of u x 1, x 2 about x 1 0.

Similarly, the other component of velocity (No audio from 46:53 to 47:11), also x 1 0 plus plus b 1 x 2 plus b 2 x 2 square plus 1 and for very thin body, it is possible to neglect all terms containing x 2 that is in this expansion, we can neglect this, all these terms and here also you can neglect all these terms as a consequently, as a consequence this comes as u body is same as u 0, x 1 of course we are not writing here, and u 2 on the body is same as u 2 0 (No audio from 48:53 to 49:10) (Refer Slide Time: 48:10).

Now, when we come to the cylindrical system, let us say for that x r system (No audio from 49:28 to 50:00) cannot be expanded about r equal to 0, since the velocity gradients are singular. Eventually, it should be noted that these coefficients here, these a 1, a 2, b 1, b 2, they represent the gradient of the velocity component or rather this a 1 is related to d u d x, a 2 is related to second derivative of u, similarly b 1 and b 2, they relate to the first and second derivative of u 2.

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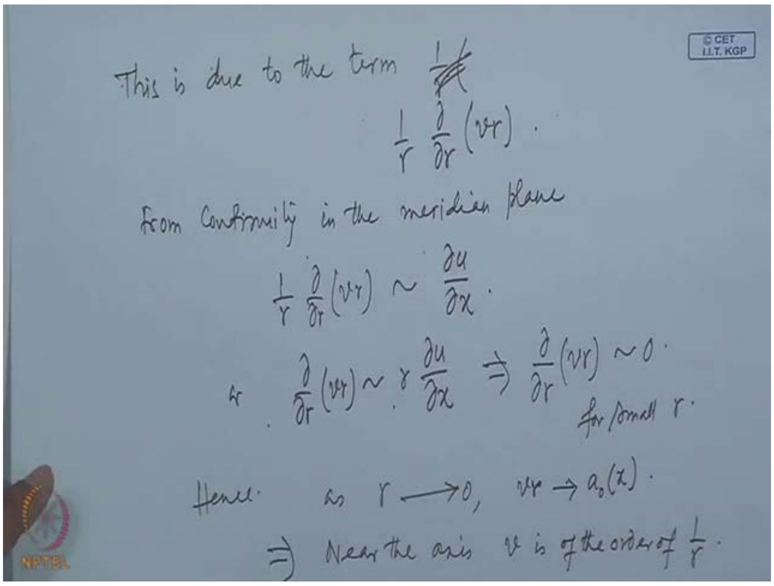

And this velocity gradients are singular, this is due to the presence of the term (No audio from 51:09 to 51:17) 1 by r sorry 1 by r d d r of v r; however since in the continuity equation the sum, these two terms sum to 0. So, they are of the same order, from continuity in the meridian plane plane 1 by r d d r of v r, it is of the same order as d u d x or (No audio from 52:23 to 52:41) and (No audio from 52:43 to 52:53) for small r. Hence as r approaches 0, v r approaches (No audio from 53:12 to 53:38), so what we find that near the axis, v is of the order of 1 by r and since 1 by r is extremely large near the axis, v is very large near the axis and it falls very rapidly.

And hence it is not possible to replace v on r equal to on the body surface to be be same be as on the axis, because in this from the axis, v approaches to a very large value, almost an infinite value while on the body surface it reaches to a finite value, and the fall is very very rapid, extremely rapid.

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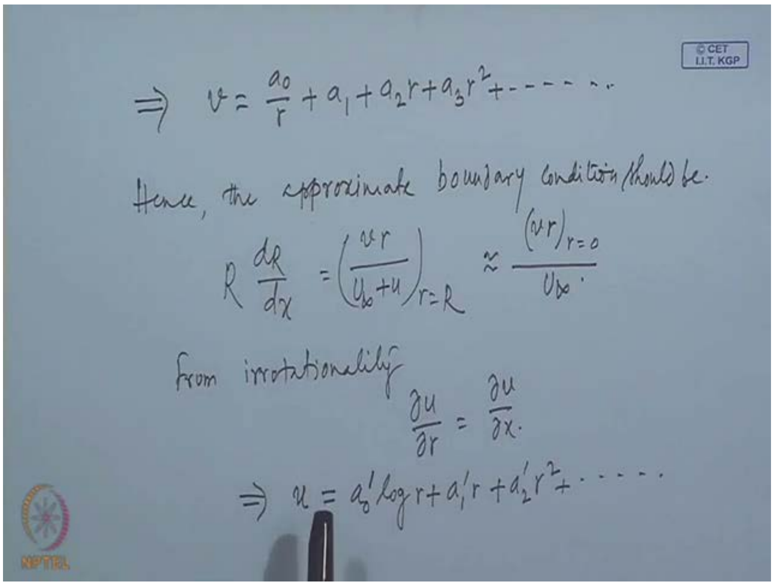

So, from here we can write this v is a 0 by r plus a 1 plus (No audio from 54:48 to 55:01) (Refer Slide Time: 54:40). So, we see now, it is not possible to take v equal to a 1 which is the case, hence this, the approximate boundary condition should be hence the approximate boundary condition should be, we see that v we cannot take as, v on the body we cannot take as v on the axis

However, we can take that for v r and to do that we multiply this by R d R d x that is equal to v r by U infinity plus u, r equal to R and now we can make it (No audio from 56:27 to 56:46) (Refer Slide Time: 56:25). Now, from irrotationality, from irrotationality we have d u d r equal to d v d x and this implies (No audio from 53:12 to 57:36) (Refer Slide Time: 53:12), so on (No audio from 57:38 to 57:49).

So, what we have seen today is that, we have derived the continuity equation in the cylindrical system, and hence the potential equation in x r theta system or the cylindrical system, and also we have seen that y for bodies of revolution, the standard two dimensional approximation of the surface boundary condition or flow tangency boundary condition, cannot be applied. That is, the velocity on the body surface cannot be approximated as the velocity on the axis that is simply because in this case, the radial component of the velocity approaches infinity near the axis, it is extremely large and consequently that approximation cannot, first order approximation cannot be applied in this system.

However, we have obtained what should be the correct approximate form of the surface flow tangency boundary condition, and also what should be the general form of the radial component and the axial component of velocity. We will subsequently move for solving some problem of practical interest in our next class.