High Speed Aerodynamics

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Module No: # 01

Lecture No: # 35

Similarity Rules for High Speed Flows (Contd.)

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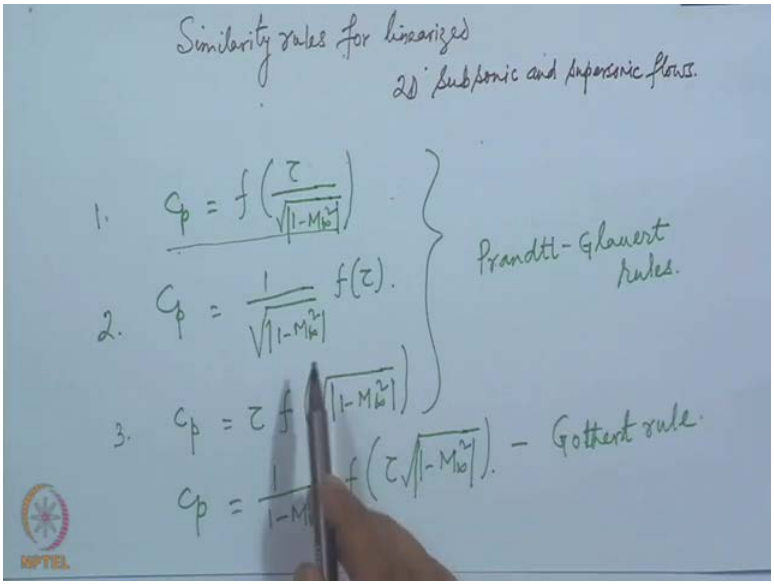

See, in our earlier lecture, we discussed about the similarity rules for linearized 2 D subsonic and supersonic flows and where we had an arbitrary A, and for various choices of A, we got different rules. And in particular, we choose four different values of A and obtained four different similarity rules which are; that c p is function of tau by root over 1 minus M infinity square, and as we mentioned that, this is valid for both subsonic and supersonic flow if we make this term sign independent. And for another choice of A, we obtained ((No audio from 01:13 to 01:28)) tau, and for another case, we had tau function of root over 1 minus M infinity square. And all these as you mentioned that they are called Prandtl Glauert rules. ((No audio from 01:51 to 02:07))

We also had a Gothert rule which gives c p as 1 by 1 minus M infinity square function of tau root over 1 minus M infinity square; which is called the Gothert rule. Now, let us see what these rules in particular say.

Let us consider this first rule, that is, this particular, this simply say that the pressure coefficient on an airfoil or a two dimensional geometry in subsonic and supersonic flow is function of tau by root over square 1 minus M infinity square absolute.

It simply says that if this parameter, that is, tau divided by square root of absolute 1 minus M infinity square is invariant, then c p also remain invariant, that is, for airfoil if we consider airfoil, airfoil of same family, if the thickness varies in such a way that t by 1 minus M infinity square is constant, then the pressure distribution on the airfoils will remain constant.

Meaning; let us consider a subsonic flow. We know in subsonic flows, with increasing M infinity, this says that thickness will increase as proportional to 1 by root over 1 minus M infinity square. And this rule then says that as the mach number; free stream mach number increases in such a fashion that the thickness is also changed by this factor or by this multiple 1 by root over 1 minus M infinity square, then the pressure distribution will remain constant.

And see that for a same family of airfoils, we can have pressure distribution over different thickness at different mach number if we know the pressure distribution on a particular thickness at a particular mach number. So, see that is the integrity of this first rule.

The second rule clearly says; for a given member of a particular family of say, the pressure coefficient increases with M infinity as 1 minus M infinity square to the power minus half or as 1 by root over 1 minus M infinity square, that is, if we have a fixed airfoil about particular family, then as free stream mach number increases, the pressure coefficient on that or the pressure distribution on that airfoil increases by a multiple of 1 minus M infinity square to the power minus half.

So, you see that if you have a particular airfoil, let us say a four digit symmetric section NACA 0 0 1 2; we know its pressure coefficient at say an incompressible flow which corresponds to mach number 1, then for the same airfoil, at a different mach number, the pressure coefficient is obtained simply multiplying that incompressible flow pressure coefficient by the factor 1 minus M infinity square to the power minus half.

