High Speed Aerodynamics

Prof. K. P. Sinhamahapatra

Department of Aerospace Engineering

Indian Institute of Technology, Kharagpur

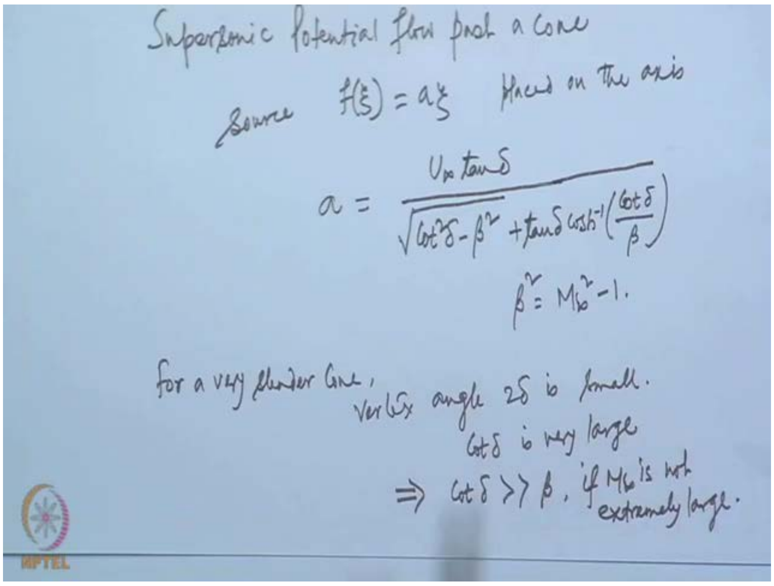
Module No. # 01

Lecture No. # 31

Linearized Flow Problems (Contd.)

In our last class, we have considered supersonic flow past a cone.

(Refer Slide Time: 00:41)

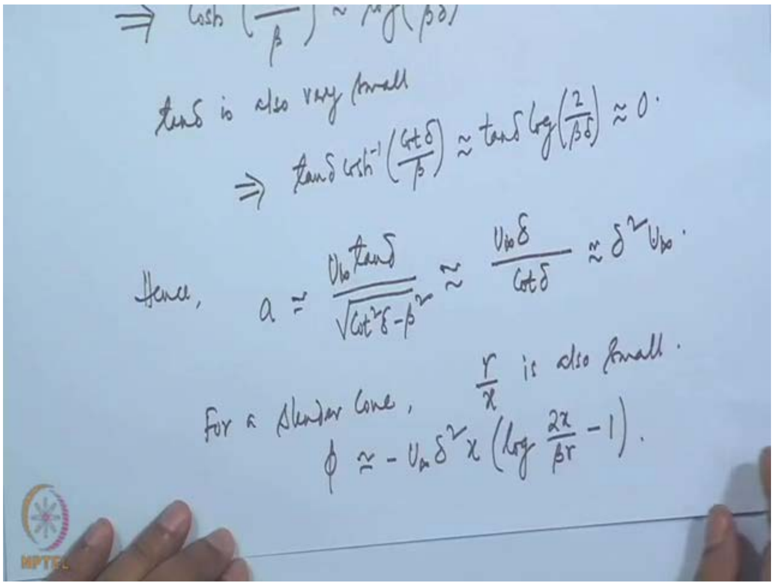


And we have seen that a source distribution given by supersonic f xi equal to a xi (No audio from 00:35 to 00:46) past a cone is given by a source distribution f xi equal to a xi placed on the axis (No audio from 00:57 to 1:08) and we found that for a given (( )), given system mach number, this constant a is given by U infinity time delta by root over cot square delta minus beta square plus tan delta cos hyperbolic inverse cot delta by beta, where we have beta equal to beta square equal to m infinity square minus 1, delta is the semi vertex angle of the cone. However, and once this source distribution is known, the complete potential, and the velocity components, and pressure can computed using the appropriate formulae.

Now, this expression for the constant a is little bit (( )) and we mentioned these expression can be simplified if we have a very slender cone. So, for a very slender cone, vertex angle 2 delta is small and then cot delta is extremely large. And if m infinity is not very large, then cot square delta is, cot delta is much larger than beta (No audio from 03:28 to 03:37), very large and cot delta is much larger than beta, if m infinity is not very large.

