High Speed Aerodynamics

Prof. K. P. Sinhamahapatra

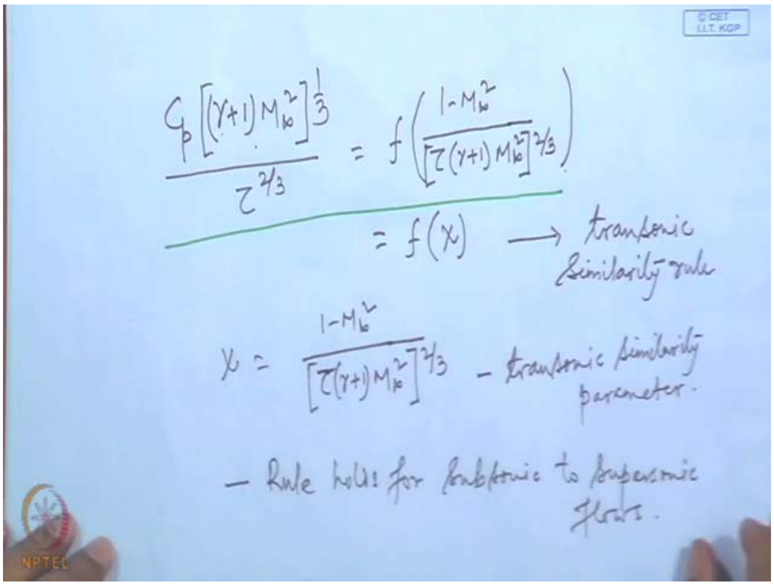
Department of Aerospace Engineering

Indian Institute of Technology, Kharagpur

Lecture No. # 36

Similarity Rules for High Speed Flows (Contd.)

(Refer Slide Time: 00:29)



In the last lecture, we derived similarity rule for transonic flow, and we saw that the transonic flow similarity rule takes the form of c p into gamma plus 1 M infinity square to the power 1 by 3 by tau to the power 2 by 3 is a function of 1 minus M infinity square by tau into gamma plus 1 into M infinity square raise to the power 2 by 3. And this argument of this function; we denoted by chi, which we called the transonic similarity parameter.

So, this is transonic similarity rule, No audio from 01:31 to 01:45 and chi equal to 1 minus M infinity square by tau into gamma plus 1 M infinity square raise to power 2 by

3 is called transonic similarity parameter. No audio from 02:10 to 02:28

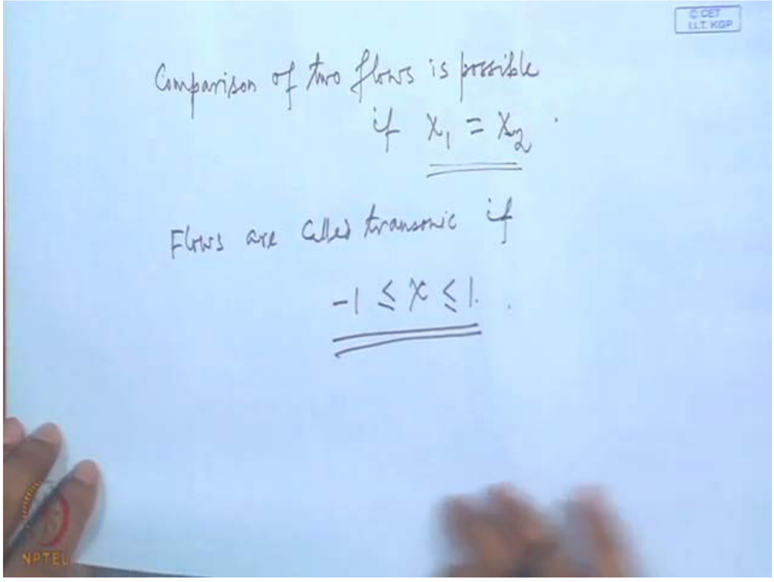
So, we see here that a transonic similarity rule involves the gas property; gamma, and these are coming because of that fixed value of the constant A is no longer arbitrary as in case of a linear subsonic or supersonic flow; We have also mentioned that since the linearized subsonic and supersonic governing equations are subset of the transonic equation; that is, if the right hand side of the transonic flow equation is set to 0, we get the linearized subsonic or supersonic flow equation.

Consequently, those linearized subsonic and supersonic flow equations are contained within the transonic small perturbation equation, and hence those linearized similarity rules are also contained within this transonic similarity rule. So, this transonic similarity rule is valid for subsonic to supersonic range. So, this rule holds for subsonic to subsonic to supersonic regime; that is, the so called Prandtl-Glauert rules or Gothert rules; they are also contained within this rule itself. No audio from 04:05 to 04:17

Since the transonic similarity parameter is of this form, due to non arbitrariness of A, what we can see here is that we cannot compare same body at different mach number or different bodies at given mach number which we could for subsonic or supersonic flows; that we can compare same body at different mach number or different bodies at same mach number which satisfies certain relationship.

