High Speed Aerodynamics

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Lecture No. # 29

Linearized flow problems (Contd.)

In the last class, we discussed about flow about slender bodies or bodies of revolution. We derived the governing equation and also found the appropriate approximate form of the surface boundary condition. And we show, why the usual two dimensional approximation, that the normal component of the velocity on the wall cannot be transferred to the axis as in case of a two dimensional flow, which comes to because of the system present in the continuity equation, which results in a extremely large radial component of velocity on the axis. And we show, how that boundary condition can be approximated appropriately for cylindrical system.

Now, in this context we would like to look again to the pressure coefficient. It may be recalled that, while discussing about the small perturbation pressure coefficient. We made certain approximation and neglected the terms based on linearization. And in that, context we mention that, some terms cannot be neglected, when the flow is axisymmetric (( )) first order accuracy. So, we will look back to that again.

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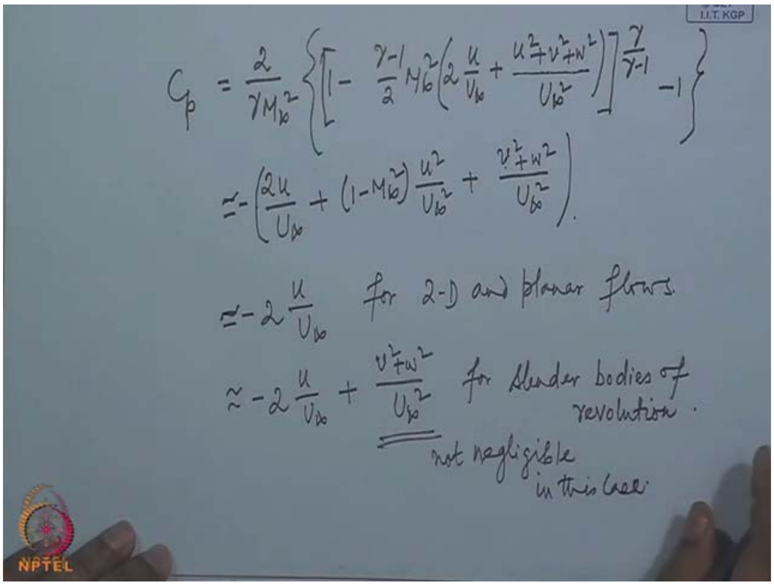

So, for that let us say, we will write the pressure coefficient as before which we had, before all these approximation were applied that, gamma M infinity square 1 minus gamma minus 1 by 2 m infinity square into 2 u by U infinity plus u square plus v square plus w square by U infinity square whole raise to the power gamma by gamma minus 1 minus 1. If this term is expanded in binomial series and higher order terms are neglected then, this comes to be minus 2 u by U infinity plus 1 minus M infinity square u square by U infinity square plus v square plus w square by U infinity square.

Now, for two dimensional planar flows, these terms are second order and can be neglected. So, this becomes minus 2 u by U infinity for 2-D and planar flows, this is these are all approximated. But, for cylindrical system for bodies of revolution as you have seen that, these cannot be neglected. Because, these is quite large very near to the on the body surface, which is very close to the axis, these terms are very large and they cannot be neglected for cylindrical system or for bodies of revolution.

So, however, this of course, can be neglected here also (Refer Slide Time: 05:22). So, this is for slender bodies of revolution (No audio from 05:31 to 05:43), this is not negligible in this case not negligible in this case. So, for bodies slender bodies of revolution this is the additional that has to be kept in mind. Now, we will look into solution of certain problems or the method of solution for this linearized general linearized problem.