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So, we see that this is also a utility of this form of the Prandtl Glauert relation that for same particular airfoil, if we know the pressure coefficient at an incompressible flow or at any low mach number, we can get it at other mach number simply by multiplying the pressure coefficient at any particular point with the factor 1 minus M infinity square to the power minus half.

The third rule can be interpreted that the pressure distribution or the pressure coefficient at any point is proportional to the thickness, if the mach number remain fixed. So, for a fixed mach number, the pressure coefficient at any point is simply proportional to the thickness.

Finally, the Gothert rule; it says that c p increases with mach numbers as 1 minus M infinity square to the power minus 1, if the thickness also increases with as 1 minus M infinity square to the power minus half; that is, the c p will increase as a multiple of 1 by 1 minus M infinity square, if the thickness increases as multiple of 1 minus M infinity square to the power minus half.

That is, for a given shape or given family of shape, if the thickness increases by 1 minus M infinity square to the power minus half, then the pressure distribution on that same geometry will increase by a factor of 1 minus M infinity square to the power minus 1. As you will see later that Gothert rule applies to axial symmetric flow as well.

We must remember that these similarity rule which are valid for linearized 2 D subsonic and supersonic flows, become less accurate as we approach the transonic range or the hypersonic range. Since in both the cases, that is, in transonic flow as well as in hypersonic flow, the linearized form of the equation is not valid, and consequently the accuracy of the similarity rules either the Prandtl Glauert rules or Gothert rules also diminishes as we approach 1 of these ranges. So, to get a reliable result, this rule must be applied for only those mach numbers which are strictly in the subsonic or supersonic range. ((No audio from 09:56 to 10:15))

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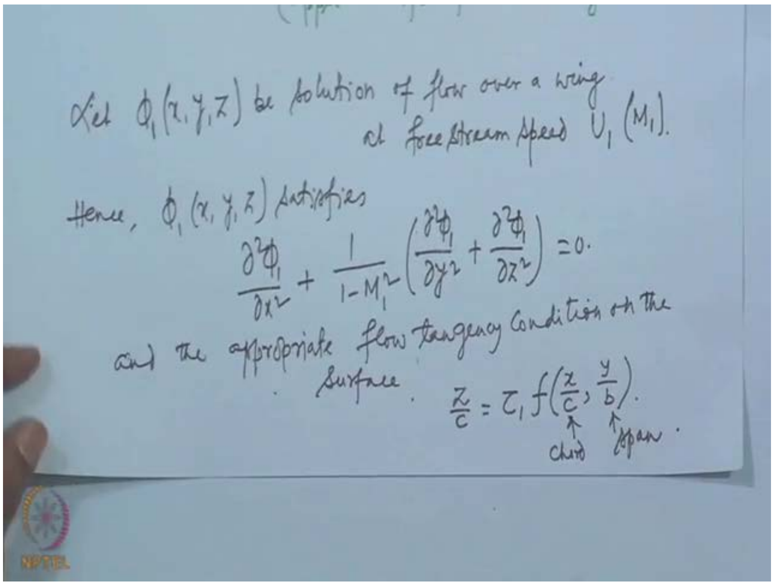

Let us now consider the three dimensional planar flows, ((No audio from 10:20 to 10:32)) similarity rule for 3 D planar flows; which are quite important because these are applicable for flow over wings. ((No audio from 10:55 to 11:10))

Let the three dimensional planar boundary; let us say that let phi 1 x y z be solution of flow over a wing, flow over a wing at free stream speed u 1 which of course, corresponds to mach number M 1. So, phi 1 x y z; this satisfies ((No audio from 12:21 to 12:37)) ((No audio from 12:46 to 13:10)) the governing equation which can be written and also the appropriate flow tangency condition, flow tangency condition on the surface which is now given as ((No audio 13:39 to 14:23)), where c is chord and b is span; c is chord and b is span and tau 1 is as before, the thickness ratio.