However, in this case, we cannot compare a same body at different mach number or a different body at same mach number. Only when for two flow, if the chi; if the parameter chi become same, then only we can compare and we can say that this particular parameter will be same.

(Refer Slide Time: 05:25)

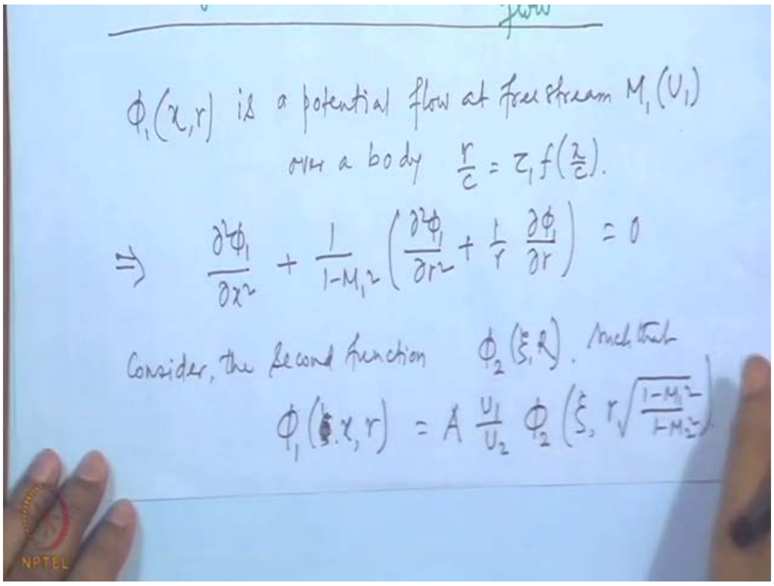


So, for two flows, if chi 1 equal to chi 2, so, you see that comparison of two flows is possible if this parameter for the two flow is same; that is, for two bodies of different thickness ratio at different mach number possibly in different gases such that this is true, only then, we can compare and then we can say that the left hand side modified pressure parameter will become same.

And flows are called transonic if… No audio from 06:33 to 06:58 So, based on this transonic similarity parameter, we also define specifically what is transonic flow, that the transonic flows are those flows in which this transonic similarity parameter lies between minus 1 to plus 1. So, even though we usually take as a thumb rule that when the mach number is somewhere very close to 1, say as an example between 0.8 and 1.2, the flows are transonic, but specifically, a flow will be really transonic when this parameter lies between this range; of course, the flow being small perturbation flow.

So, this is the most important of the similarity rules, as we mentioned in the beginning itself that, for linearized flow cases, we can have an explicit solution based on superposition because the equations are linear; however, in case of an non-linear case, there is no solution readily available and the similarity rules are extremely important. To conclude this discussion similarity rules, next we will consider the similarity rules for linearized axisymmetric flow.

(Refer Slide Time: 08:37)

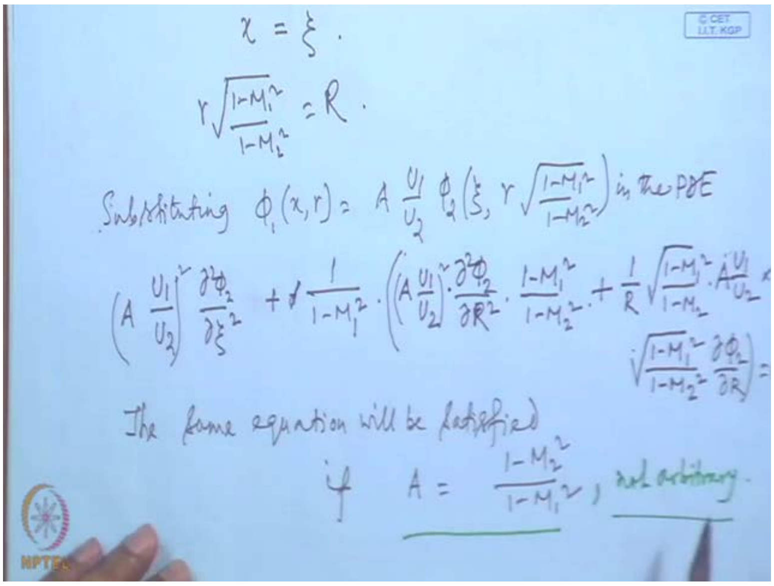


So, similarity rule for linearized axisymmetric flow; No audio from 08:33 to 09:06 previously mentioned, when we discussed about similarity rule for two dimensional flows that the Gothert rule that we developed for two dimensional case is also valid for axisymmetric flow, but that we will now see explicitly.