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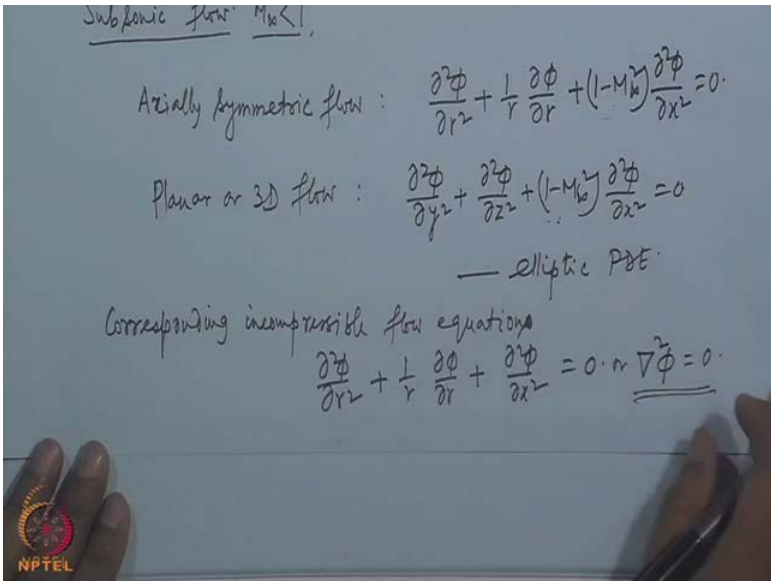

(No audio from 06:45 to 07:03) As we have already seen that, the equations and the nature of the flow changes depending on whether the flow is subsonic or supersonic. In subsonic case, the governing equation is elliptic and in supersonic case, the governing equation is hyperbolic, which needs different application of boundary conditions and also solution methodologies and the nature of flow is also quite different.

So, this method of solutions also we will see in for subsonic and supersonic flow. In the earlier example that is flow over a (( )) shaped wall or the wall is completely defined mathematically, we could find the complete solution by analytical approach as we have seen earlier; and that solution, we could extend to general airfoil and wing problem.

However, the complete solutions for phi and velocity components are possible only, when the body shape can be expressed mathematically otherwise, they cannot be found explicitly. And incase of a subsonic flow we saw that, the solution to (( )) shaped wall are such simple problems cannot straight forward be extended to airfoils or rather bodies.

So, in this case we will now consider the solutions completely. So, first of all, let us think about subsonic flow. The governing equation for axially symmetric flow governing equation for axially symmetrically flow as we have derived is d 2 phi d r square plus 1 by r d phi d r and plus 1 minus M infinity square d 2 phi d x square. If we compare it with the full equation, which contains even the theta derivative term, but since we have considering axially symmetric flow, so there is no variation along theta. And consequently, that theta derivative term is neglected. So, this becomes the equation for axially symmetric flow and for general planar or 3D you can write this equation in (No audio from 10:42 to 11:02) and these equations are elliptic PDE, elliptic Partial Differential Equation in this case.

Now, let us to look to the corresponding incompressible flow equation incompressible potential flow equation, corresponding incompressible incompressible flow equation is for the where M infinity approaches 0. And consequently, these terms these coefficient become 1 and the equations are let us write only the axisymmetric case, which is the well known Laplacian, phi equal to 0 or this is the form of the Laplacian operator in cylindrical system or axial flow in cylindrical system.

And similarly, for the general three dimensional case also the equation is, this becomes 1 and the this become then the form of Laplacian operator in cartesian system.

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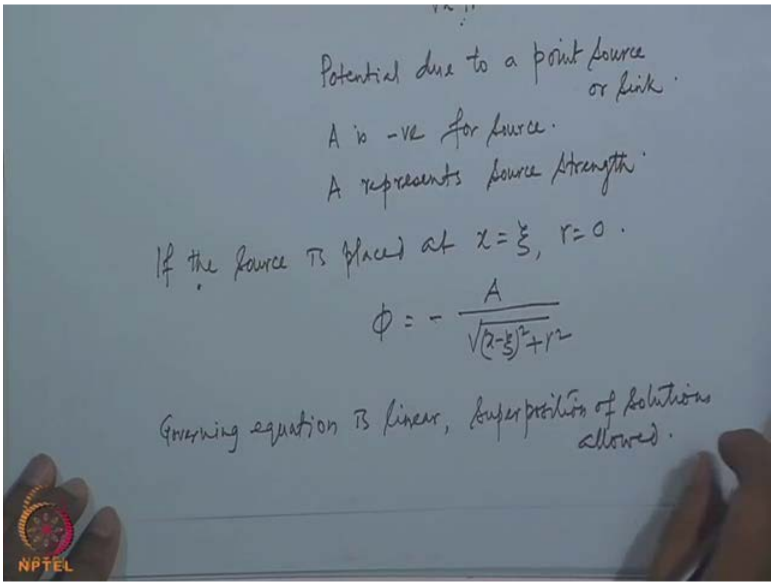

Now, we know that, this equation has a basic solution this equation has a basic solution (No audio from 13:16 to 13:44) and other higher power functions of this this, that is the solution, phi at x r is simply the distance of that point x r from the origin 1 inverse, inverse of the distance from the origin. And this is interpreted as potential due to a point source or sink due to a point source or sink, when A is negative for source, but positive for sink.