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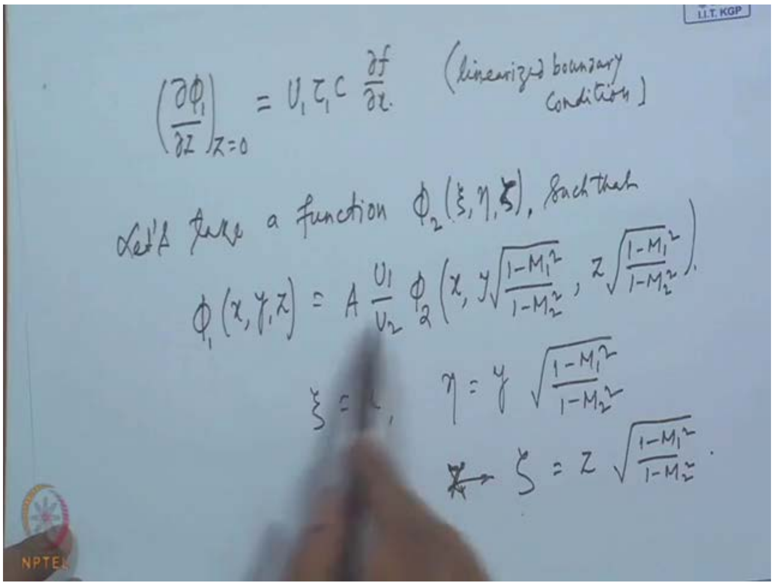

And then the boundary condition is or the linearized boundary condition; the linearized boundary condition. ((No audio from 14:41 to 15:35))

Now, as before, let us consider a second function, let us take a second function, function phi 2 xi eta zeta, say xi eta zeta, such that as before, we have phi 1 x y z is equal to A into as before A into u 1 by u 2 phi 2 x y into root over 1 minus M 1 square by 1 minus M 2 square and z is also...

That is, we are applying this transformation that xi equal to x eta equal to y into root over 1 minus M 1 square by 1 minus M 2 square, and z, sorry zeta equal to z into root over 1 minus M 1 square by 1 minus M 2 square.

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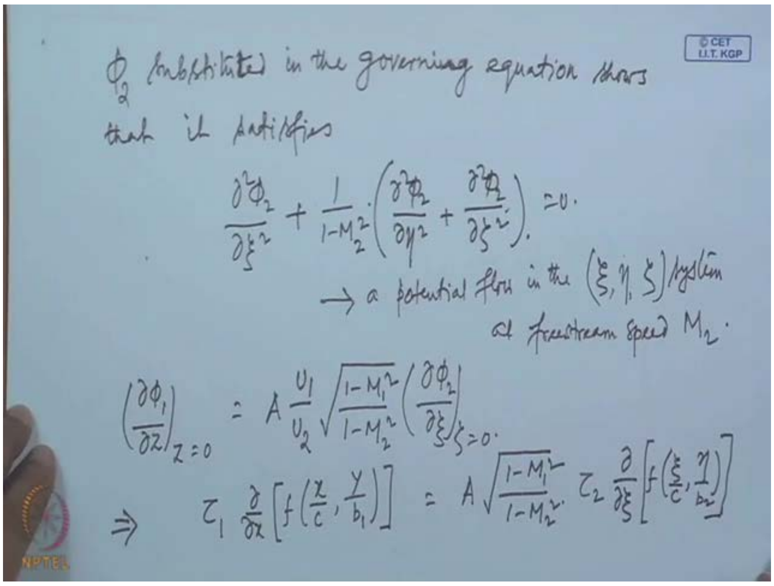

Now if we substitute this in the governing equation, ((No audio from 18:01 to 18:14)) so, phi 2 substituted, phi 2 substituted in the governing equation. ((No audio from 18:27 to 18:49)) so that it satisfies it satisfies... ((No audio 18:58 to 19:34)). That is, it satisfies a potential flow in the xi eta zeta system; a potential flow in the xi eta zeta space at free stream at free stream speed M 2. So, phi 2 satisfies the potential equation or the potential flow at M 2. Now if we try to relate the boundary conditions, what we get is d phi 1 d x at z equal to 0 is same as A into u 1 by u 2 root over 1 minus M 1 square by 1 minus M 2 square d phi 2 d xi, at zeta equal to 0.

Now, if we equate the boundary conditions, if we equate the boundary conditions, ((No audio from 21:33 to 21:44)), tau 1 d dx of function of x by c into y by b 1 equal to A into root over 1 minus M 1 square by 1 minus M 2 square into tau 2 d d xi f of xi by c and eta by b 2.