Once again, let us say that phi 1 as a function of x and r is a potential flow; potential flow at free stream M 1 which corresponds to velocity U 1 over a body given by r by c equal to tau 1 f x by c. Now since phi 1 is a solution of the potential flow; axisymmetric potential flow, then phi 1 satisfies the axisymmetric governing equation. So, we can say that d 2 phi 1 dx square plus 1 by 1 minus M 1 square into phi 1 dr square plus 1 by r d phi dr.

Now once again, the first step that we will consider a second potential function or second function, consider the second function phi 2 xi R which is related to this phi 1 No audio from 11:53 to 11:54 sorry phi 1 x r is A into U 1 by U 2 root over phi 2 xi R into root over 1 minus 1 square minus 1 minus M 2 square.

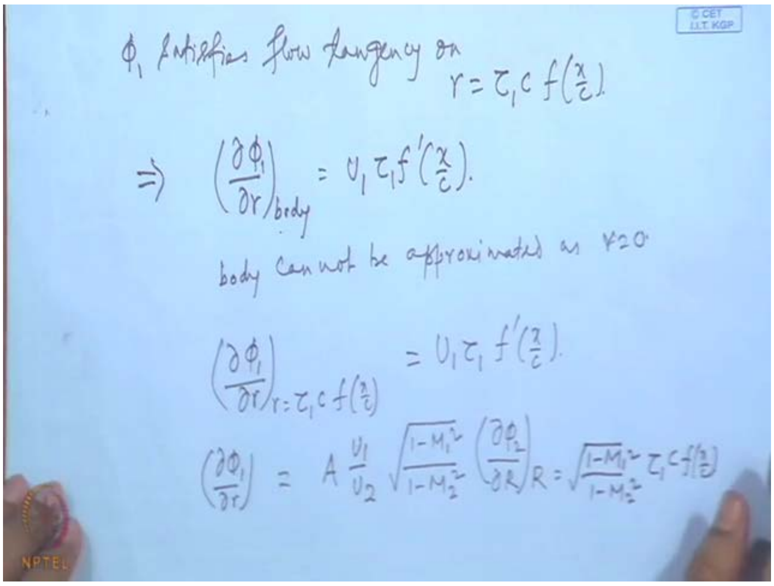
(Refer Slide Time: 12:44)



The same definition for the second function as we have used earlier, so, the associated transformation is, the associated transformation is x equal to xi and R into root over 1 minus M 1 square by 1 minus M 2 square equal to R. Now if we substitute phi 1, if we substitute this phi 1, phi 1 in this equation, then this gives No audio from 13:45 to 14:16 1 minus M 2 square. In the governing equation, that is, in the PDE, it gives that, let us substitute it fully.

So, we get d phi 1 dx can be written d phi 1 d xi d phi 2 d xi No audio from 15:04 to 15:27 plus 1 by 1 minus M 1 square A U 1 by U 2 into No audio from 16:13 to 16:52 plus 1 by R, that leads to again 1 by R No audio from 17:19 to 17:50 into No audio from 17:52 to 19:30 And you can see that this will, No audio from 19:38 to 19:51 this same equation will be satisfied, the same equation will be satisfied if A equal to 1 minus M 2 square by 1 minus M 1 square. Once again, we see that A is not arbitrary. So, not arbitrary. So, even in this axisymmetric case, which is of course a linearized problem, we again see that A is not arbitrary.

(Refer Slide Time: 20:58)



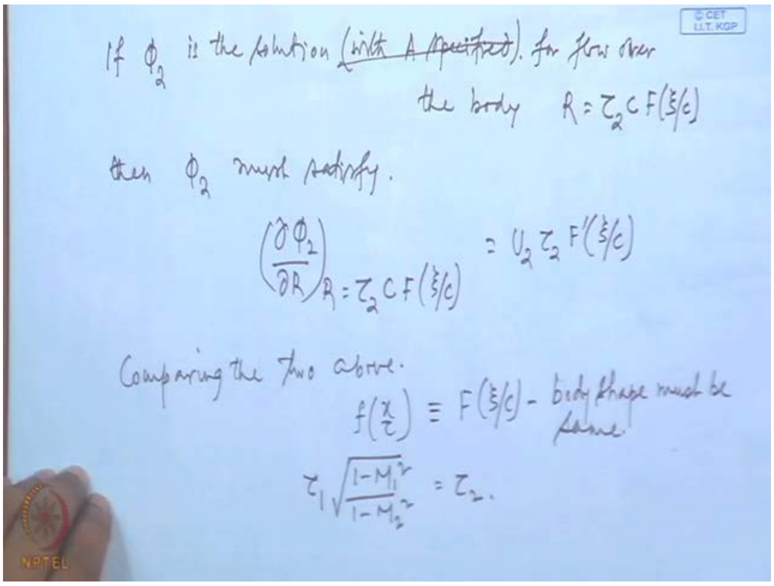
Now let us see their boundary condition. Now phi 1 satisfies the boundary condition on the body. Now phi 1 satisfies flow tangency on the body r equal to tau 1 c f x by c. Now the boundary condition is d phi dr on the body U 1 into motion of these, tau f prime x by c.