And (No audio from 14:54 to 15:15) eventually it may be recalled that, if the strength of the point source is say, M then A is basically M by 4 pi and this represents the distance (Refer Slide Time: 15:30), from the point of origin, where this sink is placed. If the source is at if the source is placed at (No audio from 15:51 to 16:12), now if the source is placed at x equal to Xi and r equal to 0 that is along the axis, but not at the origin little away from the origin, then the phi becomes (No audio from 16:35 to 16:47) and we are putting this negative to explicitly state that, it is a source.

Now, since the governing equation is linear. Any superposition is also a solution. Governing equation is linear, so superposition of solution superposition of solution is allowed and hence, this is also a solution.

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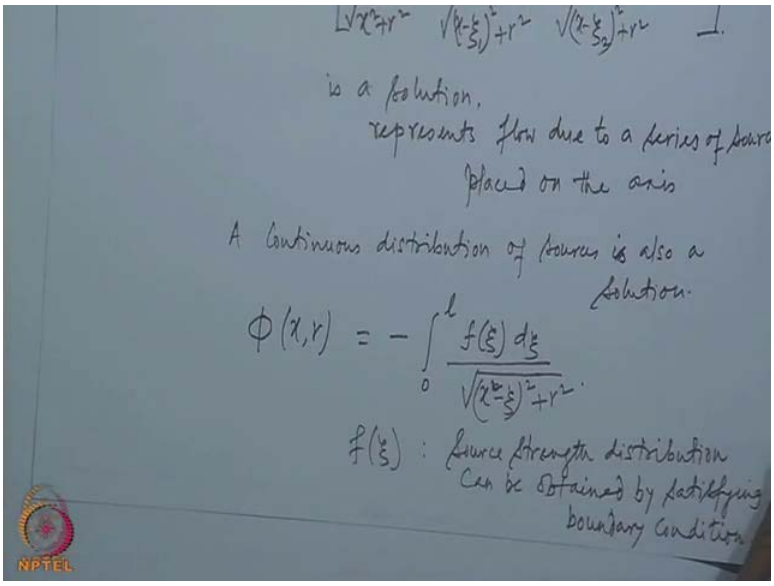

So, a point source placed at origin plus another point source placed at Xi 1 0, another placed at Xi 2 0 and so on, as many as we need. This is also a solution, this is a solution and it represents flow due to a series of sources placed on the axis. Now, instead of having a discrete number of point source, we can also have continuous distribution of point source.

So, a continuous distribution of source is also a solution (No audio from 19:58 to 20:20) and which then becomes (No audio from 20:24 to 20:57); and this f Xi now represents source strength (No audio from 21:02 to 21:28) and can be obtained from boundary condition can be obtained can be obtained by satisfying boundary condition. The wall boundary condition to be precise that is fluid tendency or 0 normal flow boundary condition, if satisfied that will finally give f Xi.

Since, the body can be arbitrary and may not be even possible to express explicitly in mathematical form in that situation, this solution has to be numerical, where a finite number of source and sinks can be used to obtain to satisfy the boundary condition at finite number of points and to obtain the source strength at those points.