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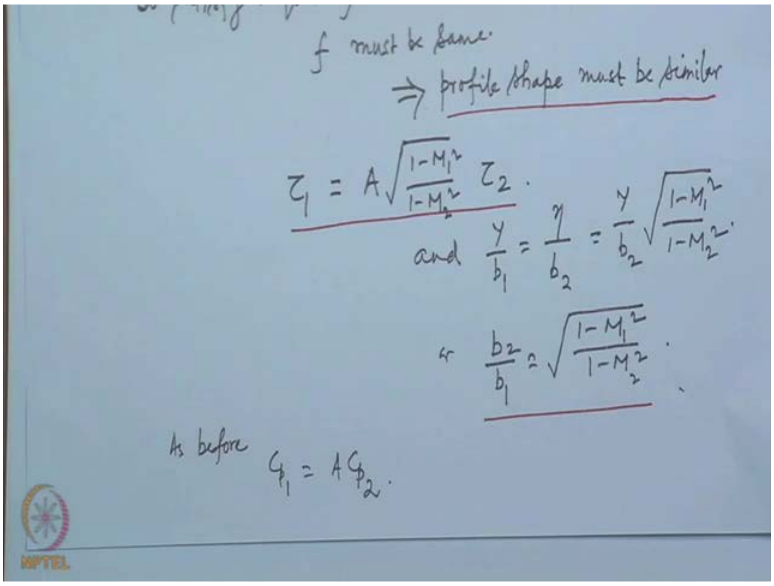

Now this equality will hold, this equality will hold if first of all f is same. So, to satisfy the equality, f must be same which actually implies that the profile shape must be same; must be similar, that is, the wing profile must be similar, the two wings must be of similar.

We also have that tau 1 must be A into root over 1 minus M 1 square 1 minus M 2 square into tau 2, as well as y by b 1 equal to eta by b 2 which is y by b 2 root over 1 minus M 1 square by 1 minus M 2 square or b 2 or b 2 by b 1 equal to root over 1 minus M 1 square by 1 minus M 2 square.

So, see that the phi 2; the way we have chosen or the transformation that we have applied will satisfy another potential flow over body which has same profile shape whose thicknesses are related by these relation and whose span are related by this relation. So, we see that the flow are similar, that the solutions are similar, if the geometry is of similar, the profile shape are similar, the two thickness of the two different planar body or the two wings relate by this relation, and the span of the two wings relate by these relation.

Now the pressure coefficient as before, we have c p 1 equal to A c p 2, and in this case also, we have seen that the A remains arbitrary. It has not been possible to find a specific value for A. So, whatever choice we can have for A, the result will be same and this is also follows from the same fact that the governing partial differential equation in this case is also homogeneous.

So, any solution multiplied by a constant coefficient remains the solution as far as the boundary condition is concerned. So, A is arbitrary. So, once this is known, then we can have the similarity rule. ((No audio from 27:57 to 28:07))

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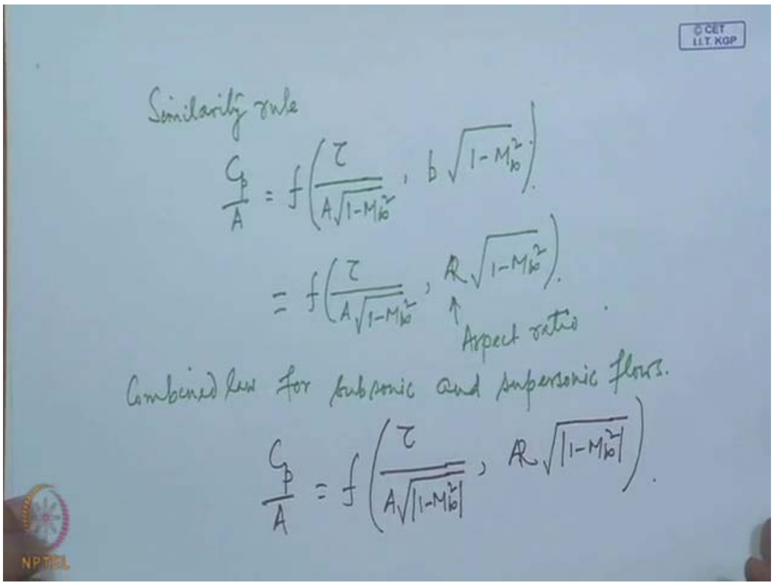

So, the similarity rule is c p by A is a function of tau by A into 1 minus M infinity square into b into root over 1 minus M infinity square. Since b is proportional to aspect ratio, this can be written as ((No audio from 29:05 to 29:31)) and this is the aspect ratio. And combined law for subsonic and supersonic flows is c p by A as function of tau A absolute of 1 minus M infinity square as well as aspect ratio. ((No audio from 30:40 to 30:53))

Here also you can have different choices for A as before, and the resulting rules can be interpreted accordingly. However, we will be seeing that there are two parameters here; the thickness ratio as well as the aspect ratio, and both are to be adjusted accordingly.