Now, we have a little difference here from the two dimensional analysis because we have seen that in two dimensional analysis, this body can be replaced by the axis itself, that is, y equal to 0 can be replaced; however, as we have discussed earlier that for the axisymmetric case, we cannot replace the body by r equal to 0; that is, body cannot be approximated by r equal to 0 which you have shown earlier why it cannot be. So, we have to use body cannot be approximated as r equal to 0. So, we have d phi dr sorry d phi 1 d r on r equal to tau 1 c x by c equal to U 1 sorry tau 1 f prime x by c.

Now, if we again substitute phi 1 equal to that A into U 1 U 2 by A 1 A into U 1 U 2 phi 2 xi R, then this can be written as, now d phi 1 dr can be written as A into U 1 by U 2 into root over 1 minus M 1 square by 1 minus M 2 square into d phi 2 dR, where R now becomes 1 minus M 1 square by 1 minus M 2 square into tau 1 x by c.

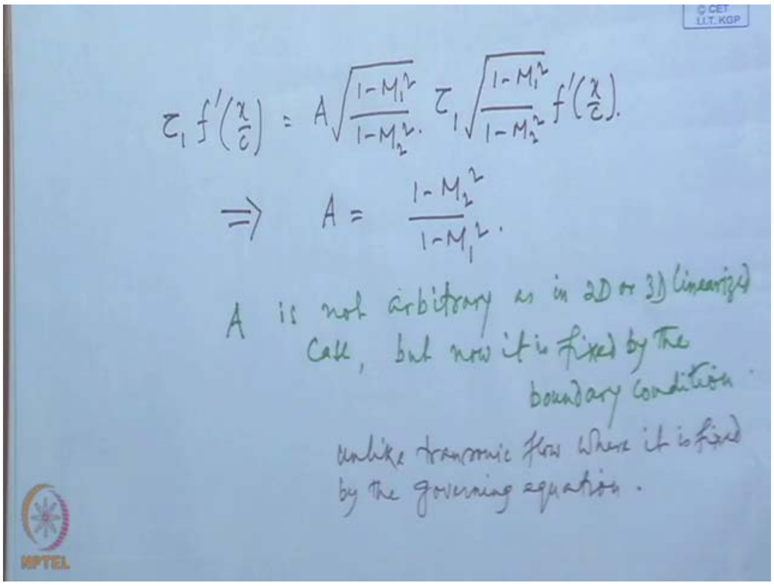
Now, let us say that the phi 2 which satisfies the governing equation subjected to a specific A, satisfies the boundary condition on another body.

(Refer Slide Time: 24:51)



So, No audio from 24:42 to 25:36 the body, then phi 2 must satisfy the boundary condition; the boundary condition R tau 2 c f xi by c is U 2 tau 2 f prime xi by c. Now if we compare these two, so, comparing these two, first we get f x by c is f xi by c; that is, the body shape must be same, body shape must be same and also we get tau 1 into root over 1 minus M 1 square by 1 minus M 2 square equal to tau 2. No audio from 27:42 to 28:23

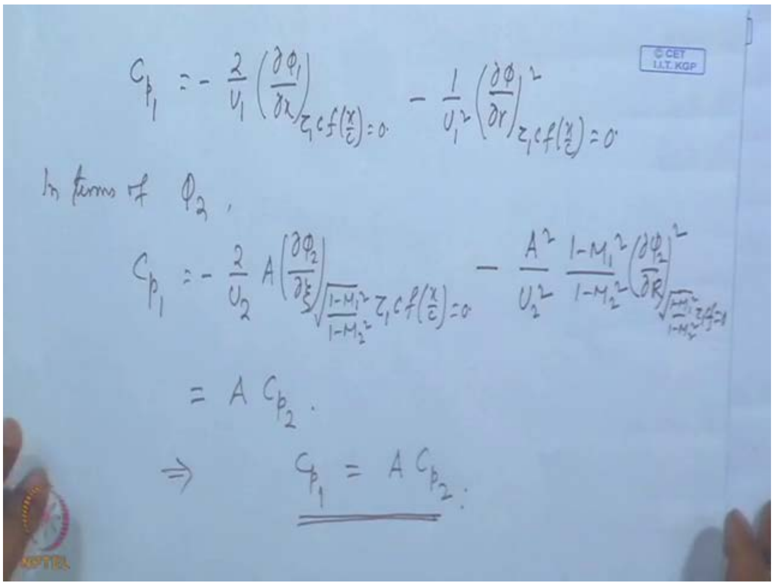
(Refer Slide Time: 29:10)