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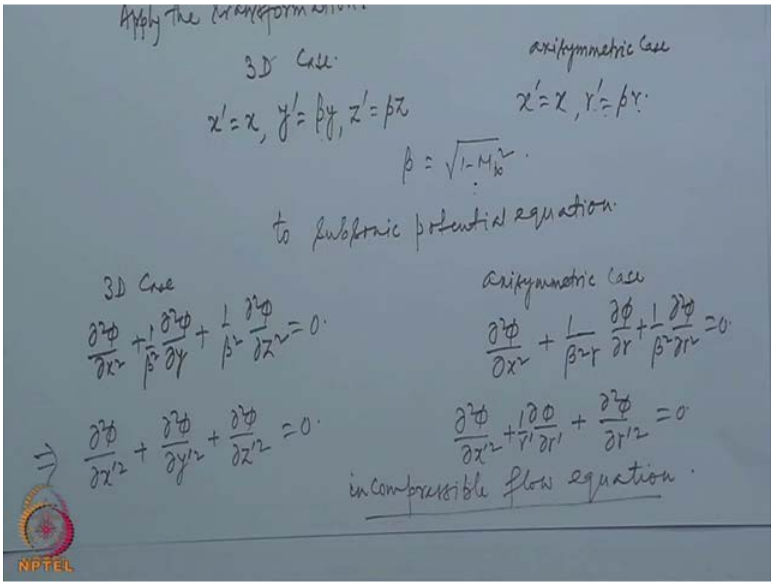

Now, since we know the basic solution of incompressible potential flow. Let see that, now let us use the transformation (No audio from 23:30 to 23:49), so a 3 D case x prime equal to x, y prime equal to beta y, z prime equal to beta z and axisymmetric case x prime equal to x, r prime equal to beta r, beta is as before square root of 1 minus m infinity square.

Now, apply this transformation to subsonic potential equation. For the 3 D case, the equation as we know is d 2 phi d x square plus d 2 phi I am sorry 1 by beta square d 2 phi d y square plus 1 by beta square d 2 phi d z square; and axisymmetric case, we have d 2 phi d x square plus 1 by beta square d 2 phi d x square plus 1 by beta square r d phi d r plus 1 by beta square d 2 phi d r square equal to 0, sorry (Refer Slide Time: 26:12).

Now, if this transformation is allowed here (Refer Slide Time: 26:15), so this now becomes (No audio from 26:20 to 26:41) and in this case also (No audio from 26:44 to

27:18), so we see, this transformation makes this equation for the incompressible flow equations. So, in both the cases the governing equation transforms to equation for the incompressible flow. So, now let us to look to this transformation, what in a sense we have done? In both case, we have seen that, the axial coordinates remain unaltered; however, the radial coordinate in this case and this (( )) and normal coordinate in this case, they are multiplied by a factor beta.

And beta is of course, less than 1. So, what happens due to application of this boundary condition, the body has become thinner. So, this transformation (( )) the body shape as it is however, it is becomes thinner; and also for three dimensional case, the span also become smaller.

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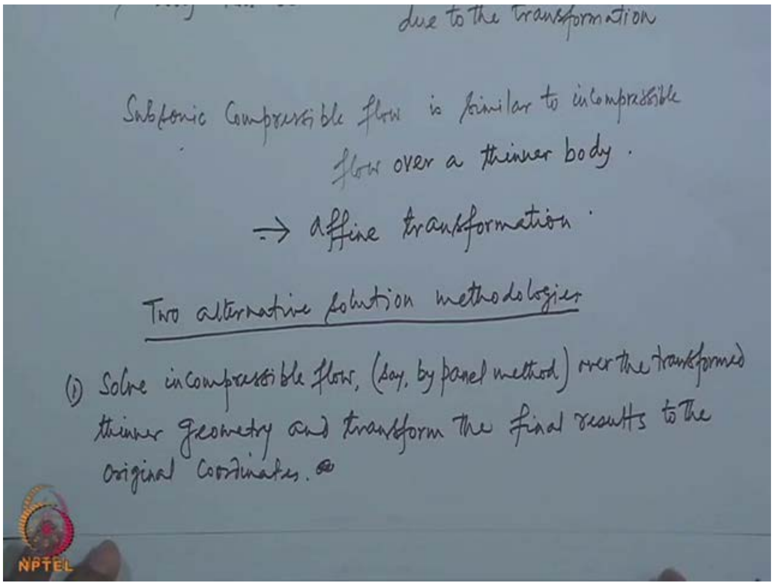

So (No audio from 29:10 to 29:24), the body has become thinner due to the transformation or body. And also we can see from here that the subsonic compressible flow (No audio from 29:56 to 30:08) is similar to incompressible flow over a thinner body. Since, same applies in two dimensional and three dimensional case also, so we can say that, a compressible flow at mach number M infinity over and airfoil is same as the incompressible flow over a thinner body, whose thickness is beta times the thickness of the original body.

