High Speed Aerodynamics

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Lecture No. # 37

Similarity Rule in hypersonic flow

We have earlier discussed about, similarity rules in subsonic transonic and supersonic flows and for both, two dimensional and three dimensional flows as well as, axially symmetric flows and also, we mentioned that in axially symmetric flow, there is no transonic similarity rule. In general and however, just to for the sake of completion of the similarity rules. We will next discuss, little bit about similarity rules in hypersonic flows.

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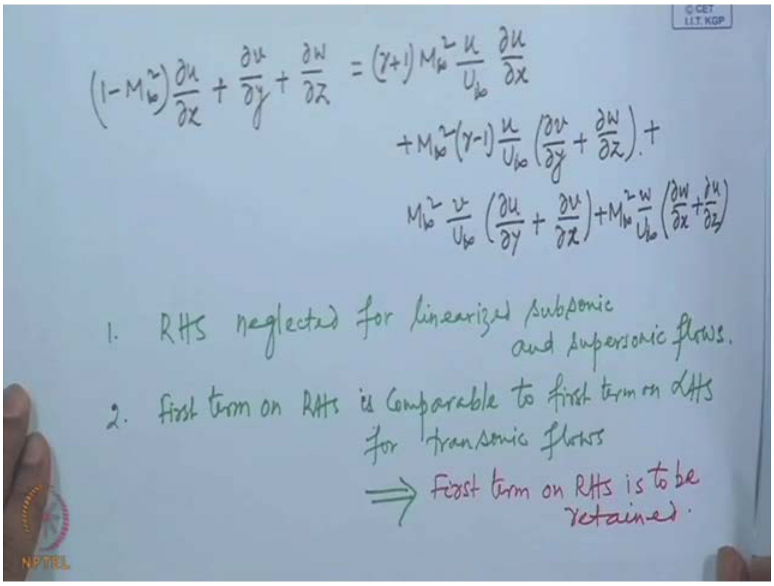

Now, while deriving the small perturbation equation, neglecting the product and higher order terms in small perturbation. We obtained the following equations, 1 minus M infinity squared d u d x that time; we wrote the equation in terms of x 1 x 2 x 3, but now, let us write them in terms of x y z d v d y plus d w d z gamma plus 1 into M infinity squared u by U infinity d u d x plus M infinity squared into gamma minus 1 u by U infinity d v d y plus d w d z plus M infinity squared v by U infinity d u d y plus d v d x plus M infinity squared w by U infinity d w d x plus d u d z .

And, we neglected the entire right hand side for linearized subsonic and supersonic flow or we said that, the perturbations are small and consequently, these terms are also negligible. So, RHS completely neglected, linearized subsonic and supersonic flow. So that, the perturbation velocity themselves are do not appear, anywhere in the equation. Only the gradient of the perturbation velocity appear on the left hand side, making the equation linear. The first term is not negligible for linear; for transonic small perturbation equation.

So, the first term on right hand side is comparable to first term on left hand side, comparable to first term on left hand side for transonic flows. Which result, the first term on the left right hand side is not negligible; first term on the right hand side is to be retained, which makes the equation non-linear for transonic flow. However, when the flow becomes hypersonic and M infinity is very large? You can see that, on the right hand side; all the terms are a product of squared of the mach number and perturbation velocity.

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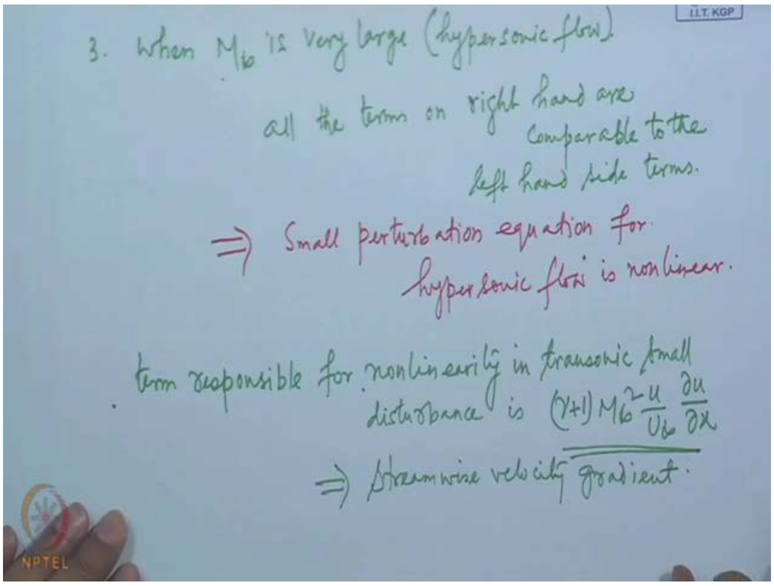

