**High Speed Aerodynamics**

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**Module No. # 01**

**Lecture No. # 32**

# Linearized Problems – Forces on Slender Bodies

We have discussed the solution of axially symmetric flow over slender bodies of revolution and in that context we consider an example flow over a cone and for which we got the complete solution and for any cone within the framework of small perturbation theory. And then approximated that solution for cylinder bodies, however instead of getting solution for a particular problem and approximating it for slender bodies, it is possible to introduce the approximation in the solution process itself and get a complete solution for any arbitrary slender bodies.

And we will use that method to find the solution about any arbitrary profile in first axially symmetric flow and then possibly in a symmetric flow with cross flow, however you know that cross flow solution can straightaway be obtained from the axially symmetric solution and we will use this approach to evaluate the forces that are acting on in cylinder body. So, first of all let us consider the forces that are acting on a axially symmetric flow or bodies immerged in a axially symmetric flow.

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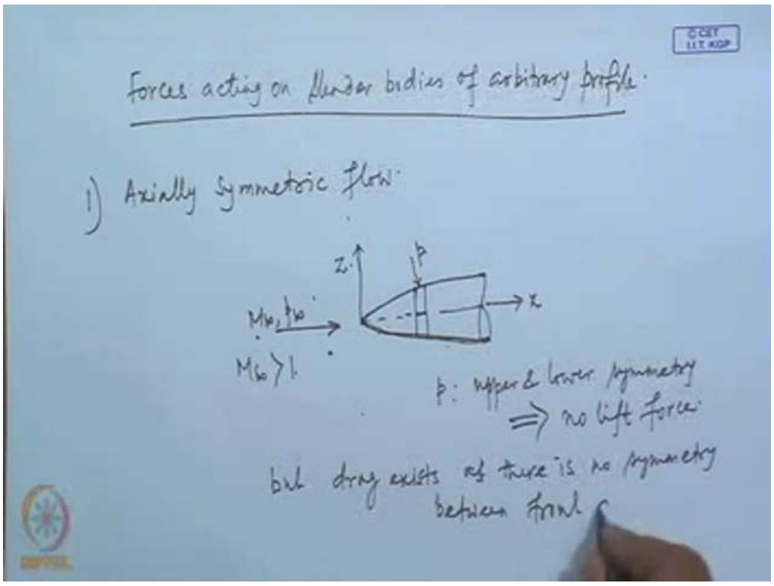


So, forces acting on slender bodies of arbitrary profile first, let us consider axially symmetric flow (No audio from 02:42 to 03:06), assume this body to be axially symmetric and the flow is along the axis. Now if we consider any small axial section over which the radius or you can say the cross-sectional area is increasing slightly, then we can see that the pressure that is acting on this strap where we have m infinity is a free stream speed when the free stream pressure is p infinity.

And on this section the pressure is acting p and we have seen from the earlier solution that the pressure varies, gradually over this length in an axially symmetric flow over a cone, which we can say may be the general result for a general profile. Now we can see that the pressure distribution is symmetric about the upper half and lower half that is pressure is pressure has a top and bottom symmetry, since pressure has top and bottom symmetry since the pressure has top and bottom symmetry consequently.

There is no lift force (No audio from 04:57 to 05:17) consequently there will be no lift force, however we see clearly that the pressure is not symmetric about front and rear but, drag exists as there is no symmetry between front and rear.

And so in this case the force evaluation of the force is basically determination of this drag force that acts on this body and we will now try to evaluate this drag force (( )) the general solution for this problem, as we have already evaluated (No audio from 06:45 to 06:58) 0 to x minus beta r function of xi d xi the source strength distribution divided by the kernel square x minus xi square minus beta square minus r square where beta square is M infinity square minus 1.

Now, if we recall that we introduced a substitution and simplified this integration earlier because this integration has a singularity at the upper limit it is an improper integral and use of that substitution make it possible to evaluate this integration. However for any arbitrary slender bodies in this context, we do not make use of that substitution to evaluate this integral rather; we split this integral in two part we split this integral in 2 part phi which are omitting this functional representation x r all the time for convenience (No audio from 08:25 to 08:35) is that means the upper limit.