And since, due to this transformation the body as becomes finer, this transformation is also known as affine transformation. Now, at this stage we can now follow two approaches of solution, in one compute the solution for incompressible flow over the thinner body and transform the final result to the original coordinate system. Alternately, we can transform the basic solution for incompressible flow to a basic solution for the compressible flow and then, solve the compressible flow equation using that basic solution.

So, this suggests two alternative solution methodologies (No audio from 32:35 to 32:55). The first, solve incompressible flow say we can use panel method, which is discussed in earlier courses on aerodynamics, flow over the transformed thinner geometry and transform the final results to the original coordinates. And that is what, we can do in this case is that, we apply this transformation to the geometry we get the transformed geometry and on this transformed geometry we solve the incompressible flow using say, the most suitable method is of course, is the panel method.

So, the advantage in this case that, if we have a panel method developed for an incompressible flow we can use that, straight away to find compressible flow just applying the transformation to the geometry; and the solution that we obtained from incompressible flow that is the incompressible flow over the thinner body, the solution in terms of the phi the velocity components, pressure coefficients they now can be transformed to the original coordinate. (No audio from 35:37 to 35:55)

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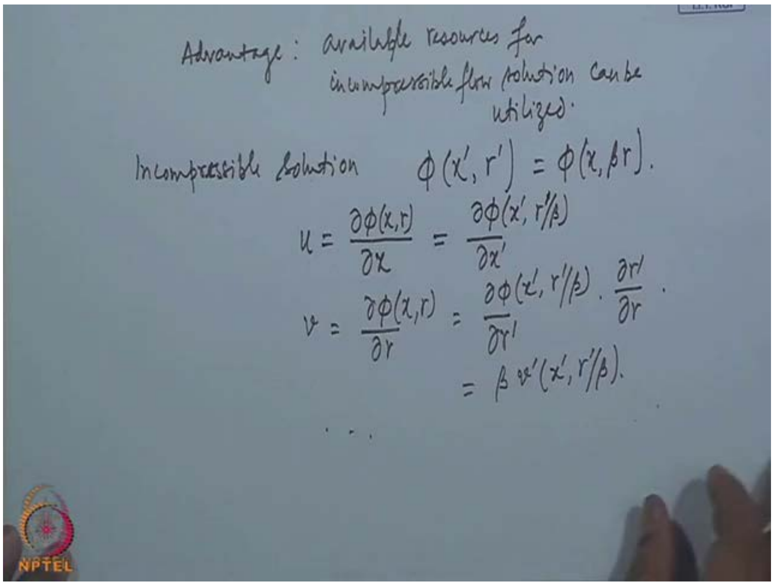

So, the main advantage of this is (No audio from 36:02 to 36:16), available resources for incompressible flow solution can be utilized. Now finally, for the transformation that incompressible solution will give us, which is phi in terms of x prime, r prime that equal to phi x, beta r. And so, that is the way the potential function can be transformed to the original plane, the velocity components the incompressible flow, so this is the compressible (No audio from 37:52 to 40:00) and so on.

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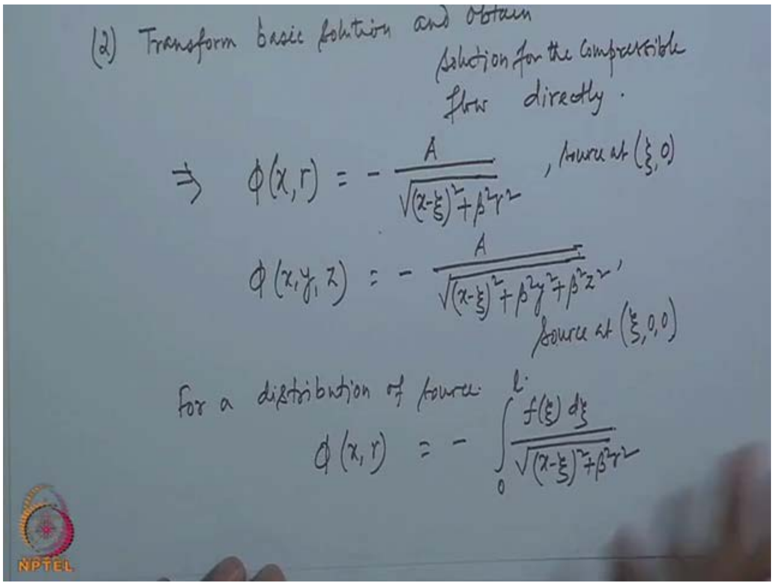