So, when M infinity is extreme very large; when M infinity is very large, all the terms on all the terms on right hand side comparable to the left hand side term. And so, the result is that, small perturbation equation for hypersonic flows are also non-linear. Equation for hypersonic flow is non-linear, and it is strongly non-linear and as the equation shows that, the linearity comes from different sources. In case of transonic flow that is, transonic flow is also non-linear; hypersonic flow is also non-linear however, the source of non-linearity are different.

We can see, from this equation that; for the transonic flow, the non-linearity for the transonic flow the non-linearity comes from this term only. This is the term, responsible for responsible in transonic, small disturbance flow is gamma plus 1 into M infinity squared u by U infinity d u d x that is a stream wise velocity gradient the stream wise velocity gradient is a streamwise velocity gradient is or in other way, the changes in the streamwise direction causes the non-linearity in transonic small disturbance flows.

However, in case of the hypersonic flow, the gradient in the other directions are also important. Rather, it can be easily seen that, which we will mention right now that, the non-linearity is mostly due to the gradients in the other directions; that is in the transverse reaction. Let us see what? Since, we will not be going a detail in hypersonic flow.

We will just have a very, very, very extremely deep discussion on hypersonic flow, without going much into the mathematics and also of course, in the physics with the flow. Let us see, what happens in case of hypersonic flow? When the flow is at very high mach number, the shock, even if they are oblique shock and belongs to the weak solution of the M theta beta relation that is corresponding to small deflection, shock of small wave angle, which belongs to the category of weak shock solution. However, the mach number being too large; the shock strength even for this weak solutions is quiet large.

As a result, there is large changes in pressure and temperature, associated with the oblique shocks in hypersonic flow, because of very high mach number. (( )) of the mach shock and also this; since this shocks are usually oblique and the angle is very close to the mach angle and what happens, because of this strong shock? The flow is very close or confined to a very thin layer near the body; bounded by the shock, which lies very close to the body.

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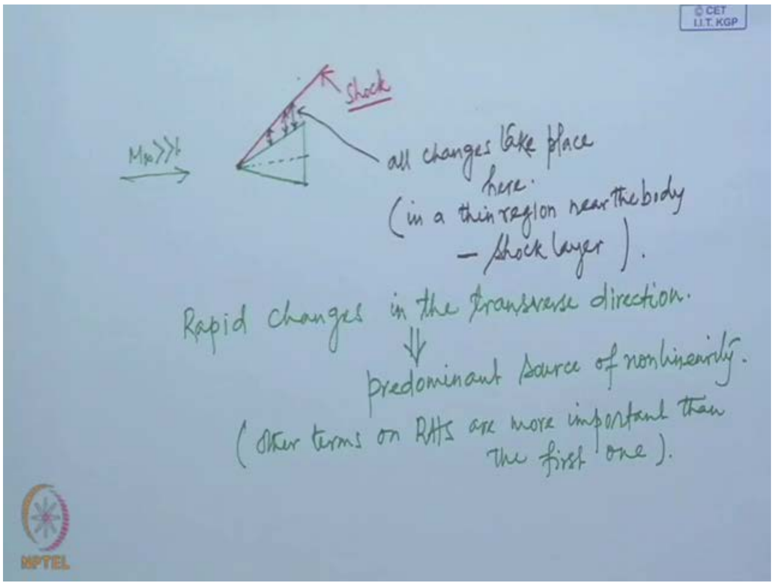

Say as an example, let us consider; again, flow about; flow past a wedge of small wave angle. Let us say in hypersonic flow, the shock will lie very close to the body. Since, as we know that, there will be no disturbance ahead of this shock. So, flow (( )) up to these is basically, this undisturbed free stream and all changes to the flow, takes place between this region; this is the region where all changes takes place. All changes take place here. That is; in a thin region near the body, which can be called as shock layer and in a real situation, where the flow is viscous? It is even extremely difficult to differentiate between the shock layer and viscous layer; they are almost merge each other.