And No audio from 28:25 to 28:59 this also gives us that tau 1 f prime x by c is A into root over 1 minus M 1 square by 1 minus M 2 square into tau 1 root over 1 minus M 1 square by 1 minus M 2 square equal to f prime x by c, and this gives that A equal to 1 minus M 2 square by 1 minus M 1 square No audio from 30:00 to 30:12 and also see that A is not arbitrary as in 2 D or 3 D linearized case, but now it is fixed by the boundary condition, but now it is fixed by the boundary condition and of course, unlike transonic flow, unlike transonic flow where it is fixed by the governing equation.

So, we see that for axisymmetric case also, this A is not arbitrary, A has a fixed value; however, here the value is fixed by the boundary condition which against, we could not linearize because of the specific requirement of axisymmetric flow. So, we have arbitrary A as in case of a two dimensional transonic flow, but in that case that specified A is specified by the governing differential equation itself, while in case of axisymmetric flow, the A is again specified not arbitrary, but in this case, this value of A is specified by the boundary condition; not by the governing equation.

(Refer Slide Time: 33:07)

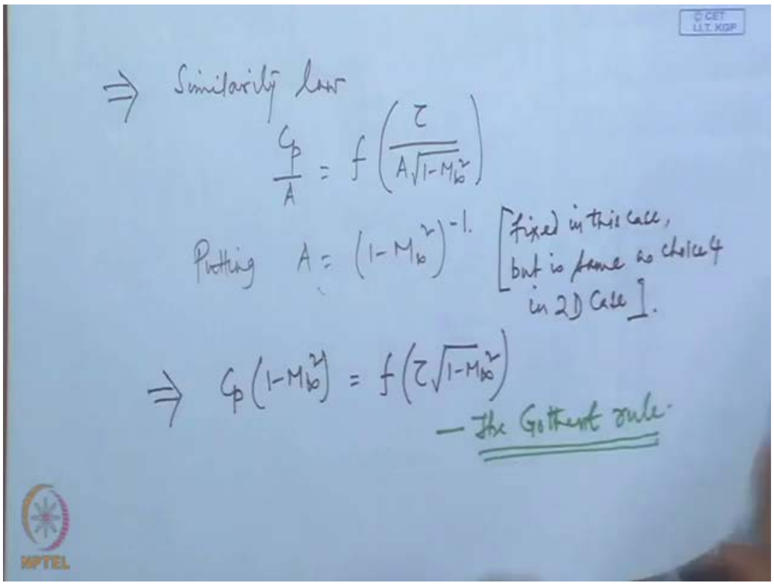


We know that pressure coefficient is also little different from the two dimensional or three dimensional cases. The linearized pressure coefficient in case of axisymmetric flow contains a second term and hence the pressure coefficient also should be compared. So, for the pressure coefficient, C p 1 which is minus 2 by U 1 d phi 1 dx from the body tau 1 c f x by c that is equal to 0 minus 1 by U 1 square d phi dr square. Again body surface is approximated to be 0.

Now, if we substitute phi 1 in terms of phi 2, so, in terms of phi 2, this becomes phi 2, now c p 1 becomes minus 2 by U 2 A d phi 2 d xi root over 1 minus M 1 square by 1 minus M 2 square into tau 1 c f x by c equal to 0 equal to minus A square by U 2 square into 1 minus M 1 square by 1 minus M 2 square d phi 2 dR square 1 minus M 1 square by 1 minus M 2 square tau 1 c f equal to 0. No audio from 35:24 to 35:44 Now this can be written as A into C p 2.

So, once again, we have the same pressure coefficient relation C p 1 equal to A into C p 2. So, even though the pressure coefficient have different formulae, the final comparison becomes the same; C p 1 equal to A into C p 2; however, A is not arbitrary.

(Refer Slide Time: 36:47)



So now, we can express the similarity law as, the similarity law now becomes C p by A equal to function of tau by A into root over 1 minus M infinity square, and substituting A equal to No audio from 37:18 to 37:35 which we see that for the two dimensional case, this is what our choice number four, but in this case, this is not choice, this is fixed, No audio from 37:44 to 37:56 This is fixed in this case, but is same as choice four in 2 D case, which is the Gothert rule.