Now, the second alternative the second alternative transform basic solution and obtain solution for the compressible flow directly. This of course, is that, so the most standard method for solving the flow in this approach or using the potential flow approach is the panel method. So, in this case, since we are transforming the basic solution we are having basic solution for compressible flow which of course, will be different from the incompressible flow basic solution; and consequently, the influence coefficients that is the velocity induced at a point due to a singularity placed at another point, that influence coefficient matrix has to be completely reformulated and computed.

So, this needs a complete development of the panel method algorithm. However, the method is very straight forward. So, the basic solution this leads to the basic solution in the original coordinate basic solution in the original coordinate to be say, minus A by (No audio from 42:17 to 42:34). Or in 3 D phi x, y, z (No audio from 42:44 to 43:10) source at Xi 0 and in this case, source at Xi 0 0; however, this need to be changed as depending on the location of the source, where it is placed.

And for a distribution (No audio from 43:34 to 43:46), if you write it for the axially asymmetric case phi x r equal to minus f Xi d Xi by (No audio from 44:02 to 44:22), this of course, now has to be placed at number of discrete points and then, compute the influence coefficient matrix. That is what would be the velocity induced at all the points, where you are pressing the sources due to a source at a particular point and that has to be repeated for all the points giving rise to a n by n matrix, if we have source placed at n number of points and the boundary conditions will be satisfied at n number of points.

So, the final solution or the complete solution is of course, numerical in nature, only in case this f Xi can be obtained mathematically for any given particular geometry then, the solution can be completed analytically. But in general, the f Xi source distribution which of course, depends on the boundary conditions, the source strengths would be such that, the boundary condition on the wall must be satisfied (No audio from 45:43 to 45:55). Now, let us (( )) supersonic flow.

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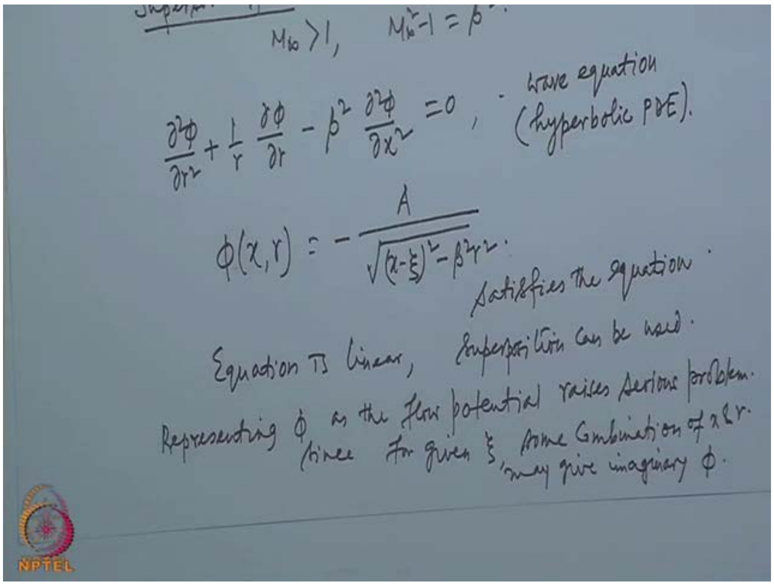

(No audio from 46:03 to 46:18) And we will consider now M infinity square minus 1 equal to beta square, so that the governing equation considering axisymmetric case is d 2 phi d r square plus 1 by r d phi d r minus beta square d 2 phi d x square equal to 0. This happens to be the wave equation in cylindrical coordinate system and which is a hyperbolic partial differential equation. Now, if we consider an analogy with the subsonic flow then, we can first of all check that, phi x r if we modify it to be in this fashion (No audio from 47:39 to 47:52). Now, this of course, satisfies the equation (Refer Slide Time: 47:54).