And in this context, you may again remember that if m infinity is extremely large, the linearized equations themselves are not applicable. So, within the framework of linearized flow theory, this is quite correct.

(Refer Slide Time: 04:44)

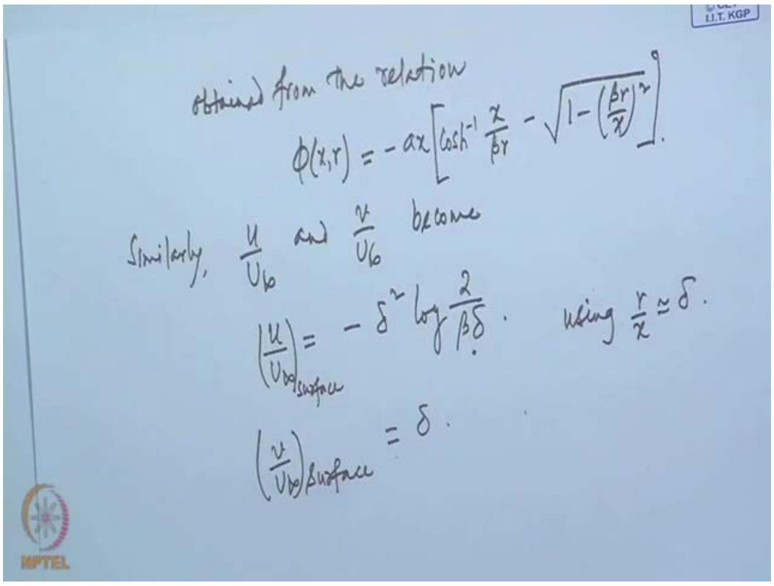


And then this cos hyperbolic cot delta by beta can be approximated (No audio from 04:34 to 04:44), cos hyperbolic inverse cot delta by beta is log of 2 by beta delta and this multiplied by tan delta (No audio from 05:08 to 05:21) small this implies tan delta (No audio from 05:29 to 05:49) that is this second term on the denominator is negligible (Refer Slide Time: 05:23).

And this then becomes, hence the coefficient becomes tan delta by root over cot square delta minus beta and tan delta for small delta is approximated to be delta and cot square delta minus beta square. Since, beta square is much less than cot square delta, this can be written as simply cot delta or this becomes tan square delta or delta square, of course u infinity is still there, so this become (Refer Slide Time: 06:50).

For a slender cone r by x is usually small and the potential function then becomes, the perturbation potential function then becomes phi is minus U infinity delta square into x log 2 x by beta r minus 1.

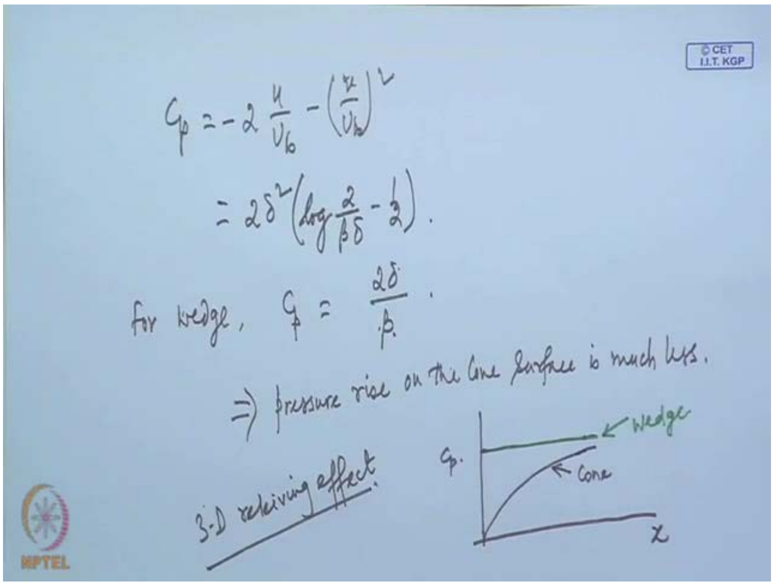
(Refer Slide Time: 08:18)



This is obtained from the general expression (No audio from 08:10 to 08:19), obtained from a relation which we evaluated in the last class that phi x r is minus a x cos hyperbolic inverse x by beta r minus 1 minus beta r by x square. So, simplification of these expression gives with a equal to minus infinity delta square gives this expression for phi for a slender cone.