So, as one particular rule, the first rule of first Prandt Glauert rule in the earlier case, that c p remains invariant over the profile, if tau by square root of 1 minus M infinity square remain invariant. Same rule applies here also; if tau by root over 1 minus M infinity square and aspect ratio into square root of 1 minus M infinity square; both parameter remain constant, that is, as mach number increases, if we increase the thickness following this and reduce the aspect ratio following this, then the pressure coefficient on the wings surface will remain constant. And once again, you can see that knowing the pressure distribution for a wing at a particular mach number can give us the pressure distribution at different mach number or for a different thickness on different aspect ratio wing at the same mach number. ((No audio from 32:33 to 32:51))

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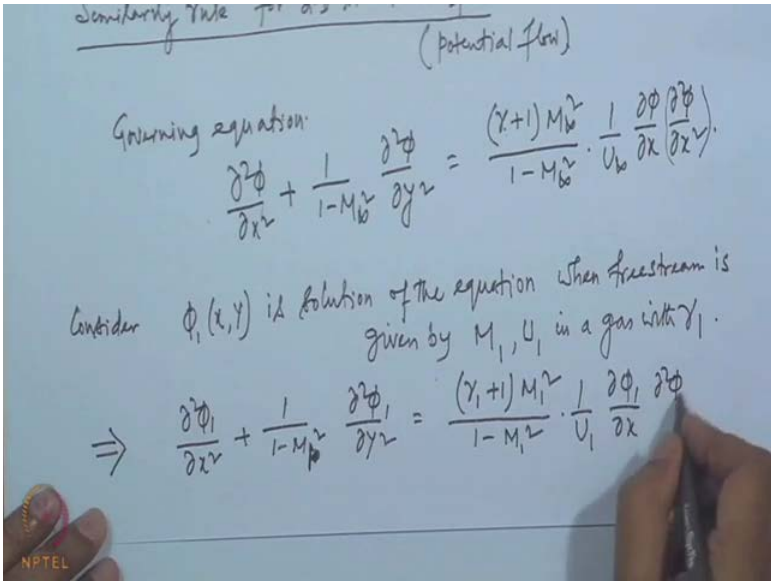

Let us now consider the similarity rule for transonic flow in two dimensional case. ((No audio 33:02 to 33:41)) 2 D transonic flow of course, linearize potential flow; small disturbance potential flow. You may recall the governing equation, the governing equation we had d 2 phi d x square plus 1 by 1 minus M infinity square d 2 phi d y square is gamma plus 1 into M infinity square by 1 minus M infinity square into u by u 1 u by u infinity or let us say, 1 by u infinity d phi d x d 2 phi d x 2.

You may recall that in deriving the small perturbation equation, we mentioned that when M infinity is close to 1, this coefficient of d d 2 phi d x 2 on this right hand side is comparable to the coefficient of d 2 phi d x 2 on the left hand side, and consequently they are comparable and are not negligible. This is true when M infinity is close to 1. Consequently, we see the equation is non-linear here and closed form solution is not readily available. So, in this case, this similarity rule will have much more usefulness than they are in case of subsonic and supersonic flow.

Now to derive the similarity rule, we will follow the same steps; that is, first of all, we will consider a particular solution of this flow over our chosen geometry, and then we will consider a second function related to this first flow solution which is of the same form, and then see that what are the under what conditions that second function will be a solution of this governing equation. Once we obtain those conditions required so that the chosen function is also a solution of this equation, we will try to frame the similarity rule.

So, first of all, we will consider a solution corresponding to free stream speed of u infinity or M infinity in a gas since in this case, the gas itself is also present in the equation. So, we have another additional option or you can change the gas also.

