High Speed Aerodynamics

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Module No. # 01
Lecture No. # 33
Linearized Problems - Forces on Slender Bodies (Contd.)

Last time we consider forces on bodies of revolution of arbitrary profile in axially symmetric flow. And we computed the drag force that acts due to the pressure distribution of for this flow, of course, there is a base drag component which we said cannot be evaluated from any in viscid flow theory. And we saw that, there for a pointed body the drag force depends on the variation of cross sectional area.

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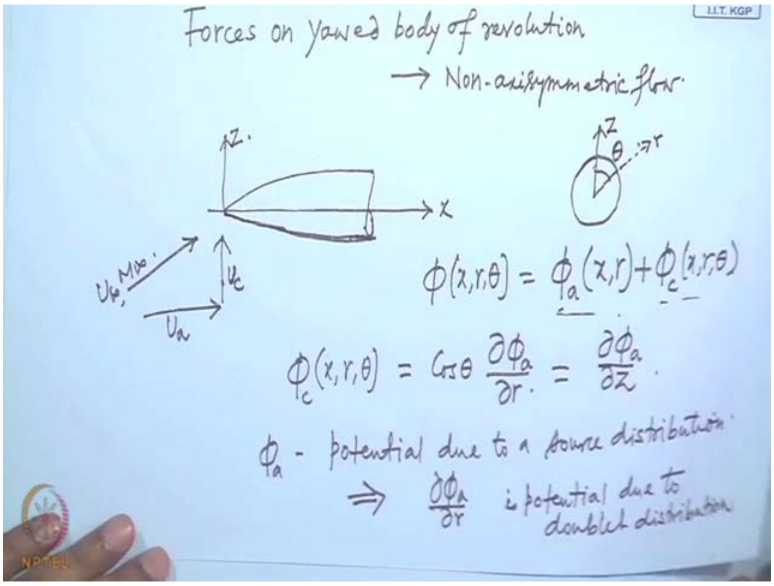

Next we will consider, when the body of revolution of any arbitrary profile is not in asymmetric flow that is the body is non-asymmetric flow or we will consider the forces that acts, when the body of revolution is in (( )) flow and (No audio from 01:27 to 01:47).

So, we would we will be determining, what are the forces acting on this body, when the flow is like this, however the geometry of the body still will be considering symmetric but, the flow is not along with axis. Consequently as we have seen earlier that it has a cross flow component and an axial flow component, which we earlier mentioned as U a and U c and you saw that the solution is the complete solution is axial contribution and a cross flow component.

So similarly, the force also can be written, that force due to this axial component of the flow which as before will be the drag force only, which I have already evaluated and again force due to this cross flow component. Now, we have earlier seen that the cross flow solution is given as, the cross flow solution is given as Cos theta d phi a d r that is once the axial flow solution is known, we can straight away get the cross flow solution.

Now, in this context you may remember that, this axial contribution that is phi a which the solution for asymmetric flow represents potential due to a source distribution, phi a this represents potential due to a source distribution (No audio from 03:47 to 04:06). Now, since the cross flow potential is simply that a derivative of this cross flow potential, which is potential due to a source and we know that the derivative of potential due to a source represents potential due to a doublet.

So, this implies d phi a d r is potential due to doublet (No audio from 04:42 to 04:59) and since this cos theta d phi a d r (No audio from 05:02 to 05:35), so cos theta d phi a d R can also be written as d phi a d z, which implies that this doublet distribution has it is axis along z (No audio from 05:51 to 06:01). So, d phi a d z this is potential due to the doublet with axis along z (No audio from 06:31 to 06:44) or we get that the cross flow potential, then is represented by a doublet distribution with axis along z and as we have seen here, that the axial flow do not produce any lift force but, produces on of course, drag force in this case but, no lift.

And from this cross flow it is obvious that this cross flow is going to produce even a lift and we come back to our known fact from incompressible flow, that a source distribution can be used to represent a non-lifting flow, while to model or represent a lifting flow a doublet distribution is require for the doublet axis is normal to the flow direction.

Now, to carry on further let us say the velocity at any point velocity at any point on the flow at any point, say P is given by V which is the undisturbed axial component of the free stream plus perturbation along x along the axial direction plus the cross flow component or in the radial direction plus perturbation component in the radial direction plus tangential component or swapped component of free stream as well as the perturbation. This is of course, V square and we have already that U a is U infinity cos alpha and U c is U infinity sin alpha are the cross flow and axial component of the undisturbed stream.