We make it x minus beta r minus epsilon, where epsilon is a small number (No audio from 08:42 to 09:05) minus (No audio from 09:07 to 09:30). This is just to avoid the singularity at the upper limit, we have reduced the upper limit by a small number epsilon a small number which we have you introduced, so that there is no singularity in this integrant, however in this part the singularity is still there.

Now, since this integrand is not singular, we can expand it in the power series of beta square r square; so with this the integrand is not singular not singular in it is domain (No audio from 10:35 to 10:50) and in powers of beta square r square (No audio from 10:54 to 11:08).

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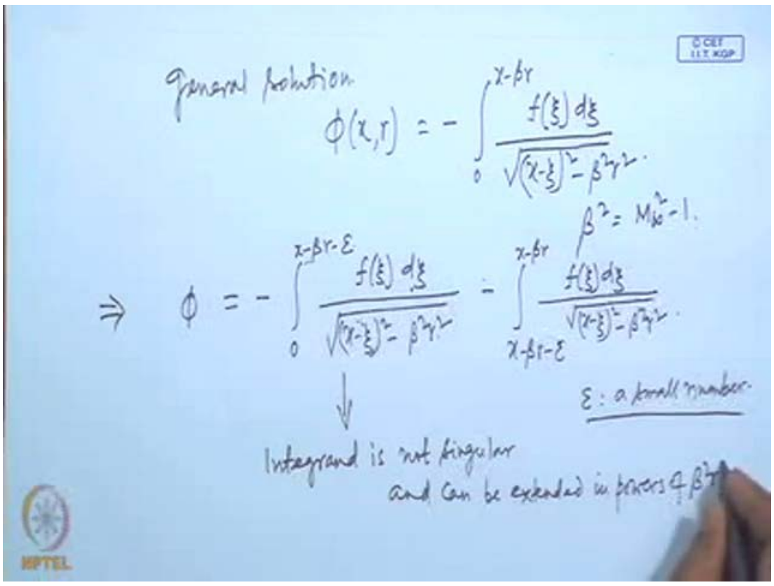

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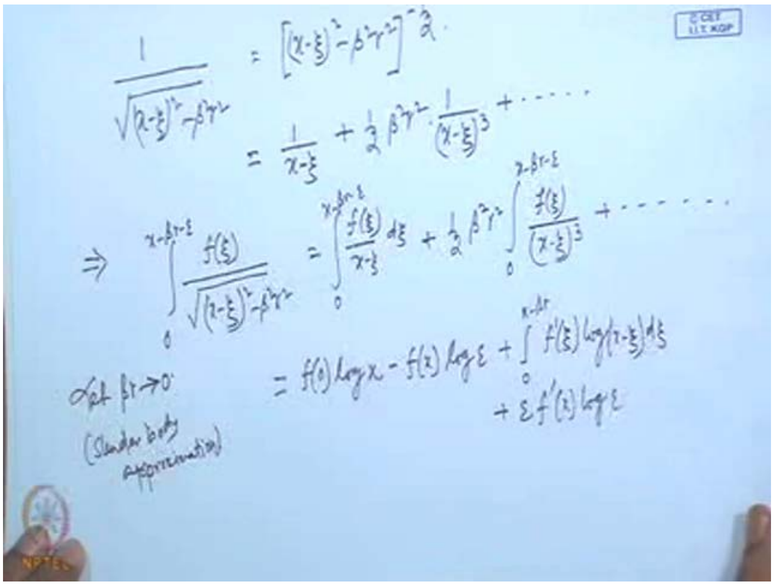

Now, if we integrate this function, that is 1 by root over x minus xi square minus beta square r square; that is x minus xi square minus beta square r square to the power minus half this can be expanded as a binomial series. And assuming slender cone that higher power of beta square r square can be neglected, this gives 1 by x minus xi plus half beta square r square into 1 by x minus xi to the power 3.

Now, then this first integral can be integrated term by term that is (No audio from 12:41 to 12:59) we had the (No audio from 13:09 to 13:59) and this becomes, then f 0 log x minus f x log epsilon plus (No audio from 14:25 to 14:45) f prime xi log x minus xi d xi plus epsilon f prime x log epsilon this is where the case. When beta r approaches 0 that is for slender bodies; so this where, we introduce slender body approximation (No audio from 15:33 to 15:47).