Even many a times, shock layer may lie completely within the viscous boundary layer. Be that, as it may; this changes takes place over a very small distance in the transverse region. So, rapid changes in the transverse direction and we see, this is the predominant cause of; predominant source of non-linearity and it so happens that; in many situation, the first term on the right hand side which gives the non-linearity in transonic flow can often be neglected, in case of hypersonic flow; the other terms are more important. Other terms on right hand side are more important than, the first one; so in that, the term that is responsible for transonic non-linearity can possibly be neglected.

In case of, a hypersonic flow because the other terms are much larger. There is very rapid change in the transverse reaction. Now, one more thing happened that, in case of it hypersonic flow; the mach number being very large consequently, even for an oblique shock of weak category solution. The temperature rise may be extremely large and as a consequence, in many such flows due to very high temperature increase, there are some sort of chemical and thermal changes in the gas itself. As an example, the vibration modes of the molecules are excited; there might be some chemical reactions like dissociation, ionization, and so on. However, we will be not considering all those phenomena here.

But one more, most important thing; in case of, a hypersonic flow is the rapid temperature rise or very high level of aerodynamic heating. This aerodynamic heating becomes so important, that usually the hypersonic bodies or hypersonic vehicles will have rounded leading edge; instead of sharp leading edge. So, we will come back to that rounded leading edge, as in case of incompressible or low subsonic speed again, but this time because of aerodynamic heating.

It can be shown that, aerodynamic heating is considerably less. If the leading edge is blunt or rounded, which gives rise to a bow shock wave and subsequently, a subsonic flow downstream of that bow shock wave and in that case, the aerodynamic heating will be much less than, if it were a sharp leading edge as in; as we prefer, in case of a supersonic flow. The rounded leading edge of course, will give much larger drag due to the presence of the bow shock wave, but in this case, the aerodynamic heating is much more important and to avoid that heating, a larger drag is accepted.

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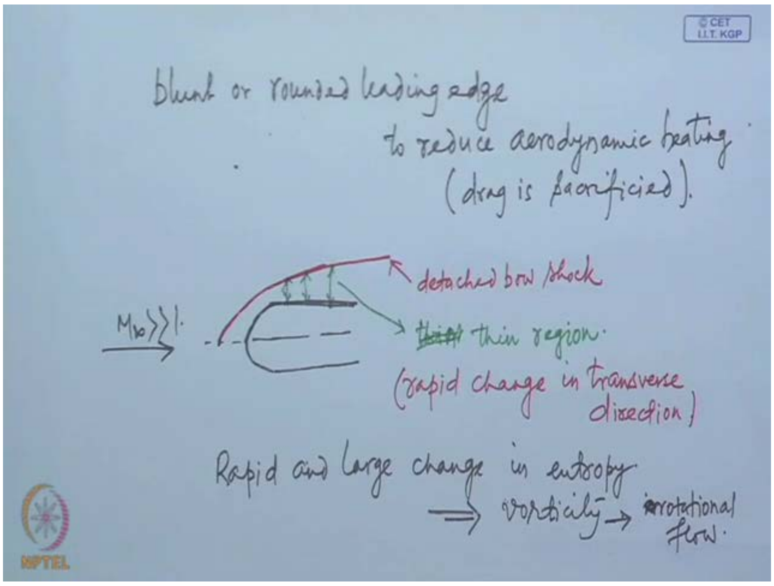

So, we have a general characteristics that, blunt or rounded leading edge to reduce aerodynamic heating. Drag is sacrificed that is, we sacrifice a large amount of drag and consequently of fuel consumption, just to avoid the aerodynamic or not to avoid completely, just to reduce the aerodynamic heating in this case. Now, if the bodies are rounded then, of course, a question comes that, is that thin region or that large changes are occurring in the transverse reaction over a thin region, is that now true. That remains true again, that is even if say, a body is thin; a body is blunt.

Let us say a this type of blunted body, we will have a detached oblique shock, but again, this detached bow shock. However, this region still a thin; so you see whether, we have a blunt nosed body or a sharp nosed body that is, whether we have a detached bow shock or an attached oblique shock. In case of a hypersonic flow, the shock is always very close to the body surface, giving rise to a very thin region.

In which flow changes rapidly, so here also, rapid change in the; again, rapid change in transverse direction and as we mentioned that, in case of a real flow, then this implies a very strong interaction between these shock layer and viscous layer. One more special phenomena that you must mention here; that is, the large entropy change across this across this strong shock; there is a large change in entropy across this oblique shock. Because as I mentioned already, that the mach number being very large, this oblique shock is also quiet strong and hence, a large change in entropy occurs.