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So, first step, consider phi 1 x y is solution of the equation when free stream is given by M 1 u 1 in a gas with gamma 1. Since phi 1 is a solution for this equation corresponding to M infinity equal to M 1 u, infinity equal to 1 and gamma equal to gamma 1, we have that these phi 1 satisfy this equation that d 2 phi 1 d x square plus sorry 1 minus M 1 square d 2 phi 1 d u y square equal to gamma 1 plus 1 into M 1 square by 1 minus M 1 square into 1 by u 1 d phi 1 d x d 2 phi 2 d x square.

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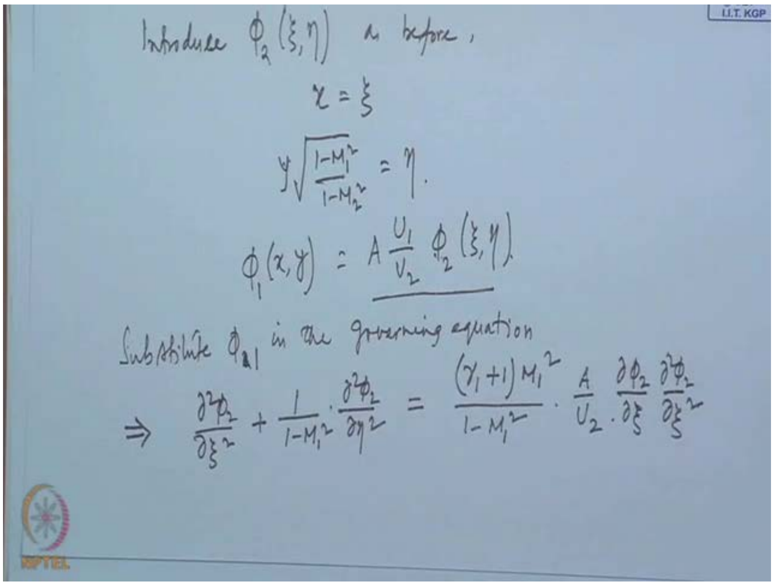

Again introduce that phi 2, introduce phi 2 xi eta as before, that is, x equal to xi sorry and y root 1 minus M 1 square by 1 minus M 2 square equal to eta, and phi x y equal to A into u 1 by u 2 phi 2 xi eta, sorry phi 1.

Now, if we substitute this phi 2 in the governing equation, substitute phi 2 in the governing equation; ((No audio from 40:36 to 4103)) oh sorry substitute this phi 1 in the governing equation and what we get then, get that that is, your substitutive this expression for phi 1 in this particular equation, putting this phi 1 here, what we are getting is d 2 phi 2 d xi square by 1 by 1 minus M 1 square d 2 phi 2 d eta square into gamma 1 plus 1 square by 1 minus M 1 square into A by u 2 d phi 2. ((No audio 42:26 to 43:21))

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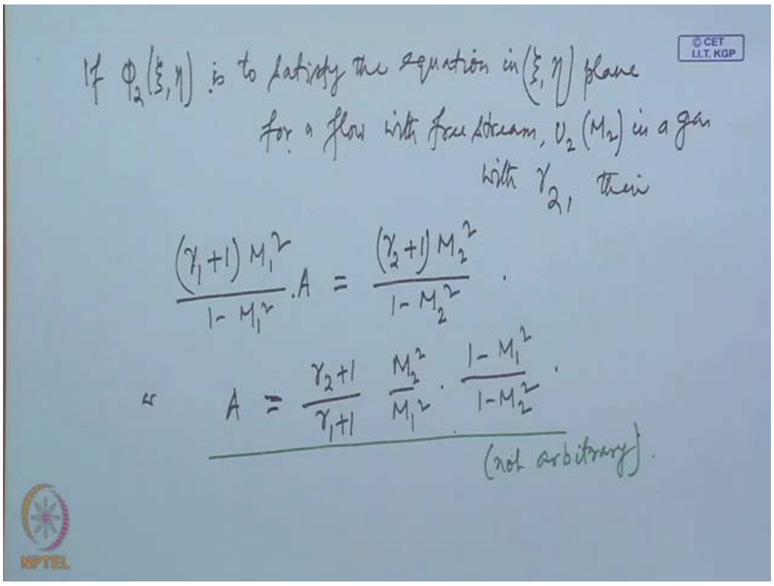