For the second integral for the second integral we introduce same substitution, which we introduced earlier that is (No audio from 15:58 to 16:10) xi equal to x minus beta r cos hyperbolic sigma. So, for second integral (No audio from 16:16 to 16:27) use xi x minus beta r cos hyperbolic sigma as before and this gives x minus beta r minus epsilon to x minus beta r by f xi (No audio from 17:12 to 17:35) cos hyperbolic inverse beta r plus epsilon by beta r equal to function of x minus beta r cos hyperbolic sigma d sigma.

This integration of course, we carried put earlier, so we can use those result and this gives f x cos hyperbolic inverse beta r plus epsilon by beta r d sigma minus beta r into 0 to cos hyperbolic inverse beta r plus epsilon by beta r into f prime x cos hyperbolic sigma d sigma plus higher order terms in beta r (No audio from 19:05 to 19:17) assuming beta r 0; that is for slender body approximation, this gives (No audio from 19:25 to 19:35) log by 2 beta r plus f x log epsilon minus epsilon f prime x.

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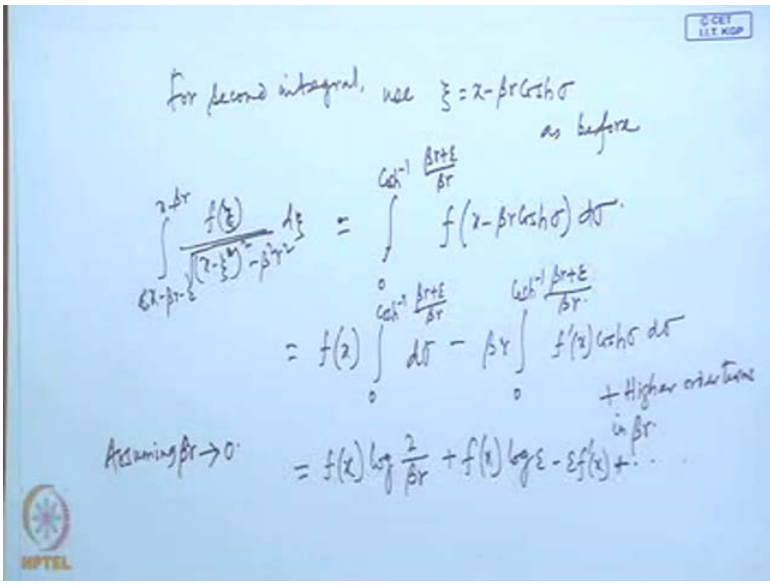


Now, if we combine this two if we combine this two we get the potential, so (No audio from 20:07 to 20:19)  two; we have phi x r equal to minus see, all this terms in both the integration what we have carried will become negative; because, in the definition of phi the integral r preceded by a negative sign. So, this makes minus f x log 2 by beta r minus 0 to x f prime xi log x minus xi d xi (No audio from 21:15 to 21:45) and see that the terms containing epsilon can be made to vanish if epsilon is approaching to 0.

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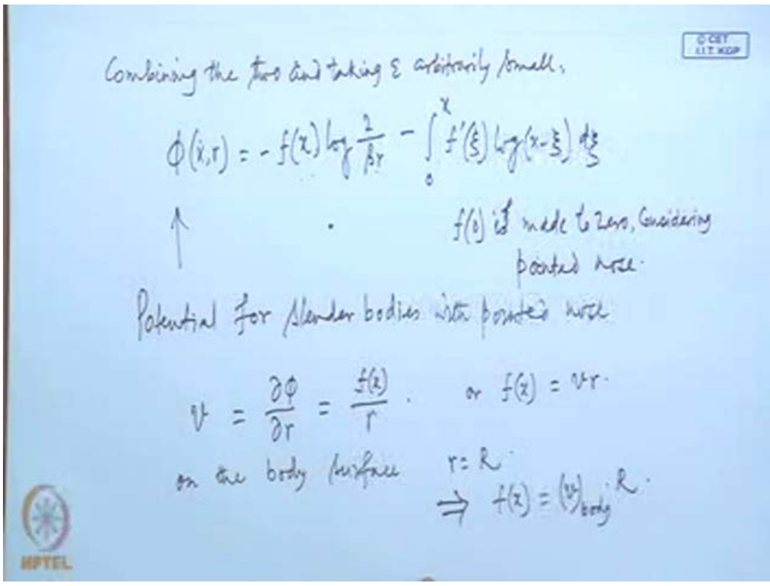


Now, if we consider a pointed body (No audio from 21:58 to 22:18) made to 0 considering pointed nodes and this eliminates the f 0 term present in the first integration the term f 0 log x present in the first integration is made to 0 by assuming that the body has a pointed nodes; so that f 0 is 0 and this then is the, so what we get is the potential for slender bodies with pointed nodes and here it is for any general arbitrary slender bodies.