And this then gives us the similarity rule to be C p into 1 minus M infinity square into function of tau root 1 minus M infinity square which is same as the Gothert rule; this is the Gothert rule. No audio from 39:15 to 39:25 So, we see that the Gothert rule which was found for two dimensional case for a specific choice of the constant A, has now become the rule for axisymmetric flow also.

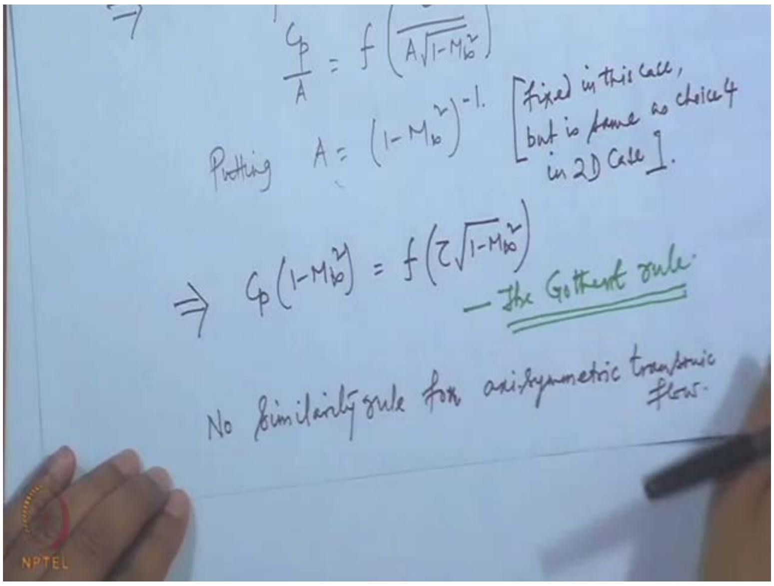
So, we have obtained a similarity rule for axisymmetric case as well and as it happen that this is same as the Gothert rule for two dimensional or three dimensional cases. And the steps we have followed the same; we have first of all considered two solution or two possible solution and we have seen that the choice of the second solution satisfies the governing equation.

However, we have seen that the boundary condition; if is to be solved satisfied, then in the both the cases, the body shape or the body profile must be same as in case of the two dimensional similarity rules or three dimensional similarity rules. Also the thickness will have similar relation as in two dimensional case, and in addition, we have seen that the satisfaction of boundary condition over an axisymmetric body fixes the value of parameter A.

So, A is no longer arbitrary like transonic flow; however, the specific value of A in this case comes from the boundary condition, not from the governing equation as in case of a transonic flow; however, the governing equation is satisfied for any arbitrary value of A as in case of two dimensional flow, but the specific boundary condition is not.

Finally, for comparing the pressure coefficient over in the two flows, we have seen that the pressure coefficient satisfies the similar relation; that is, C p 1 equal to C p A into C p 2 and finally, we get axisymmetric similarity rule which happens to be the same Gothert rule as in case of two dimensional flow. And it may be mentioned in this context that for axisymmetric transonic flow, no similarity rule can be derived in this fashion.

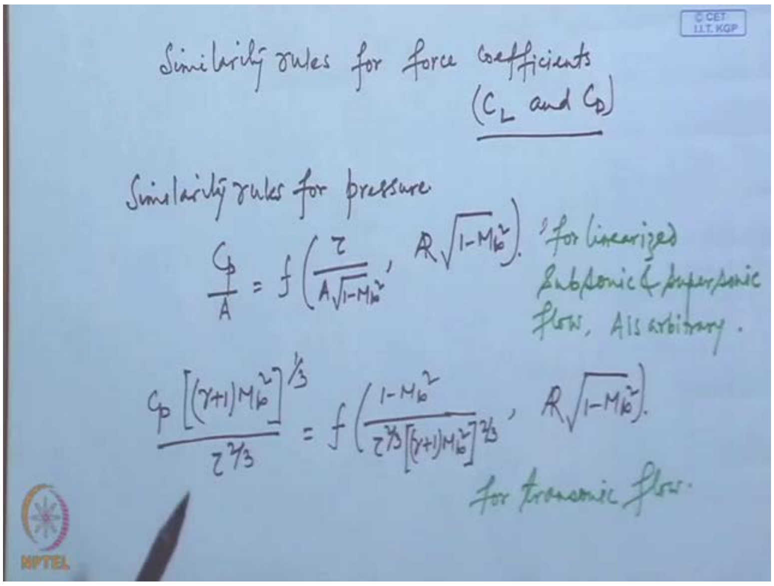
(Refer Slide Time: 42:05)



So, we can say no similarity rule for, no similarity rule for axisymmetric transonic flow.