And again in analogy with the subsonic or incompressible case, we can call this as supersonic point source located at Xi 0, again this equation is linear (Refer Slide Time: 48:24), so here also a superposition is possible. But then we see that, there is some problem in representing this (( )) flow potential that is because that, for some combination of x and r, this can be negative.

So, (No audio from 49:01 to 49:46) representing the function phi phi as the flow potential raises serious problem. Since for given Xi, some combination of x and r some combination of x and r may give imaginary phi, because when Xi is fixed then, that is the source look for a fix source location at some point x r, the potential will be imaginary, this can be negative. And this of course, is not possible since the in the physical problem imaginary potential is not possible.

So, there is some difficulty in a using this as the potential function as we have used in case of subsonic flow, because if we use this type of source distribution, then for some distribution of source at some points in the flow field, the flow solution is imaginary.

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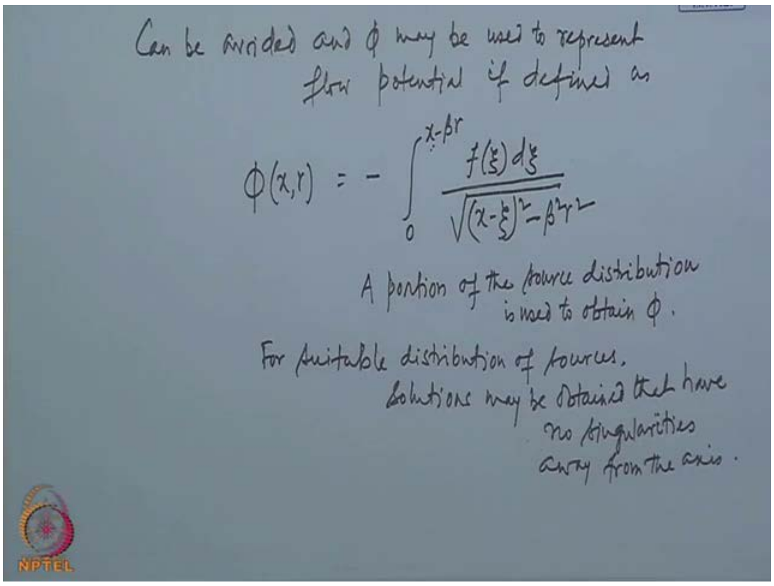

However, this is can be avoided if we can be avoided and phi maybe used to represent flow potential if defined as (No audio from 52:43 to 53:23). See, that the redefinition is in the use of this upper limit, in the subsonic case we considered 0 to l, where l is the length of the axisymmetric body or say, the chord of the airfoil or wing in case of a two or three dimensional geometry.

And the entire source distribution over the axis of the body is used to define the potential at any particular point. However, in case of a supersonic flow you see that, if we want to use this basic solution, then we have to make this modification, that the potential at any point will be defined by the source strength, not over the entire length of the body; but, source distributed over a certain portion of the length. And in this case, first we if we have suitable distribution of sources we may get solution, which is no singularities of the axis.

So, a portion of the source distribution is used to obtain potential phi. And of course, for suitable distribution because this still contains a (( )) that at Xi equal to x minus beta r, this is a single singular integral. So, for suitable distribution of sources, solutions may be obtained that have no singularities away from the axis. The axis is of course, left out.

So, in this case, there will be we may have some suitable distribution, which will which may give no singularities of the axis or in the flow field. However, a portion of the source distribution is to be used. Now, we would we will be seeing subsequently that, what is the meaning of this particular portion? And is it permitted that in case of a subsonic flow, while the entire singularity distribution is used to define the potential, only a part of the singularity distribution is used. Is there any physical justification? Or it is just mathematical plot? Because, if there is no physical justification then of course, it is difficult to define these points.

So, we will subsequently look, what is the meaning of this particular source distribution or meaning of this limit placed on the portion of the source distribution, that can be used to obtain the flow potential. However, that part we will continue in our next class.