Now, we will make an assumption that alpha is small (No audio from 09:49 to 09:58) and consequently this term wise, let us say term wise U a plus d phi d x we have not yet split it in two part that is axial and cross cross flow potential but, anyhow we will subsequently do it and let us say we are interested in the body, that is on the surface, this of course, is independent of the body coordinate (No audio from 10:39 to 10:57) on the body surface.

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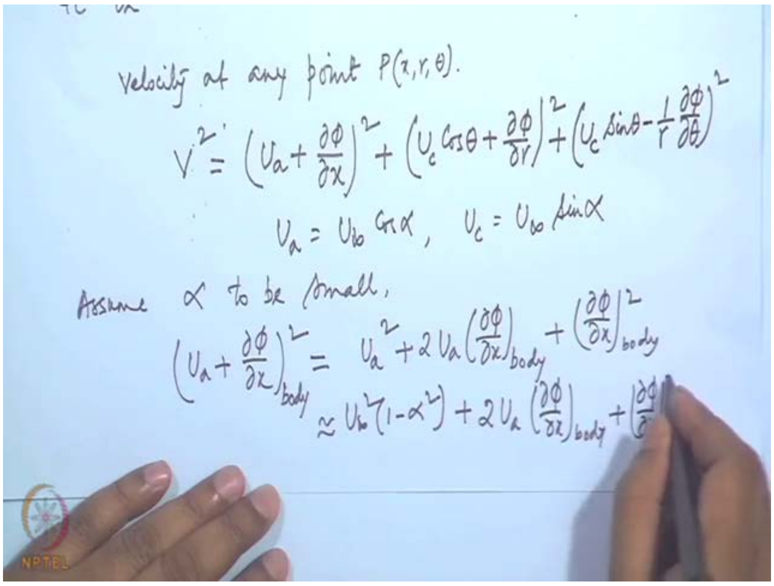

Now, U a we substitute U infinity cos alpha and making small assumption Cos square alpha can be written as 1 minus alpha square, so this term becomes (No audio from 11:14 to 11:30) plus these are written as it is (No audio from 11:34 to 11:53) but, the radial component similarly, U c Cos theta plus d phi d r and this can be written as U infinity d r d x square this we obtained using boundary condition (No audio from 12:42 to 12:53). Now, we have already seen that for slender bodies (No audio from 13:00 to 13:10) this phi c which is cos theta d phi a d r and which for general slender bodies, we have seen is given by f x by r cos theta.

However, if you remember that for a cross flow for the axial flow, this effect represented the source distribution, however now for phi c this must represent the doublet distribution. And we now give them standard notation for doublet strength, that is mu x cos theta (No audio from 14:06 to 14:16) which is doublet strength distribution (No audio from 14:18 to 14:29) and as we have seen that this source strength, in case of the axial part of the flow can be evaluated, using the boundary condition and which relates to the radius of the cross section or the cross sectional area.

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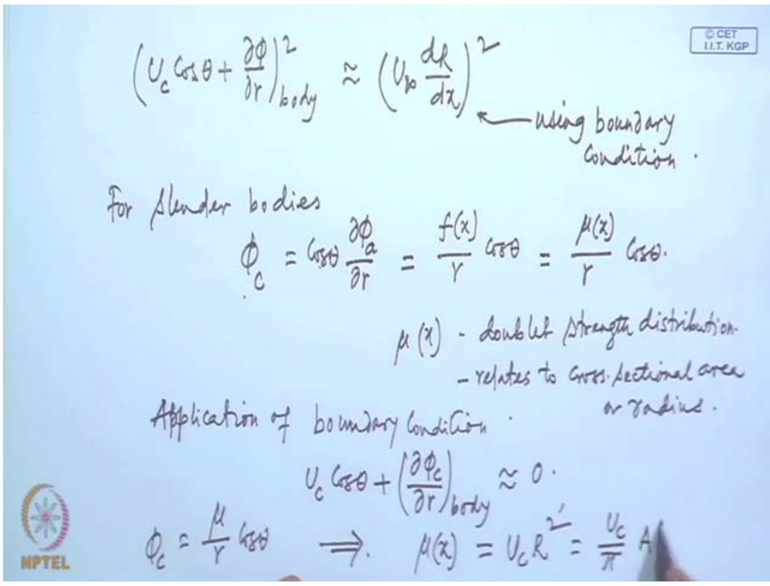

Similarly this doublet strength will also depend on the boundary condition or can be evaluated from application of the boundary condition and it must also relate to the cross sectional area of the and it relates to cross sectional area or or radius. Now if we apply the boundary condition (No audio from 15:39 to 15:53) application of boundary condition gives us that U c Cos theta plus d phi c d r on the body is approximately 0.