Similarly, the velocity components can also be. this u by U infinity and v by U infinity, they become on the cone surface on the cone surface minus delta square log 2 by beta delta and v by U infinity surface is delta, in this we have (No audio from 10:13 to 10:28).

(Refer Slide Time: 10:44)



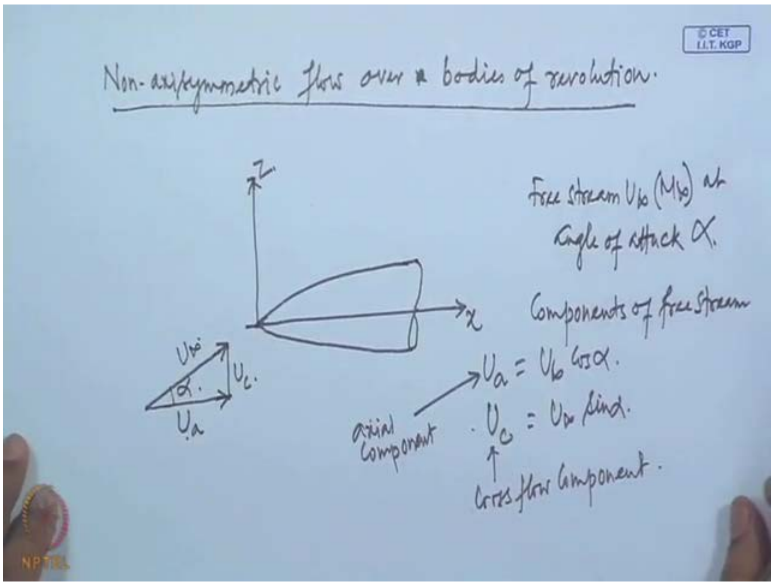
The pressure coefficient then becomes the pressure coefficient then becomes, minus 2 u by U infinity minus v by v infinity square equal to 2 delta square log of 2 by beta delta minus half.

Now, for this slender cone, the equivalent two dimensional (()) is a slender wedge and we have earlier seen that for slender wedge (No audio from 11:25 to 11:38), you remember 2 by root over m infinity square minus 1 into theta, where theta is semi vertex angle which you have denoted here by delta, so it is 2 delta by beta. If you compare these two relations, what you see that pressure rise on the cone surface is much less (No audio from 12:02 to 12:18) is much less when you compare it with your wedge. Eventually, if we (No audio from 12:27 to 12:54) distance along the axis or wedge, the pressure distribution is the pressure on the wedge surface is constant while for cone this becomes (No audio from 13:26 to 13:37) (Refer Slide Time: 13:27).

So, the pressure rise and the cone surface is much gentle and so, this is once again 3 dimensional relieving effect (No audio from 13:53 to 14:02) that is when in case of a three dimensional body that is as a cone, the flow has an additional reaction to adjust itself; while in two dimensional case, there is no such extra direction. Because in the (( )) reaction, the conditions are same at all stations that is what the meaning of two dimensional flow, as a consequence in two dimensional flow; the flow has not that additional direction available to it for adjusting; while in case of a three dimensional object there is an additional direction available in which the flow can adjust itself. And consequently, the effect of the body is much less on the flow and the pressure on this body is much less (No audio from 15:00 to 15:09).

Knowing the pressure distribution and full potential on the surface, we can compute the forces that are acting on the body, in this case the cone, or in general anybody of revolution in axial axial symmetric flow. This is obvious that in case of a axial symmetric flow, there will be no lift force, because the pressure on the upper half of the body and the lower half of the body will cancel each other, however a drag force will act which we have already mentioned as the wave drag so that drag will be present.

(Refer Slide Time: 16:25)