Now, we know that this potential function must satisfy the boundary condition, now first of all let us evaluate, what is v the radial component of velocity is obtained as d phi d r and this becomes only the first term here is a function of r. So, this can be easily differentiated and the result is f x by r or f x equal to v r, now let us say on the body surface r equal to r and this then gives v on the body into r.

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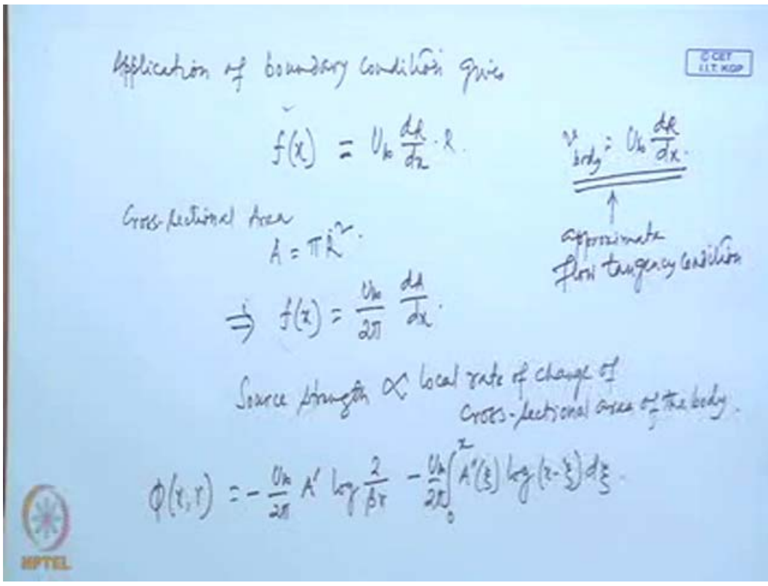


So, application of boundary condition, then gives (No audio from 25:22 to 25:42) condition gives f x equal to v body which you know is u infinity r d r d x, v body is U infinity d r d x into r (No audio from 26:06 to 26:26). This is what the approximate flow tangency condition, which we have written many times; so, this gives the distribution of source strength for any arbitrary profile and assuming that cross-sectional radius (No audio from 26:51 to 27:04) cross-sectional area A is pi r square.

So, introducing this f x is u infinity by 2 pi d A d x that is for a body in axially symmetric flow for a slender body in axially symmetric flow, the source strength distribution on the axis is simply given by the rate of area change of cross-sectional area along the axis. So, what we get in source strength is proportional to local rate of change of cross-sectional area of the body meaning that, the portion of the body which are far away from the cross section or the corresponding concerned cross section has no influence on the local condition.

That is flow at any part of the slender body, simply depends on the local cross section and it is change however and the cross section which are faraway or the part of the body, that is far away from this station, do not have influence on the local flow condition. In terms of perturbation you can see that, the rate at which the flow is perturbed due to pushing of the flow by the body away from it depends entirely on the local rate of change in area (No audio from 29:34 to 29:44).

Now, with this we can complete the solution for complete the solution for the potential over a cylinder body of revolution with close nodes and arbitrarily smooth meridional section is minus U infinity by 2 pi (No audio from 30:17 to 30:28). The rate of change of area will denote by A prime into log 2 by beta r minus sorry u infinity by 2 pi A double prime xi log of x minus xi d xi integrated over 0 to x. So, this is the potential for any cylinder body of revolution with close nodes and arbitrarily smooths meridional section in axial flow (No audio from 31:17 to 31:29).