And if substituted phi c equal to this term, so phi c equal to mu by r cos theta then gives us that mu x is U c into r square which is U c by pi a whereas, before the cross sectional area at a particular station x (No audio from 17:22 to 17:33) and this then gives the cross flow potential to be mu cos theta by r, that is U c r square by r cos theta and U c we can write as U infinity (No audio from 18:07 to 18:29).

So, third term in that expression for velocity, so hence third term in v square which we operate an earlier is U c sin theta minus 1 by R d phi d theta, we have replaced the variable R by the surface radius and d phi d theta. Since phi has no theta dependency this d phi d theta is simply d phi c d theta which it can be evaluated from here and this gives 2 U c sin theta square that is 2 U infinity sin theta square into alpha square for small alpha (No audio from 19:57 to 20:10).

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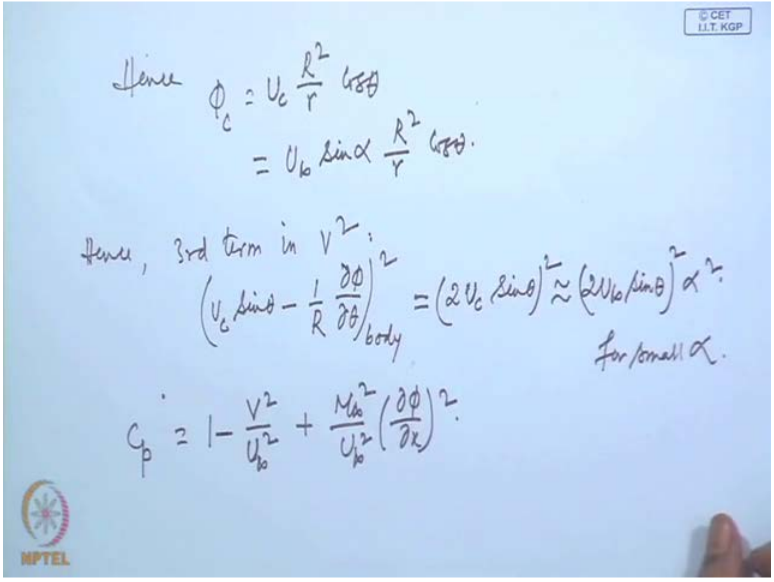

Now, knowing the velocity on the surface of the body, we can evaluate the pressure coefficient and for that let us go back to that expression for pressure coefficient (No audio from 20:32 to 20:44). Now the expression for the pressure coefficient, which we have derived earlier can be written in a more convenient form (No audio from 20:55 to 21:05) can be written in more convenient form that is 1 minus (No audio from 21:10 to 21:21) plus (No audio from 21:23 to 21:39) which is up to second order accuracy as is required for axial bodies of revolution or asymmetric flow cases.

Now, this expression can then be written as on the body surface (No audio from 22:04 to 22:18) minus 2 by U infinity d phi d x on body plus alpha square minus d R d x square minus 4 alpha square sin square theta plus beta square by U infinity square d phi d x square in the body (No audio from 23:06 to 23:16). And this is much smaller than the other term, so this can be neglected this term can be neglected, now this can also be written as that an axial contribution plus a cross flow contribution.

And axial flow contribution and a cross flow contribution and where this axial flow contribution is minus 2 by U infinity d phi a d x on body (No audio from 24:14 to 24:24) and c p c has minus 2 by U infinity (No audio from 24:33 to 24:52).