Now, if now see that the equation that phi 2 satisfies, the equation that phi 2 satisfies is not exactly the equation for, not exactly the governing equation corresponding to the free stream speed u 2. It is not exactly the small disturbance small disturbance equation for transonic flow, not exactly the small disturbance equation; however, if we want that phi 2 is a solution of the transonic small disturbance equation, so, if phi 2 xi eta is to satisfy the equation in xi eta plane for a flow with free stream u 2 M 2 in a gas with gamma 2, then this coefficient on the right hand side that must be equal to gamma 1 plus 1 into M 1 square by 1 minus M 1 square into A should be gamma 2 plus 1 into M 2 square by 1 minus M 2 square. So, if or A must be gamma 2 plus 1 by gamma 1 plus 1 into M 2 square by M 1 square into 1 minus M 1 square by 1 minus M 2 square.

So, we see now that A is no longer arbitrary. If we want this choice of our second function phi to xi eta is to be a valid potential function, then this A must be this. ((No audio from 47:20 to 47:38)) So, it is not arbitrary in this case or other way that if we have a solution phi 1 for a transonic plot M 1, and then we choose another functions similar to this function phi 1 and we want it to be a solution at M 2, then the A is no longer arbitrary as in case of a subsonic or supersonic flow, or the governing equation was linear.

In this case, this A must have this specific value so that the similar choice can also be a solution. If we consider the boundary conditions, ((No audio from 48:25 to 48:37)) as far as the boundary conditions are concerned, you see there is no change at all with the two dimensional subsonic or supersonic flow and these transonic flow. So, the boundary condition or the result that you obtain from the consideration of boundary condition will remain the same.

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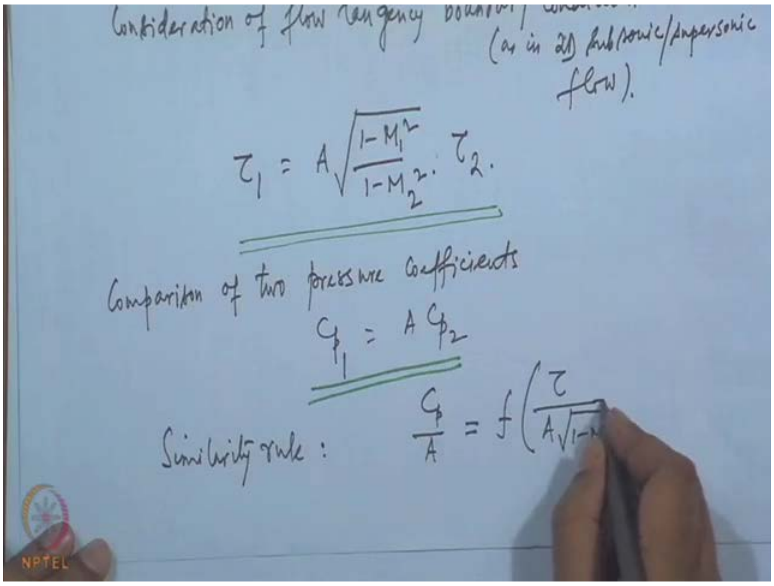

So, you can say that consideration of flow tangency boundary condition ((No audio 49:00 to 49:35)) will again give us that tau 1 equal to A into root over 1 minus M 1 square by 1 minus M 2 square into tau 2 which happens to be the same relation, but the A is no longer arbitrary.

Similarly, if we compare the pressure coefficient, ((No audio from 50:23 to 50:45)) remember we had c p 1 equal to A c p 2, or other way that two transonic flow will be similar, and their pressure coefficient will be related by this relation, if the two thicknesses are related by this relation, and the coefficient A is given by the relation that you have obtained.