So consequently, the flow as we know or as you have seen earlier, or rapid and large change in entropy and as you have seen that, whenever there is a large change or change in entropy, so it will change this in production of vorticity. Entropy changes produced vorticity and hence, the flow is irrotational; even if, inviscid flow is also irrotational. So, we have irrotational flow, rotational flow rotational flow and we cannot define a potential function, associated with this irrotational flow.

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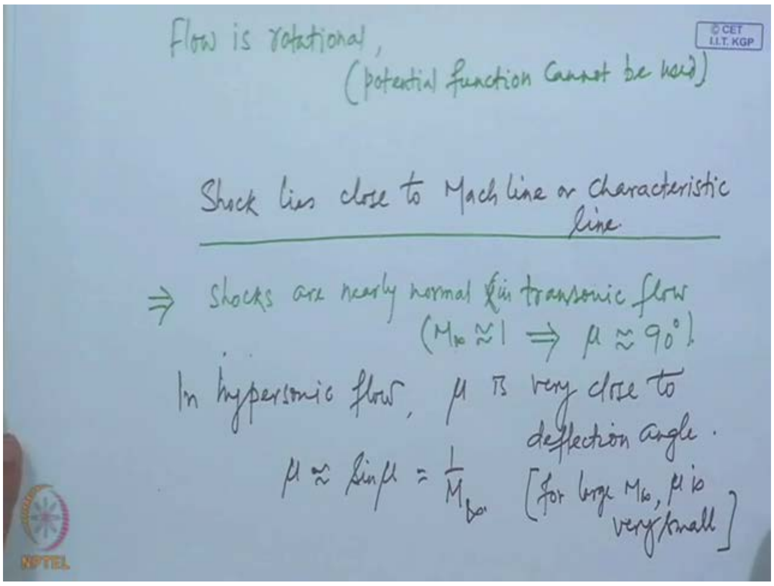

So, the viscous flow; inviscid flow is also rotational and a potential function cannot be used, that means even for small perturbation equation; we have to small perturbation equation for hypersonic flow; we have to solve in terms of the velocity and pressure, that means you have to solve the complete set of non-linear equations. All the components of Euler's equation as well as, the energy equation along with continuity equation, are to be solved. Even in case of, small perturbation hypersonic flow. Now one more thing, that we find here, that is a shock is very close to the bodies' surface.

In general, we know that; shock is very close to the mach angle in supersonic flow. Now, this is a general feature in all supersonic flow that shock; this is of course, a general feature; shock lies close to mach line or characteristic line, this is a general characteristic of supersonic flow. So, we see that in transonic flow, where the mach number is very close to unity, shocks are nearly normal in transonic flow. As M infinity close to 1 implies that, mach angle is in hypersonic flow in hypersonic flow mu, the mach angle is very close to deflection angle.

And what then its result that? Now, mu is very close to sin mu, which is 1 by M 1. This is that, for large M infinity mu is very small; that is the characteristics angle is very small, when we have very large mach number or rather, that in hypersonic flow, the characteristic angles are very small and the shock will be very close to that characteristic angle; shock will also be very close body.

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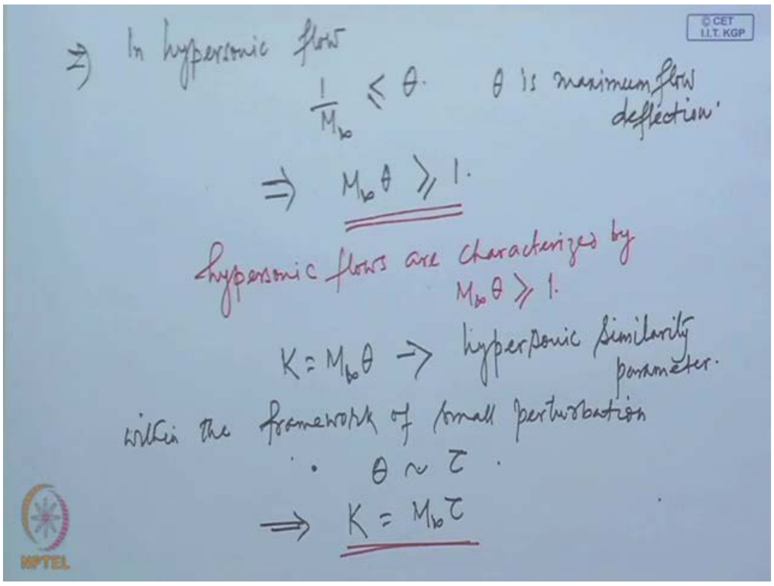