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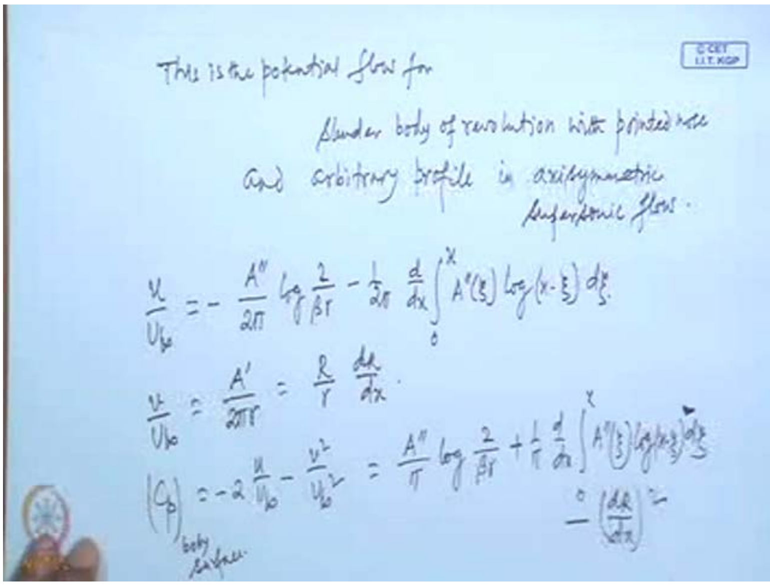


So, this is the potential for (No audio from 31:31 to 31:48) cylinder body of revolution with pointed node and arbitrary profile in asymmetric flow in asymmetric supersonic flow within the framework of small perturbation theory. Now the velocity components can be evaluated that is U by U infinity by differentiating that expression for phi with x, this gives u by u infinity is minus A double prime by 2 pi log of 2 by beta r minus 1 by 2 pi d d x of (No audio from 33:16 to 33:31) and v by U infinity is (( )) d pi d r which gives A prime by 2 pi r which becomes r by r d r d x.

Now, c p of course we can get the pressure coefficient is minus 2 u by U infinity minus v square by u infinity square. So, this gives A double prime by pi log of 2 by beta r plus 1 by pi d d x of this integration (No audio from 34:37 to 34:48) sorry log x minus xi d xi minus. There is c p on the body surface c p on body surface, so we get the pressure coefficient on any cylinder body of revolution, we pointed node in axially symmetric supersonic flow.

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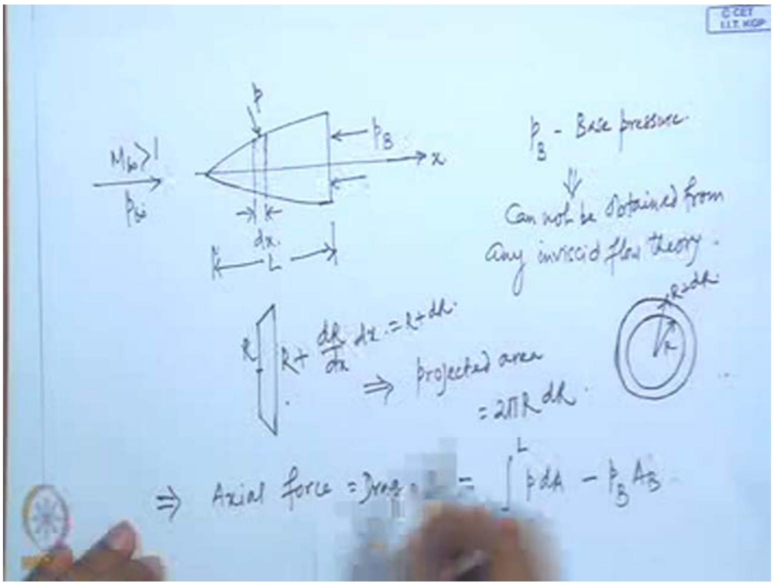


Now, let us come back to this pressure distribution once again (No audio from 35:33 to 36:13), we will consider a small section of length d x and the free stream as we have mentioned M infinity greater than the pressure here is p infinity. The pressure on the surface is p but, the back the pressure will of course, be different and will call that base pressure. Now, the pressure coefficient that we have obtained corresponds to this pressure.

However, as far as the back pressure is concerned, we cannot evaluate this back pressure based on these in viscid flow theory; because flow behind this body with a blunt base is always highly viscous even may be turbulent and cannot be evaluated by any in this in viscid flow theory. So, this p b can only be obtained using either a full viscous flow theory or by experiment, so cannot be obtained from any in viscid flow theory; we need either a complete viscous flow theory, perhaps including turbulence and or experimental determination of the base pressure.