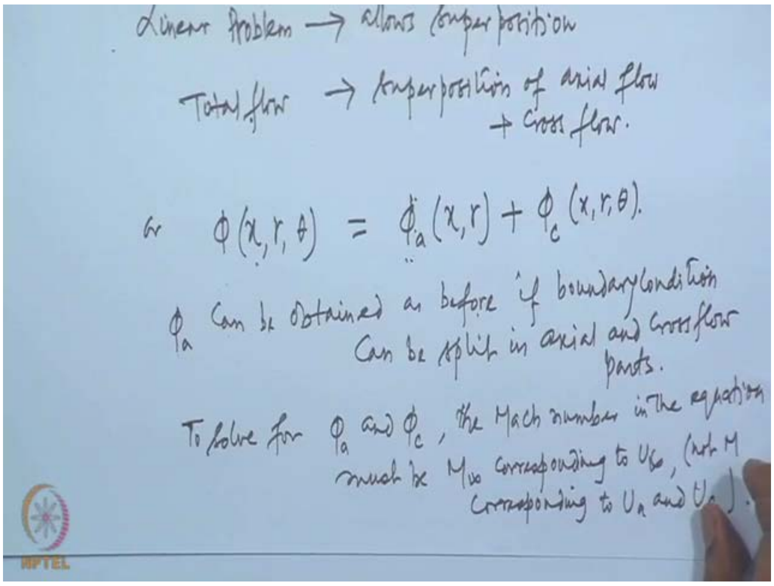


However, before we try to compute that forces acting on this body of revolution in axially symmetric flow, we will try to see, happen if the flow is not axially symmetric (No audio from 16:10 to 16:21).

So, we will now consider non axisymmetric flow flow over a body of, over bodies of revolution (No audio from 16:43 to 17:00). Let us consider again a body of revolution (No audio from 17:11 to 17:23), then the axial axis x, then normal axis is z, and let us say that the field stream is not along the axis, but aligned at an angle of a attack, we call this U infinity and the angle of attack is we call it (No audio from 17:50 to 18:09) (Refer Slide Time: 17:45).

So, free stream at an angle of angle alpha, of course corresponding to Mach number m infinity at angle of attack alpha. Now, free stream has two components, components of free stream, the axial component U a is U infinity cos alpha and the cross flow component (No audio from 18:57 to 19:28). Now, since we are dealing with linearized flow which allows superposition, we can think this flow to be a superposition of two separate flow; one axial flow with free stream speed of U a, and another cross cross flow with free stream speed of U c (No audio from 19:46 to 19:56).

(Refer Slide Time: 20:06)

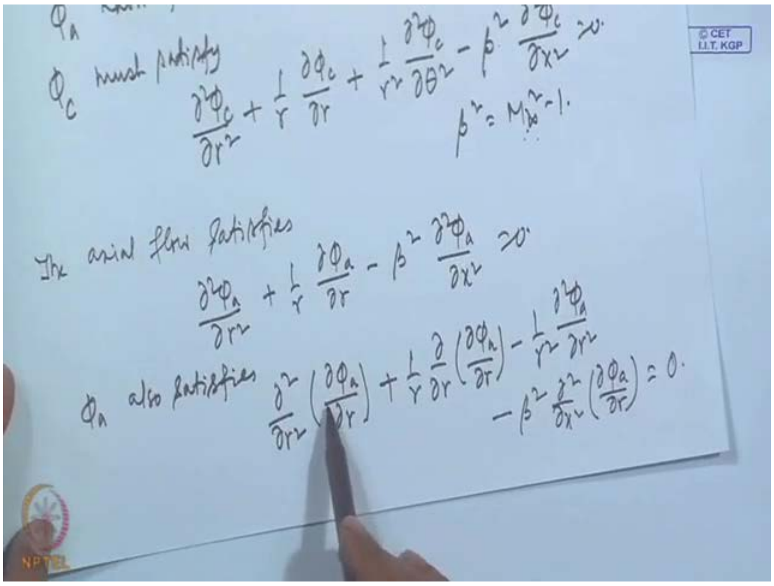


So, the problem is problem is linear linear, problem allows superposition which allows the flow is total flow is superposition of axial flow plus cross flow. So, the solution at the solution for this problem which is the solution, which is the perturbation potential as a function of x r theta is sum of two potential; one is the axial flow solution phi a x r plus phi c x r theta. Now, the axial flow solution, we have already discussed how to obtain the axial flow solution (No audio from 21:32 to 21:50). Now, this axial problem if we want solve this axial problem separately, it is essentially required that our boundary condition can also be splitted in axial and cross flow part.

So, phi a can be obtained as before, if boundary condition can be split in axial and cross flow part, in a we will see that this boundary condition can also be split in axial and cross flow part. However, at this stage we will just assume that the boundary condition can really be split and we can get a separate solution for phi a, and can obtain the solution by the method that we have discussed earlier (No audio from 23:08 to 23:19).