So, what we get? That in hypersonic flow, in hypersonic flow 1 by M infinity is theta; which implies that, M infinity theta is much larger than 1. So, this is what is? The hypersonic flows are characterized by this parameter, so hypersonic flows are characterized by this. M infinity theta is denoted by usually K and this is what? Is termed as hyperbolic similarity parameter, hypersonic similarity parameter now, within the framework of small perturbation theory, within the framework of small perturbation theory so this is the transonic similarity; hypersonic similarity parameter and this can also be used as a definition for hypersonic flow, in over a thin or cylinder body that is when M infinity tau is much larger than 1, the flow may be called as hypersonic.

Now since, we have; we are not dealing with the complete set of equation for hypersonic flow. We can derive or estimate some approximate hypersonic similarity rule, based on simple shock or expansion consideration.

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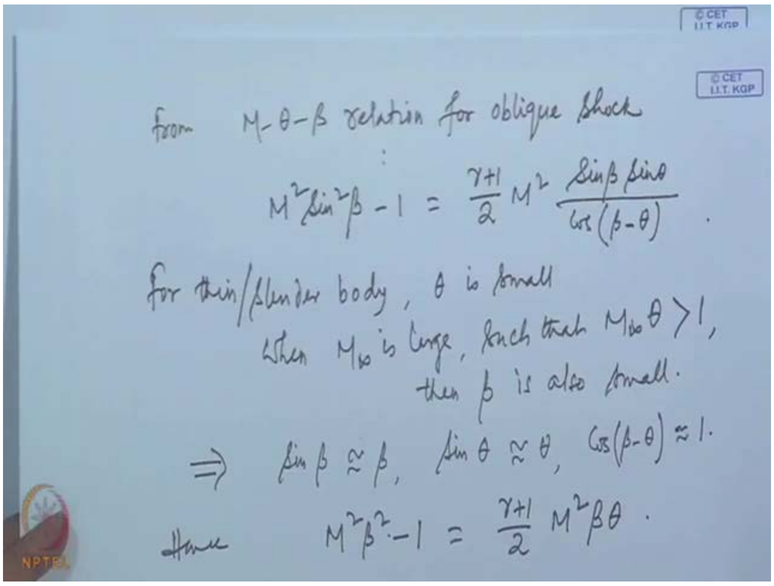

Earlier, we have seen that, from M theta beta relation for oblique shock; we had M squared sin squared beta minus 1 equal to gamma plus 1 by 2 M squared sin beta sin theta by cos beta minus theta. Now, for thin or slender geometry, theta is small and now, when M infinity is large? Such that, M infinity theta is much larger than 1 then, beta is also small.

This can very easily be verified from the theta beta curves, which shows that, when theta is small; beta is also small; if, M infinity theta is large or and then, we can have this approximations that, sin beta is nearly equal to beta sin theta is nearly equal to theta and cos beta minus theta is 1. So, we have this equation then, become M squared beta squared minus 1 equal to gamma plus 1 by 2 M squared beta theta.

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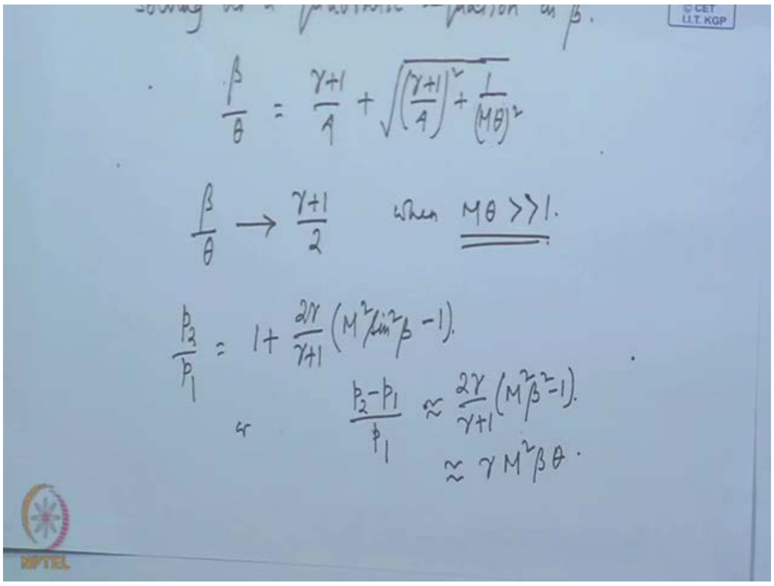