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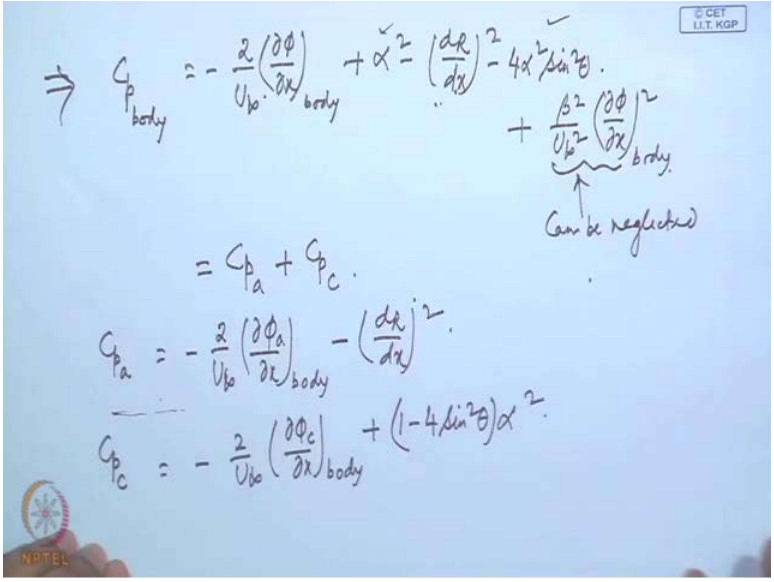

So, see this this and this (Refer Slide Time: 24:57) are just cross flow contribution and which comes only because of the body is in an angle of attack, this is of course, the familiar axial flow contribution and this has a combination of axial flow contribution and cross flow contribution. And now this axial contribution to the pressure coefficient can give only a drag force as we have seen before (No audio from 25:35 to 25:45), so C p a this gives rise to (No audio from 25:59 to 26:15) and this part we have already evaluated, which is that C d 1 we which is evaluated in the last class that comes from C p a.

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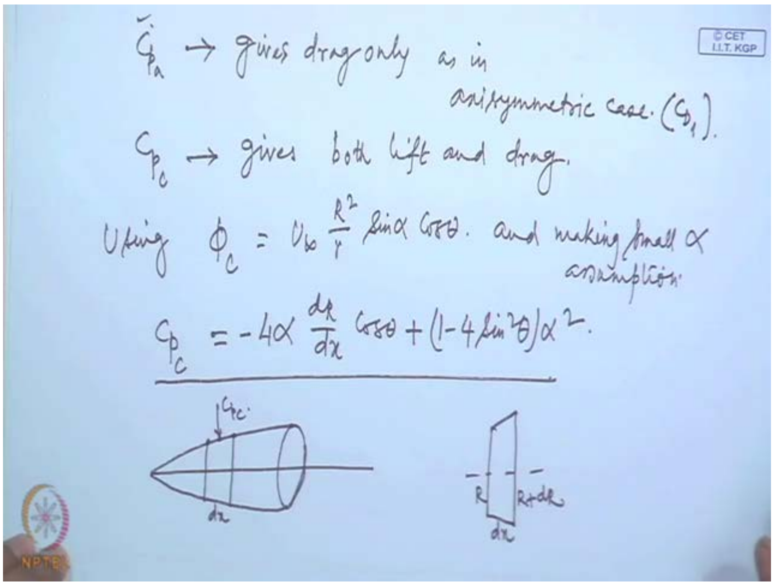

However this C p c the cross flow contribution (No audio from 26:35 to 26:46) it gives (No audio from 26:47 to 26:57) gives both lift and drag, now using that expression for phi c which we have already derived, which is (No audio from 27:14 to 27:24) r and and making small alpha assumption (No audio from 27:38 to 27:55), that C p c can be completely determined which is minus 4 alpha d R d x cos theta plus 1 minus 4 sin square theta to alpha square, now this is the pressure distribution, that or the cross flow contribution of the pressure coefficient that acts on the body.

So, once again considering a small segment of (No audio from 28:40 to 29:19) we will only consider the C p c because this is already determined and as (No audio from 29:28 to 30:00). Now because of this pressure distribution, there will be component of force which is in the normal to the surface area and which can be projected on different direction.

So, the radial component of the force force (No audio from 30:33 to 31:10) on the elemental surface area is half rho infinity U infinity square C p c R d theta d x this implies the cross force the cross force that is force in the direction of U c is half rho infinity U infinity square 0 to L (No audio from 32:15 to 32:33) this is what is the (No audio from 32:36 to 32:46) elemental area projected normal to radius.