Now, one thing in this case must remember that, while solving the problem for axial and cross flow, the corresponding mach number in the potential flow equation must be taken as m infinity that is undivided mach number, not the mach number based on the component. So, for this (No audio from 23:53 to 24:02) solve for phi a and phi c, the mach number in the equation must be m infinity, corresponding to U infinity, not m corresponding to U a and U c. That is, while solving the velocity should be taken as U a or U c, but the mach number must be taken as m infinity that is, because the flow is, the nature of the flow, and the entire flow is governed by that m infinity corresponding to the free stream mach number, not by the part mach number.

(Refer Slide Time: 25:37)



Anyway, let us see that we have the axial flow solution, then we have phi a known, now phi c must satisfy the cross flow equation or the (( )) equation d 2 phi c d r square plus 1 by r d phi c d r plus 1 by r square d 2 phi c d theta square minus beta square equal to d 2 phi d x square 0. And beta square is m infinity square minus 1 that is in this, even though the system speeds is U c, the Mach number is not corresponding to that U c, but corresponding to m infinity.

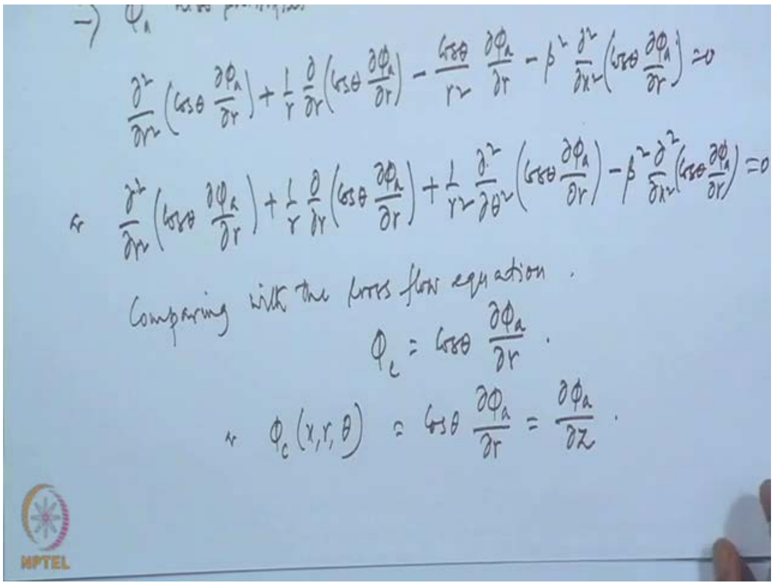
So, this phi c must satisfy this equation, one thing in this case must be remember that this supersonic flow equation is valid even if the component U c is subsonic; in provided m infinity is supersonic. So, as long as the main flow is supersonic or the total flow is supersonic, if the cross flow component is subsonic, still the equation for supersonic flow is to be taken. And once again, since in this case we have the split the problem in two parts, the axial and cross flow part. In case if if it happens that, the perturbations corresponding to axial flow or cross flow is large, is immaterial as long as the total perturbation is small.

So, the small perturbation theory may still hold, if the cross flow perturbations are not large, provided the total perturbation that is cross flow perturbation plus axial flow perturbation remains small. Now, the axial flow (No audio from 27:56 to 28:05) the axial flow satisfies the governing equation corresponding to d 2 phi a d r squared plus 1 by r d phi a d r minus beta square d 2 phi a d x square equal to 0, the axial flow solution of this equation. Now, if we differentiate this equation with respect to r, the resulting equation is also be satisfied by phi a (No audio from 28:51 to 28:57).

So, phi a also satisfies, if we differentiate this equation with respect to r, the resulting equation is also satisfied by phi a. So, phi a satisfies (No audio 29:15 to 30: 05) since phi a satisfy this equation, phi also satisfy this equation, which is obtained just by differentiating this with respect to r (Refer Slide Time: 29:15).

Now, if this d phi a by d r present in this equation is replaced by cos phi a d phi (()) d r, then that is also, phi also satisfy that equation (Refer Slide Time: 30:26). That means, if we introduce another cos theta here, phi also satisfy that, because that interruption of phi cos theta in this equation does not alter anything, which is constant for this particular equation, that is no theta derivative being present in the equation (No audio from 30:55 to 31:05) (Refer Slide Time: 30:38).