Now, looking to this area (No audio from 38:25 to 38:43), let say that in this side the radius of the cross section is r, then on this pass the radius of the cross section is R plus d R d x into delta x and projecting it on a plane. We get the projected area looks like two concentric circle (No audio from 39:17 to 39:40) projected area is basically two concentric circle (No audio from 39:43 to 40:04). So, this projected area is becoming 2 pi R d R and of course, it is a function of x it depends on x, now the pressure reacts on this projected area and consequently that of course, the axial force this implies the axial force which itself is drag.

In this case, if we consider the total length of the body is L then, minus p B A B where A B is the area at base (No audio from 41:22 to 41:38) A B is base area.

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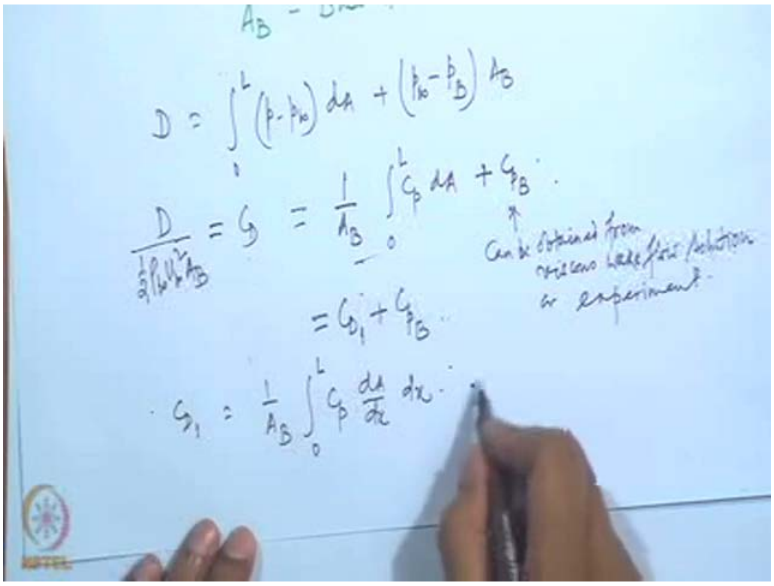

Now, this can be arranged in this fashion 0 to L p minus p infinity d a plus p infinity p B into A B, then if we divide this by that is you are defining drag coefficient based on the base area; not on the plan form area as usually done in aerodynamics of wings or incompressible flow aerodynamics that that is always based on plan form area but, in this case we can see, that that drag coefficient is based on this area.

Now, this gives when divide by half rho infinity infinity square this becomes the pressure coefficient, so 0 to L C p d A plus C p B; now this this can only be obtained from viscous wave solution or or from experiment, this flow is cannot be obtained from inviscid solution. So, we can over this part as we have already obtained can be obtained by substituting that part and let us called that C d 1 plus C p B and this C d 1, then can be written as (No audio from 44:55 to 45:27), this we can write that A B into C d 1 is 0 to L C p A prime which is of course, of function of x into this.

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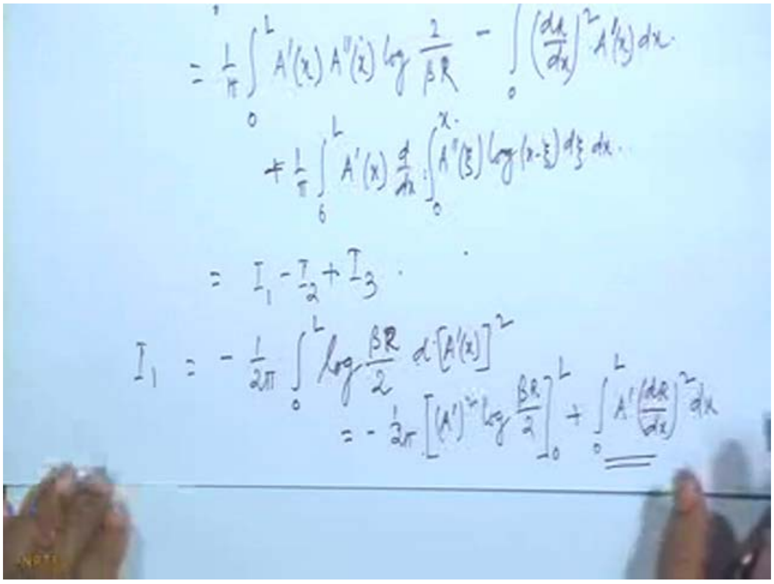