So, in this case that the two solutions are similar if A is the specific that we have obtained and the two thickness are related by this relation, and in such a situation, the pressure coefficient will be related by this relation; c p 1 equal to A c p 2. ((No audio from 51:42 to 51:53))

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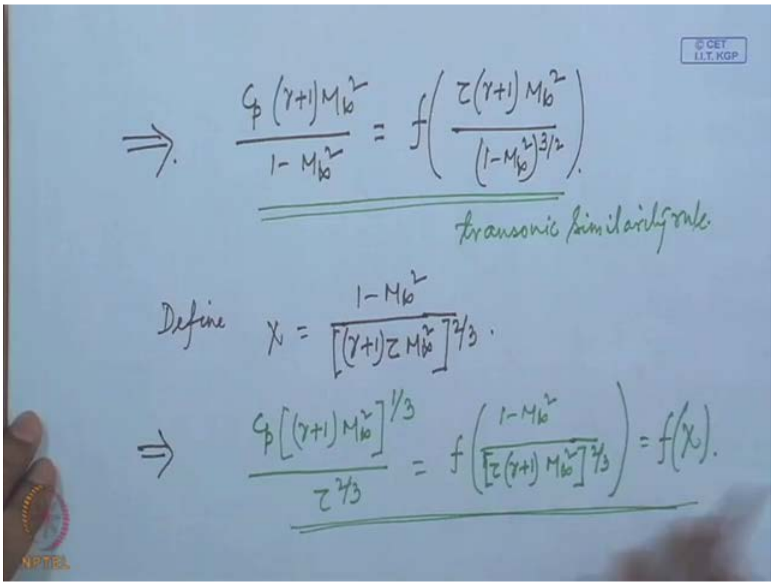

So, the similarity rule, the similarity rule gives c p by A. Before c p by A equal to function of tau by A root 1 minus M infinity square, and substituting for A, we have, for c p, you substitute gamma plus 1 M infinity square by 1 minus M infinity square is function of tau into gamma plus 1 M infinity square by 1 minus M infinity square to the power 3 by 2. So, this is what the transonic similarity rule is. This is the transonic similarity rule.

Now, we defined parameter chi which is 1 minus M infinity square by gamma plus 1 into tau M infinity square to the power 2 by 3. And multiply both side by this, then we have c p multiplied by gamma plus 1 M infinity square to the power 1 by 3 by tau to the power

2 by 3 equal to function of 1 minus M infinity square by tau into gamma plus 1 M infinity square to the power 2 by 3. That is a function of chi.

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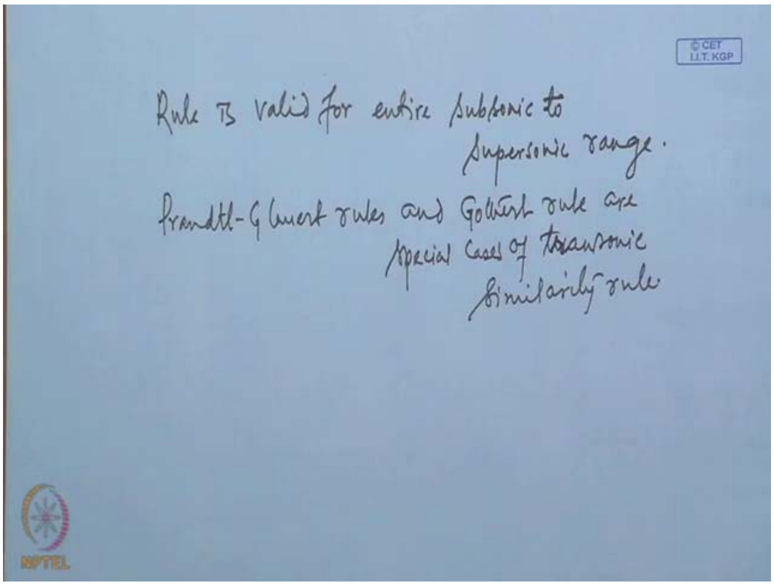

So, this is what is written as the final form of the transonic similarity rule, and what we can see is that this rule of course is valid for the entire range of subsonic to supersonic flow. A rule is valid for entire subsonic to supersonic range. So, this particular rule is valid even for subsonic as well as supersonic flow. This is in contrast to that linearized rule which is not valid for the transonic flow, but this transonic rule is also valid for subsonic and supersonic flow which is quite easy to comprehend because the subsonic and supersonic flow equations are just a special case of this particular equation. So, that Prandtl Glauert rules and Gothert rule; these are all special cases of transonic similarity rule.

Now, we will see that how can we interpret this particular rule, that is, the subsonic and supersonic rules in our next lecture. So, we have derived the similarity rule, transonic flow which is the most useful similarity rule and; however, we will discuss about this rule in the next lecture.