(Refer Slide Time: 31:13)



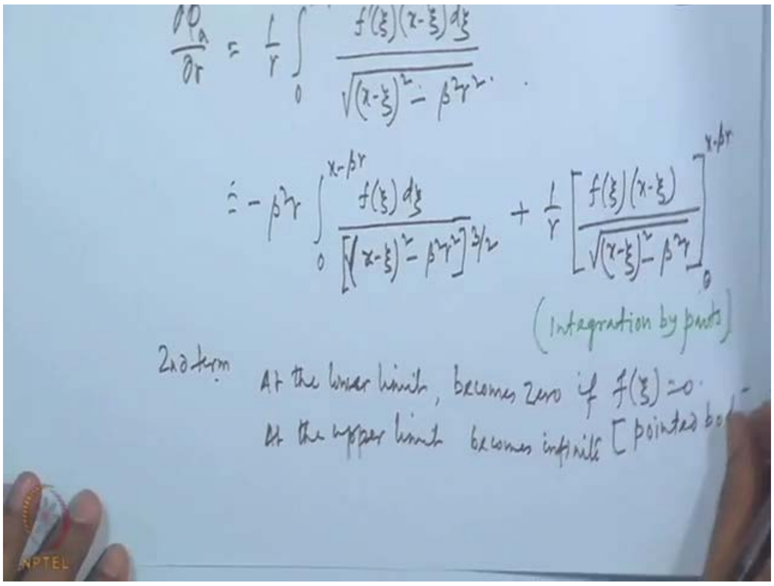
So, where this implies phi a also satisfies (No audio from 31:11 to 31:26) d phi a d r plus 1 by r d d r of cos theta d phi d r minus sorry in this in this equation, these these these are not square, this is first derivative (Refer Slide Time: 35:13). So, minus cos theta by r square d phi a d r minus beta square d 2 d x 2 cos theta d phi a d r equal to 0, or for this equation cos theta d phi a d r is the solution.

Now, this equation can also be written as or we can write this term as cos theta d phi a d r plus 1 by r d d r of cos theta d phi a d r. Now, in this this term, we can write as 1 by r square d 2 d theta square cos theta d phi a d r minus beta square d 2 d x 2 cos theta d phi a d r equal to 0.

Now, if we look to this equation, this is exactly the equation for the cross flow equation. So, comparing it with the cross flow equation (No audio from 34:08 to 34:29), what we get is that phi c equal to cos theta d phi a d r. So, what we see that, if we have the axial flow solution phi a known, then this cos theta d phi a d r satisfy the cross flow equation or the cross flow solution is cos theta d phi a d r (No audio from 34:59 to 35:08).

Or we can write phi c is a solution of x r theta is cos theta d phi a d r which is, of course a function of x and r alone, phi a or this can also be written as d phi a d z. So, it shows clearly that if we have the axial flow solution known, we can straight away obtain the cross flow solution without properly solving the equation.

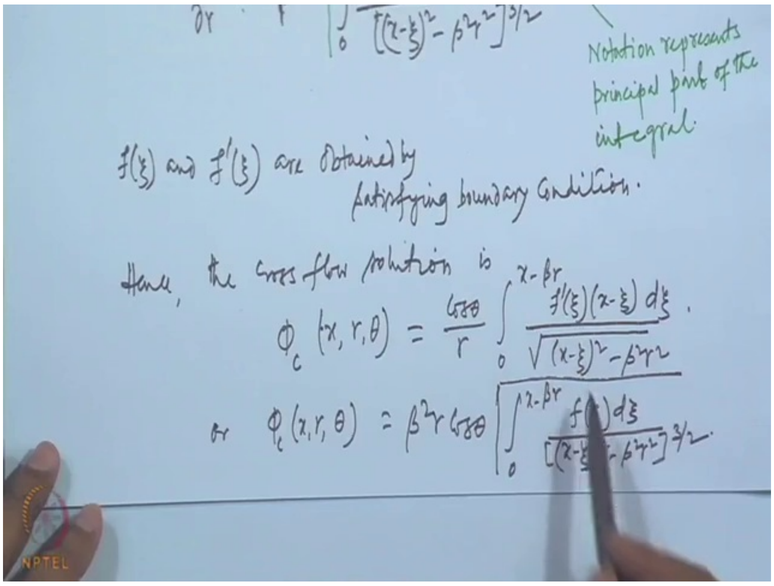
(Refer Slide Time: 36:16)



Now, to obtain that, let us see what is d phi a d r, d phi a d r is 1 by r, we have already the expression for phi or phi a, you have to x minus beta r f prime xi x minus xi into d xi (No audio from 36:32 to 36:56). This can be written by integration by parts as minus beta square r, 0 to x minus beta r f xi d xi by root over sorry by x minus xi square minus beta square r square to the power 3 by 2 plus 1 by r f xi x minus xi by (No audio from 37:57 to 38:21) (Refer Slide Time: 37:58), so this is integration by parts (No audio from 38:25 to 38:42).