And solving this equation, as a quadratic equation; solving as a quadratic equation in beta, what we get is? That beta theta ratio is gamma plus 1 by four plus, look to this particular case, that if M infinity theta is; if M theta is very large then, this term is negligible, compared to this and this becomes gamma plus 1 by 2. So, beta by theta approaches gamma plus 1 by 2, when M theta is much larger than 1.

Now, we can evaluate the pressure coefficient also, using the oblique shock relations. From oblique shock relations, we have p 2 by p 1 equal to 1 plus 2 gamma by gamma plus 1 M squared sin squared beta minus 1. Which we have, p 2 minus p 1 by p 1 and using this approximations; this goes to 2 gamma by gamma plus 1 into M squared beta squared minus 1 and M squared beta squared minus 1. If we substitute the value, this becomes to be gamma plus 1 by 2 it is cancel. So, it remains gamma M squares beta theta.

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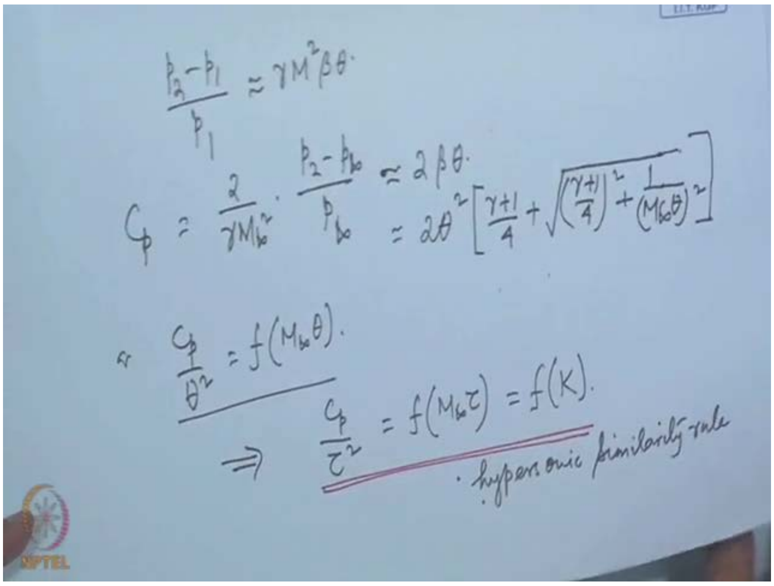

So, we have p 2 by p 2 minus p 1 by p 1 is approximately, gamma M squared beta theta and as we know that, C p equal to 2 by gamma M infinity squared into p 2 minus p 1 by p 1 or p infinity and this; then now, becomes 2 beta theta and substitute in beta by theta here, this gives 2 theta squared into gamma plus 1 by 4 plus gamma plus 1 by 4 or C p by theta squared is function of M infinity theta. Which of course, again can be written as that, C p by tau squared is function of M infinity tau or. So, this is the hypersonic similarity rule.((no audio from 46:16 to 46:48)) This is of course, a very indirect type of estimation of the supersonics hypersonic similarity rule.

Where we have not considered, the governing equation for hypersonic small perturbation equation and also have not considered the boundary condition, but based on the general feature of hypersonic flow and assuming that, the shock wave plays the important role. In case of a hypersonic flow, using simply the oblique shock relations; we have obtained the hypersonic similarity rule. However, this is the same rule that is obtained; if full hypersonic flow equations and boundary conditions are considered and a detail analysis is made. So, what in a sense we have done is that? We have discussed the very basic or very fundamentals of the nature of the hypersonic flows, over a cylinder body.

That hypersonic flows are confines to a very thin region between the body and the shock; where the shock is very close to the body surface, whether it is an attached oblique shock or a detached bow shock? Which is usually the case, in case of a hypersonic flow;

because in hypersonic flow, the aerodynamic heating is severe and to reduce that heating, what is that usually made of blunt nose or rounded nose? Which reduces the aerodynamic heating, but increases that act to a larger value, but that is a scarifies made to avoid the heating or to reduce the effect of heating. In a way, the consequence in case is that, the shock waves are very close to the body surface. This is of course, a general feature of the high speed supersonic flows, that is when M infinity is large; its mach angle is very large, mach angle is very small. And since, the shock waves lie very close to the mach angles.