And then, we substitute C p here from the earlier expression that we had to give 1 by pi 0 to L a prime x A double prime x log of 2 by beta R, R is also a function of x minus 0 to L (No audio from 46:23 to 46:42) plus 1 by pi 0 to L A prime x d d x of A double prime xi log of x minus xi d xi. You may recall that this first and third term here, has come from the axial velocity component while this second term here, we have come from the radial velocity component v square by infinity square in c p. Now, this for convenience, we will write as I 1 minus I 2 plus I 3 some of 3 separate integration.

Now, the first term can be written as the term can be written as minus 1 by 2 pi 0 to L we inverse this parameter of logarithm to be at the negative sign log beta. This must be beta R by 2 into d which on integration by parts gives us minus 1 by 2 pi square log of beta R by 2 0 to L plus 0 to L A prime d R d x square which, this term we can see cancels with the second term I 2. So, this I 1 minus I 2 becomes only this term (No audio from 49:52 to 50:06).

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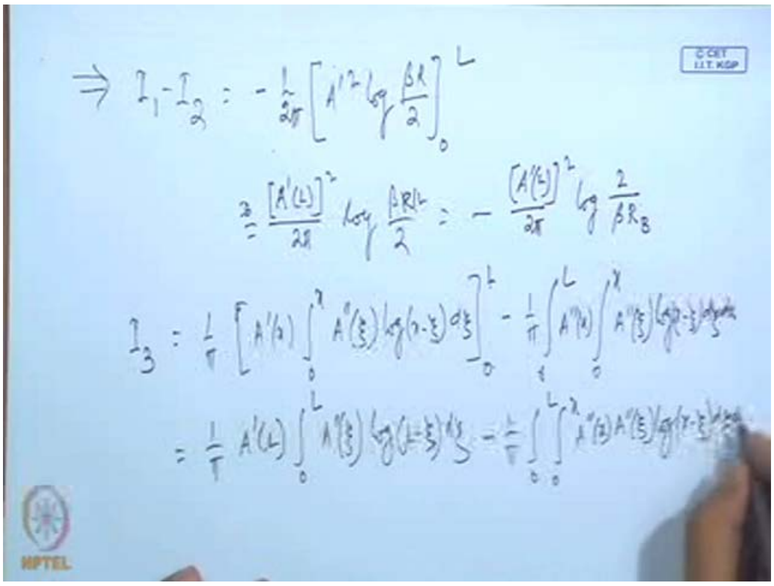

Minus 1 by 2 pi A prime square log of beta R by 2 0 to L (No audio from 50:27 to 50:41), now since we are considering bodies with pointed node, so this becomes at 0 the it become 0, so it remains only the (( )). So, this becomes A prime at L square by 2 pi log beta r by 2 that is (No audio from 51:31 to 51:52), where r be the radius at the base. The third integration as it happens is 1 by pi A prime x 0 to x A double prime xi log x minus xi d xi 0 L minus 1 by pi 0 to L A double prime x 0 to x A double prime xi log x minus xi d xi d x.

And this becomes 1 by pi A prime L 0 to L A double prime xi, where we have set the limit L for the length of the body and at 0, we have use the properties of pointed body minus 1 by pi 0 to L 0 to x A double prime x A double prime xi log x minus xi d is xi d x.