No audio from 42:11 to 42:38 A three dimensional rule can be similarly, not for axisymmetric case, for transonic case, we can write the similarity rule for three dimensional case.

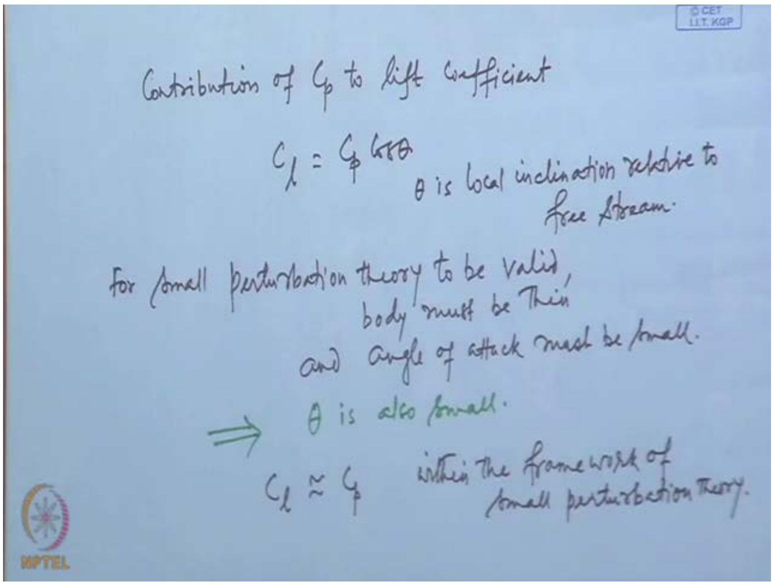
(Refer Slide Time: 43:15)



Once we have obtained this similarity rules for two dimensional and three dimensional body, let us now look for similarity rules in terms of the force coefficient. No audio from 43:15 to 43:29 So, let us now look for the similarity rules in terms of force coefficient for force coefficient, and in particular; lift and drag. No audio from 45:35 to 45:56

We have the similarity rule for say wing, similarity rule for pressure which we have No audio from 44:11 to 44:31 C p by A is function of tau by A root 1 minus M infinity square into aspect ratio root over 1 minus M infinity square. This is for linearized subsonic and supersonic flow, for linearized subsonic and supersonic flow For linearized subsonic and supersonic flow, A is arbitrary and this for transonic flow has become C p into gamma plus 1 M infinity square to the power 1 by 3 by tau to the power 2 by 3 equal to function of 1 minus M infinity square by tau to the power 2 by 3 gamma plus 1 into M infinity square by 2 by 3 into aspect ratio root over 1 minus M infinity square; this is for a transonic flow. No audio from 46:41 to 46:56

(Refer Slide Time: 47:16)

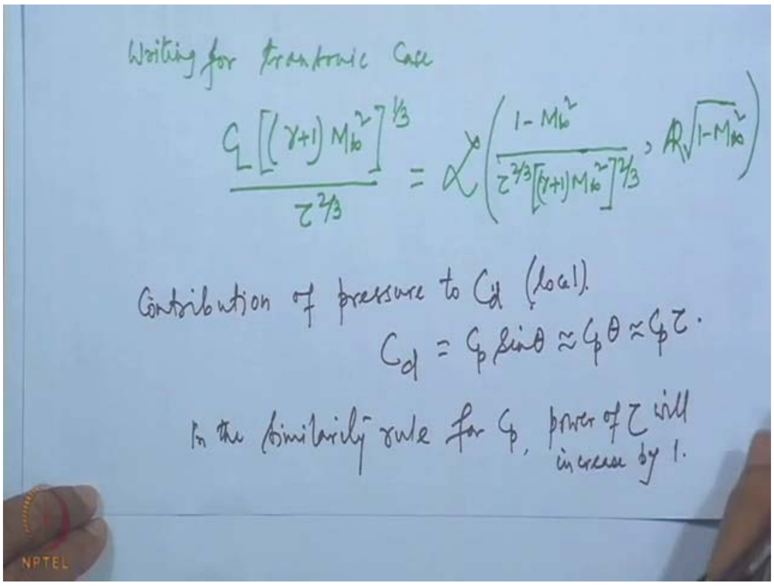


Now, from pressure, we can obtain the lift and drag coefficient like this that the continuation to lift coefficient from pressure, so, contribution to lift coefficient C p 2 lift coefficient, where C l is a local lift coefficient is C p cos theta, where theta is local inclination relative to free stream.

Now for small perturbation case, the body is thin and the angle of attack is also small, and consequently the local inclination relative to free stream will also be small. So, within small contribution framework, small perturbation theory to be valid, for small perturbation theory to be valid, body must be thin and angle of attack must be small. And when both these are satisfied, that the body is also very thin body and the flow angle of attack is also very small, then this combined effect; that theta is also small.