So, the shock angles are also very near about the mach angle and consequently, in case of a hypersonic flow, this is almost same as the float turning angle or even may be smaller than that. As a consequence, the difference between theta and beta that is, the wave angle and the float turning angle; flow deflection angle for oblique shock is very small, theta beta are very closely or approximately, the same and we have used that approximation, which we obtained from the general feature of high speed flows.

To derive a hypersonic similarity rule, which gives us that C p by tau squared is a function of the hypersonic similarity parameter M infinity tau. A similar relation can also be obtained; if we consider expansion relation, instead of supersonic oblique shock relations. So, similar relation can also be obtained, if a expansion or prandtl-meyer expansion is considered.

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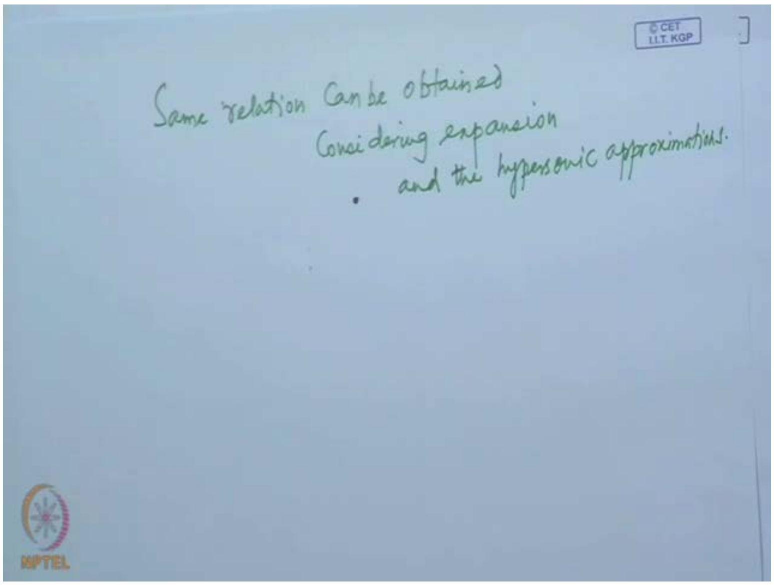

So, same relation is also obtained; can be obtained considering expansion and a similar and the approximations and the hypersonic approximations however, we will not repeat that process. So to summarize that, we have obtained a hypersonic similarity rule; without considering the hypersonic flow governing equations and boundary conditions, but by a qualitative consideration of the general feature of hypersonic flow and in doing so, we have also enumerate some of the basic difference in the non-linearities, associated with transonic small perturbation flow and hypersonic small perturbation flow.

We have seen that, in case of a transonic small perturbation, the stream wise gradient is the major reason of non-linearity; while in case of a hypersonic flow, the flow is confined within a very thin region between the body and the shock and large changes occur in the transverse reaction. While in a over a very short distance; while in the stream wise distance, the changes may be of the similar order takes place over a much larger distance and consequently, the transverse gradients are more important than the stream wise gradients. Which is contrary to the transonic flow? However, stream wise gradients are more important than the transverse gradient, because the flow extent in the transverse direction is quite large.

Also you have seen that, since, the flow is confined within a thin region. So, the viscous interaction is also always present, with the usual boundary layer approximation that, all the viscous effect are confined within a narrow boundary layer and outside it. The flow is basically inviscid or practically inviscid is not really useful here, because the entire flow is confined within a thin region.

So, the viscous and inviscid part of the hypersonic flow are very closely linked and their interaction is always significant. Also you have seen that, because of a very strong shock at the shock being at very large mach number; the entropy changes are also considerable and this may causes the change in vorticity and make the flow rotational. So that, usual irrotational potential flow assumption is not really useful, in case of hypersonic small disturbance equations and.

So, these are some essential feature of hypersonic flow, that we have brought in here and using those relations; using those qualitative discussion and some shock relations. We have derived, the hypersonic similarity rule and as we mentioned that, similar simulate rule can also be obtained. If we consider the expansion relation, but not done this of course, concludes our discussion on simulate rule which now, we have completed over the entire flow regimes. Starting from subsonic, transonic, supersonic, hypersonic; all flow regimes. Next, we will consider some fundamentals of transonic flow, so we conclude.