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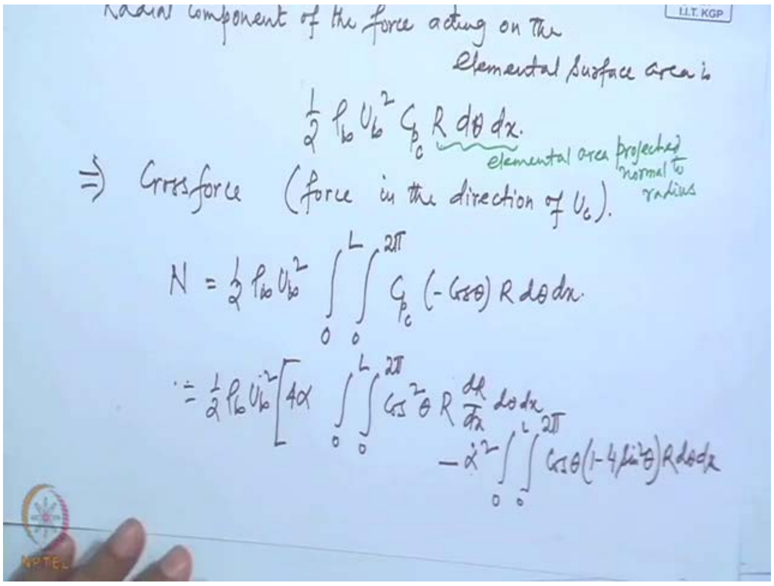

Now, this we can now substitute the expression for C p c here which gives half rho infinity U infinity square into 4 alpha 0 to L 0 to 2 pi 4 square theta into R d R d x d theta d x minus another term this half rho infinity square; we can keep like this minus alpha square into 0 to L 0 to 2 pi cos theta into 1 minus 4 sin square theta r d theta d x (No audio from 34:28 to 34:40).

The integration can be carried out completely giving N equal to half rho infinity U infinity square into alpha sorry into 2 alpha into A B, A B is as as before area of base (No audio from 35:14 to 35:28) area at L to define a normal force component again based on the base area, so that half rho infinity u infinity square A B equal to 2 alpha (No audio from 35:53 to 36:02).

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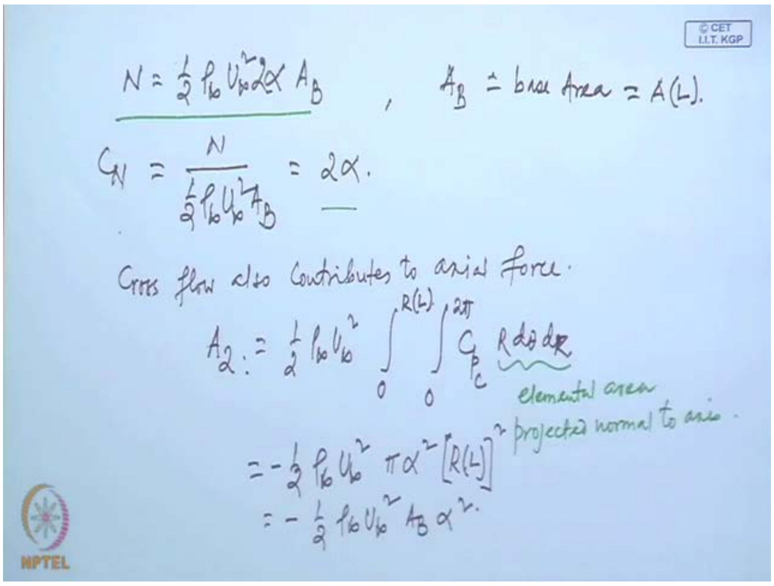

So, this is what is the normal force contribution by the cross flow part of the pressure this cross flow also contributes to an axial force, now cross flow also contributes to axial force so we see that the axial force contributes only axial force but, cross flow contributes both a cross force as well as an axial force. And the axial force due to the cross flow lets denote it by A 2 where that drag force or which we may, now call in a generalized tension axial force, due to axial contribution as A 1, then we get A 2 equal to half rho infinity U infinity square into 0 to R L, 0 to 2 pi C p c R d theta d R where R d theta d R is the (No audio from 37:50 to 38:04) elemental area projected normal to axis (No audio from 38:10 to 38:22).