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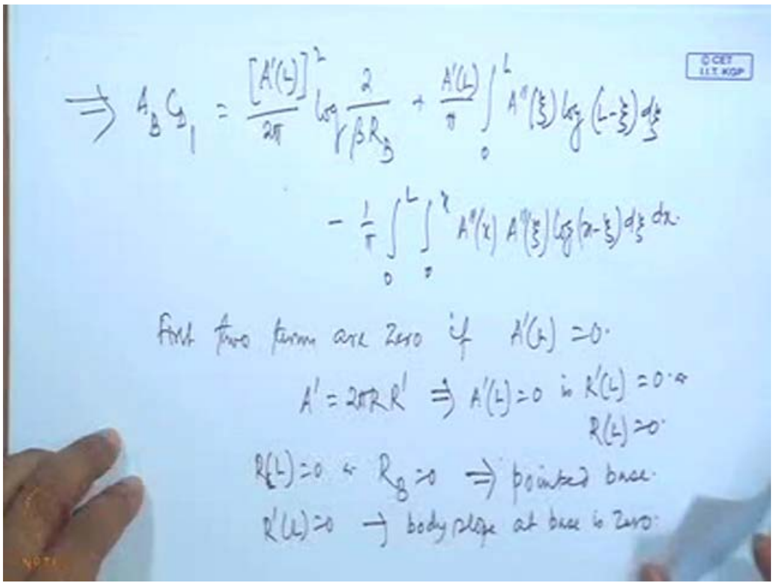


Now, combining then we get finally, when all three are combined this we have A by 2 pi log 2 by beta R B plus a prime at L by pi 0 to L A double prime xi log of L minus xi (No audio from 54:41 to 55:04). So, this is the expression for the drag that comes in asymmetric flow over a cylinder body with pointed node in supersonic flow due to the pressure distribution on the surface of the body that is excluding the base pressure drag.

Now, you can see here that, these first two terms will become 0 if a prime L that is the rate of change of cross-sectional area at the base is 0, first two terms are 0, if A prime L equal to 0. Now what is a prime at 2 pi R R prime this then implies that (No audio from 56:14 to 56:46) or R L equal to 0 (No audio from 56:50 to 57:07) pointed base; say the base of the body is also pointed then the first two terms also becomes, 0 or if R prime L equal to 0 then body slope at base is 0.

So, if the base is also pointed or the body slope at the base is 0, then the drag is given by the last term (No audio from 57:48 to 58:06) (Refer Slide Time: 57:58) 0 to l 0 to x (No audio from 58:08 to 58:28) which, happens to be that integration over a triangular area bounded by xi equal to 0 xi equal to x and xi equal to well.

So, we have obtained the drag coefficient due to the surface pressure distribution for body of revolution with of cylinder cross section with arbitrarily smooth profile and at pointed nodes. Subsequently, I have seen that, if the base is also pointed, then the drag term become simpler and similarly, if the slope of the body at the base is 0 then also we have a simpler expression for the drag coefficient.

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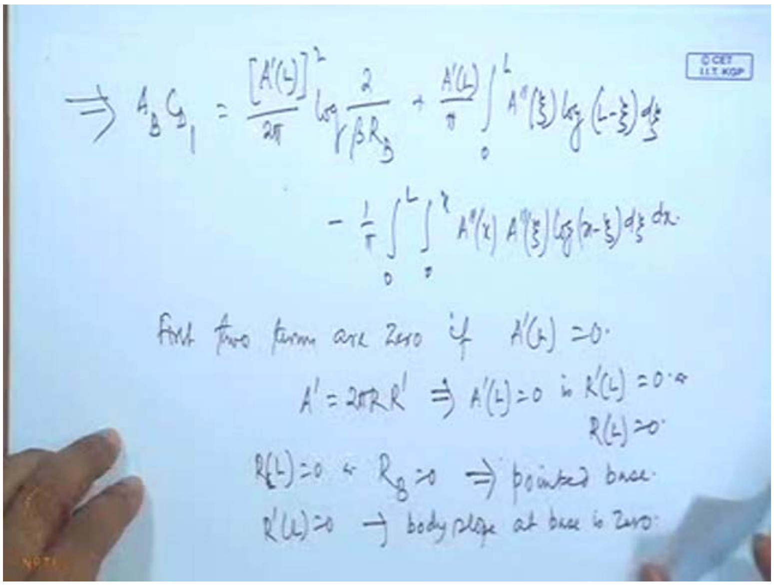


Now, this simply implies that drag coefficient will not change if the base cross-sectional area is not change that means, if the base is extended or rather if there is (( )) and after subsequently a constant area part of the cylinder body. Then, that uniform part of the body does not contribute to the drag coefficient, that is if we have say, think about a cone cylinder (Refer Slide Time: 1:00:09) (No audio from 1:00:09 to 1:00:24). So, this part where over which there is no change in cross-sectional area does not contribute to the drag coefficient (No audio from 1:00:30 to 1:00:52) and hence drag.