And consequently, so, C l is nearly equal to C p within the framework of small perturbation theory; No audio from 50:06 to 50:22 that is, the local contribution of pressure coefficient itself is contribution to the lift coefficient, local pressure coefficient itself is the contribution to the lift coefficient. And when this is integrated over the entire body, you get the overall C l, but since integration over the entire body is not going to change similarity rule, so, we have the same similarity rule applies for lift coefficient also.

(Refer Slide Time: 51:18)

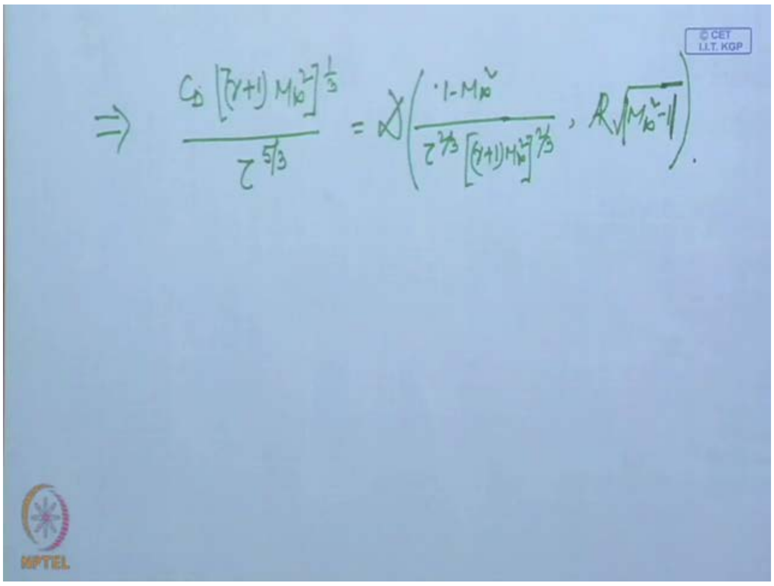


So, this applies… So, same similarity rules, same similarity rules apply for C l or if we write it let us say for the transonic flow case, as an example, we write it only for transonic flow case, similarly, we can write it for other cases as well, but for transonic case, we can write it explicitly. So, if we write it, writing for transonic cases only, we have C l into same rule gamma plus 1 into M infinity square to the power 1 by 3 by tau to the power 2 by 3. This function of course, can be of different nature. Once again, we get 1 minus M infinity square by tau to the power 2 by 3 gamma plus 1 M infinity square to the power 2 by 3 into aspect ratio root over 1 minus M 1 M infinity square.

So, similar transonic similarity rule applies for lift coefficient as well. Now for the contribution to drag force, contribution of pressure to C D, pressure to C D local; that is, C D is C p sine theta and within the small perturbation framework, this is C p into theta; local inclination and which is proportional to tau; the local thickness. So, continuation to C D, drag coefficient comes as C p into tau.

So, it is multiplied by tau. So, there will be power of tau will increase by 1 in the similarity rule. No audio from 54:43 to 54:55 So, in the similarity rule, in the similarity rule for in the similarity rule for C p, power of tau will increase by 1.

(Refer Slide Time: 55:42)



So, what we get is for similarity rule for drag coefficient, similarity rule for drag coefficient is C D into gamma plus 1 M infinity square to the power 1 by 3 by tau to the power 5 by 3; power of tau is increased by 1 from 2 by 3 to 5 by 3 is 1 minus M infinity square by tau to the power 2 by 3 gamma plus 1 M infinity square to the power 2 by 3 and aspect ratio root over… And in all these relation, we can make it…

Say for in transonic case, the supersonic sorry the free stream mach number may be either more than 1 or less than 1. Consequently these will be sign independent in all the cases; that is, here also. No audio from 57:12 to 57:24 So, this completes the similarity rule for lift and drag coefficient as well. We have seen that the similarity rule for lift coefficient remains same as it is the similarity rule for pressure coefficient; however, for drag coefficient case, the power of tau increases by 1 and we now have similarity rules which holds for subsonic to supersonic regime and also for axisymmetric two dimensional axisymmetric case.

Now, the flow regime that is left out is basically the hypersonic regime for which we have not derived any similarity rule; however, we have mentioned when we derived the small perturbation equation that in the hypersonic cases also, when M infinity is very large, there are many terms which cannot be neglected, rather the number of terms that cannot be neglected are more than the one that is not neglected for transonic flow and the equation is non-linear, and again a similarity rule will be important.

However, we will not go on explicit derivation of hypersonic similarity rule, but we will discuss a little bit about hypersonic rules and hypersonic similarity rule in the next lecture. Thanks.