And once again if we substitute this C p c the cross flow pressure, the result comes out to be minus half rho infinity U infinity square into pi alpha square R L square or R B square (No audio from 38:58 to 396:18) and (No audio from 39:20 to 39:30) and axial force coefficient C A 2 A 2 by half rho infinity u infinity square A B is minus alpha square the C A 1 is eventually that C d 1 that (No audio from 40:08 to 40:21), we have asymmetric flow case.

So, when a body is in a flow which is not along its axis or that is body of revolution, we see that there is two contribution to the axial force one coming from the axial component of the force flow and another is coming from the cross flow component of the flow and there is on normal force component coming from the cross flow. So net axial force (No audio from 41:08 to 41:24) A 1 plus A 2, which can be written as half rho infinity U infinity square A B (No audio from 41:50 to 42:00) this is as we remember C d 1 plus C p b minus (No audio from 42:12 to 42:22).

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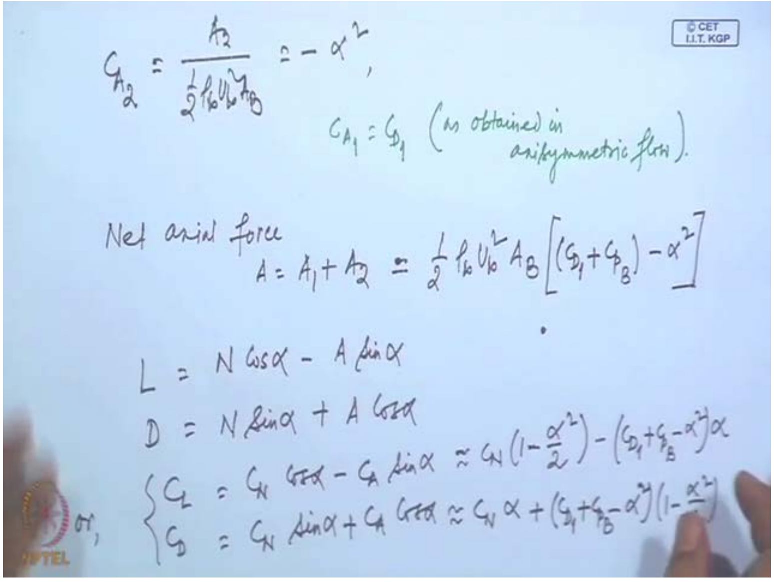

Now, the lift and drag forces can be determined, once the normal and axial forces are known, so the lift force L is N cos alpha minus A sin alpha and the drag force is N sin alpha plus A cos alpha or in terms of coefficients C L is C N cos alpha minus C A sin alpha and C D is C N sin alpha plus C A (No audio from 43:28 to 38:45) and making the usual small angle of at approximation this can be written as C N into 1 minus alpha square by 2 minus (No audio from 44:05 to 44:16) C D 1 plus C p B minus alpha square into alpha (No audio from 44:24 to 45:07).

Now, we can make further approximation to this approximation and and substitute C N equal to 2 alpha, so if we (No audio from 45:27 to 45:35) if we retain only dominion terms (No audio from 45:39 to 45:55) C L equal to 2 alpha and C D equal to C D plus p B plus alpha square. The alpha is of course, in radian now looking to this we can say that the drag force coefficient alpha square that has arises only due to the cross flow contribution or cross flow contribution also produces the lift; so this part of the drag so this part of the drag is due to the cross flow and hence due to lift.

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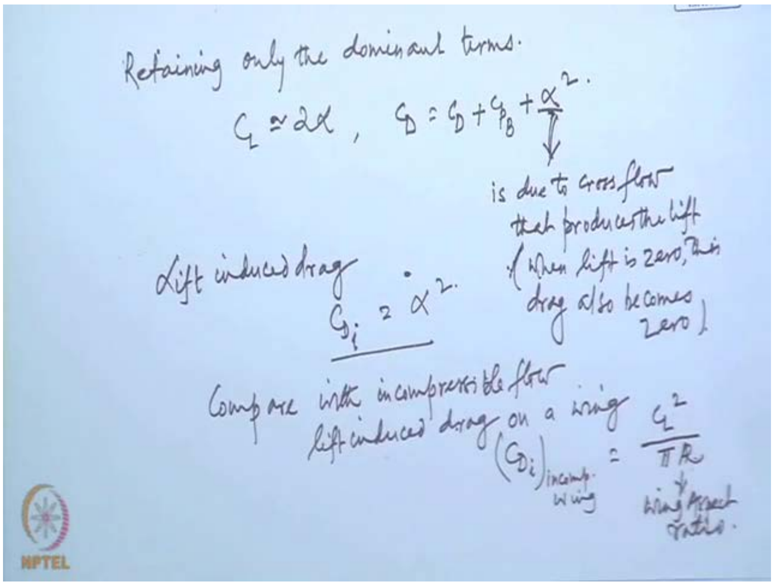