Now, look into the second term, second term becomes 0, if the body is pointed when the lower limit is 0. So, looking to the second term at the lower limit at the lower limit becomes 0, if f xi equal to 0, which is you know power pointed body. And at the upper limit this, it becomes infinite (No audio from 39:37 to 39:56) at the upper limit this term becomes infinite. So, it is rule of integration to consider only the finite part of the past integral.

(Refer Slide Time: 41:00)

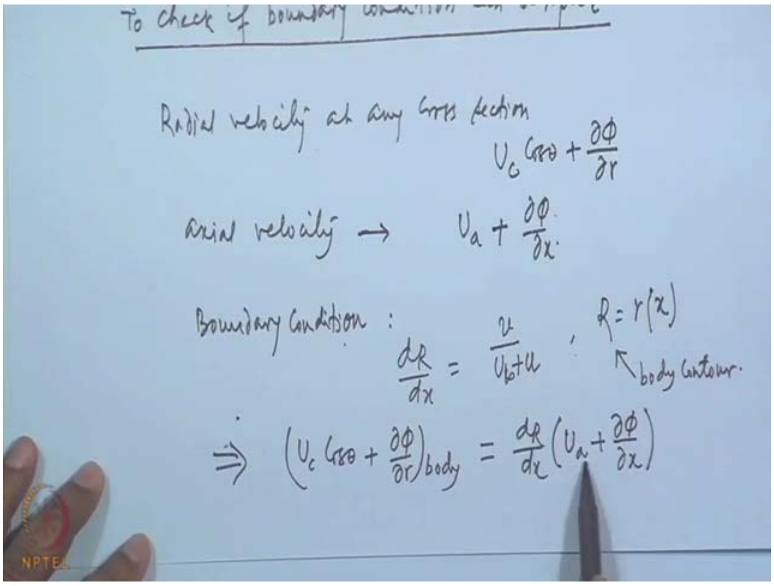


So, what we need is only the finite part or called the principle part, first term is taken and this is formally written as (No audio from 40:51 to 41:05) minus minus beta square r into (No audio from 41:12 to 41:33). And the finite part of this integral, and this symbol represents the principle part of the integral (No audio 41:49 to 42:41) (Refer Slide Time: 40:56). Now, this undetermined part this f xi or f prime xi in which ever form we use for this can be obtained by satisfying the boundary conditions.

So, (No audio 42:55 to 43:25), so finally, the cross flow solution we can write as phi (No audio from 23:30 to 43:49) phi c x r theta equal to cos theta into d phi a d r and d phi a d r we have two form. So, taking this first form becomes cos theta by r into 0 to x minus beta r f prime xi x minus xi by root over x minus xi square minus beta square r square d xi or (No audio 44:39 to 45:31) x minus xi square minus beta square r square to the power 3 by 2, the finite part of it. So, as a cross flow solution, we can use any of these to find the cross flow potential, provided we know the stretch of the singularity f xi or f prime xi (No audio form 46:04 to 46:12).

Now, we mentioned that this solution, that is the solution of axial part and cross flow part separately can be carried out, if the boundary condition can be split into two part separately; that is the boundary condition can also be split in axial and compressible sorry cross flow part and this is what now we like to see (No audio from 46:46 to 46:52).