So, this is this term is due to cross flow that produces the lift and you see when lift is 0 this also becomes 0, hence this is called lift induced drag; induced drag as alpha square (No audio from 48:14 to 48:38) this drag also becomes 0 (No audio from 48:40 to 48:57) and of we compare this with where incompressible flow compare this with incompressible flow (No audio from 49:05 to 49:40) incompressible wing (No audio from 49:43 to 49:55) C L square by pi aspect ratio, where this is wing aspect ratio (No audio from 50:00 to 50:15). And even if we consider that in incompressible flow that C L according to the linearised theory over a flat plate becomes 4 pi alpha square; so there is an aspect ratio term present here which was not here (No audio from 50:36 to 50:54).

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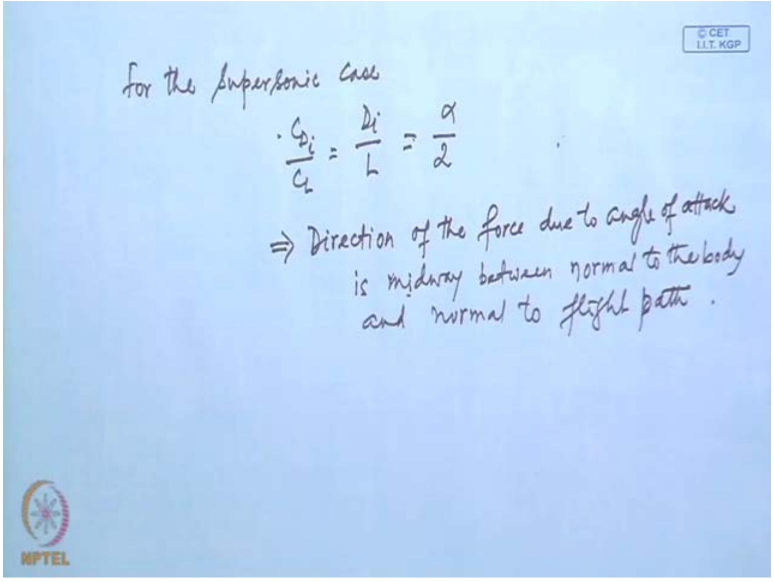

Now, we also find that the for the supersonic case (No audio from 50:58 to 51:11) we have C d i by C L that equal to D i by L is alpha by 2, so that implies the the direction of the force vector due to angle of attack angle of attack is midway between normal to the body and normal to flight path.

So, that completes our discussion on the forces that are acts on a body of revolution either in axially symmetric flow or in you had a case, we have seen that when the body is in an axially symmetric flow, then only a drag force acts; which of course, depends on the variation of the cross sectional area of the body, while if there is a cross flow present as in the asymmetric axis non-asymmetric flow case we have both axial and normal forces so the total axial force.

In a general case comes from two sources the asymmetric component of the flow and as well as from the cross component of the flow, however the normal force comes only from the cross component of the flow. And we also have seen that the induced drag, in this case is or induced drag coefficient is simply square of angle of attack expressed in radian and the lift force approximately is two times the angle of attack expressed in radian.

We also have seen that this cross flow potential can very easily be obtained from the axial flow potential eventually this cross flow can be modeled by or is represented by a doublet distribution along the axis, while the symmetric flow is represented by source distribution along the axis. Which confirms our incompressible flow analysis earlier, that non lifting flow can be modeled by our source distribution, while our lifting force is to be modeled by a doublet distribution with doublet axis, normal to the free stream reaction; and that completes our force evaluation force computation on bodies of revolution.