(Refer Slide Time: 46:37)



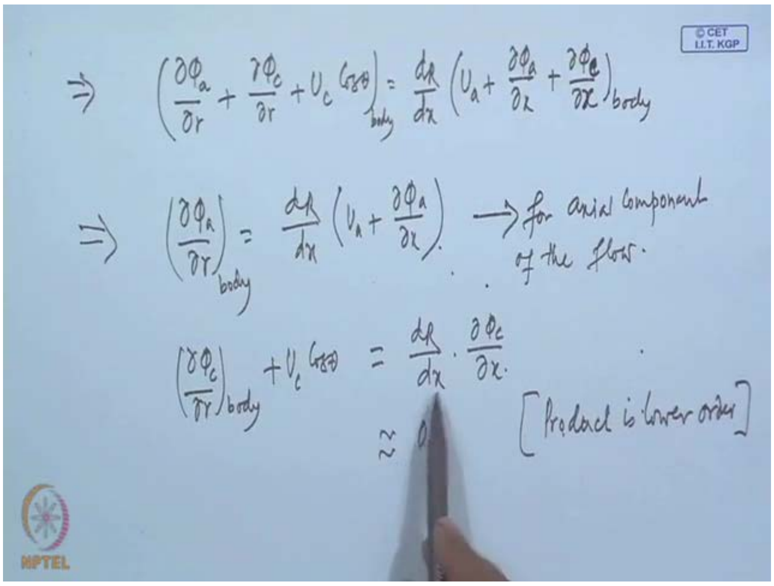
To check if boundary condition can be split (No audio from 47:07 to 47:15), now we have say radial velocity at any cross section (No audio from 47:22 to 47:32) cross section is the undisturbed stream U c cos theta plus a perturbation part d phi d r.

Similarly, the axial velocity again at any cross section is U a plus d phi d x, now the boundary condition is the tangential component need not be checked. Because, the tangential component always satisfy the boundary condition which, we have mentioned earlier that only the meridional section should be looked into the boundary condition; what we had is d r d x equal to v by U infinity plus u (No audio from 48:52 to 49:06), this is the body contour.

Now, in these we substitute these and this gives us U c cos theta plus d phi d r on the body is d r d x into U a plus d phi (No audio from 50:00 to 50:08) where this is, what is this, axial, total axial velocity? (Refer Slide Time: 49:16) Now, we substitute phi as phi a plus phi c in this expression and hence this becomes d phi a d r plus d phi c d r plus U c cos theta into d r d x into U a plus (Refer Slide Time: 50:31).

Now, see from here that we can very easily split it, this taking all the axial term in this equation sorry d (( )) we have d phi a d r, of course on the body is d r d x into U a plus, see that this contents only phi a that is only the axial flow, so this is for the axial flow and the remaining term (No audio 52:30 to 53:05) (Refer Slide Time: 51:37).

(Refer Slide Time: 50:36)



Now, for a slender body this d r d x is small and d phi c d x is also small and consequently they can be, the product can be neglected (No audio from 53:19 to 53:33). Since, this product is lower order, it can be neglected with in comparison to this term and consequently this can be taken as this (Refer Slide Time: 53:38).

So, look into what we have done today, first of all we approximated or simplified the source expression for the source distribution, when the cone is very slender for the general from the more accurate solution that we obtained in the last last class. Further then, went on to see the asymmetric flow or the when the flow is non axi non axisymmetric with respect to the body that is the flow is at an angle of the (( )).

We saw that being the the problem being linearized, it offers us a possibility to split the flow in two separate part; one being the axial flow component, and the other is the cross flow component. And provided that the boundary condition can also be splitted in these two parts, which we saw that can be done that the boundary condition can be splitted. And since, the boundary condition can be splitted, the flow can be solved as a combination of two separate flows, an axial flow and a cross flow.

And also we saw that the once the axial flow solution is known, the cross flow solution can be straight away obtained by a very simple relationship which gives that the cross flow part of the potential phi c as a function of x r and theta is simply the product of the axial flow potential phi a and sorry derivative of the axial flow potential phi a multiplied by cosine of theta that is phi c is simply cos theta d phi a d r.

And then we saw that, since phi a is known, or general solution for phi a is known when which is simply a source distribution along the axis and which still can be obtained from the satisfaction of the boundary condition. And then, once the axial flow solution phi a is known, the cross flow solution can straight away be obtained by cos theta into d phi a d r which we saw that can be expressed in two different form, one containing f prime xi as the singularity singularity strength, the other containing the singularity strength as f xi, but which is having a r q function of the denominator or equivalent to an r q function in the denominator.

And in this case, only the finite part or the principle part of the integration should be taken. And with this, we can now extend our cross flow solution over slender cone to a cross flow solution over the cone and complete the conical flow solution for any arbitrary flow, of course within the framework of linearized small perturbation theory, and we can compute the process. So, next we will compute the process that acts on the body either at axially symmetric flow or asymmetric flow.