

High Speed Aerodynamics
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Lecture No: # 04
Review of Thermodynamics(Contd.)

So, to conclude our recapitulation on thermodynamics, finally we will discuss few more important relations, that are very useful in high speed aerodynamics, and then we will conclude our revisit of thermodynamics, but as we mentioned many times repeatedly, that this is just a recapitulation, and for more details, a book on thermodynamics must be consulted, or you may consult your earlier notes on this course. Now as we have stated that, the specific heats are very useful quantity in thermodynamics.

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Specific heats in terms of entropy

$$C = \frac{\delta Q}{\delta T} = \frac{T \delta S}{\delta T} = T \frac{\partial S}{\partial T}$$
$$C_p = T \left(\frac{\partial S}{\partial T} \right)_p, \quad C_v = T \left(\frac{\partial S}{\partial T} \right)_v$$

Let's take $S = S(T, v)$

$$ds = \left(\frac{\partial S}{\partial T} \right)_v dT + \left(\frac{\partial S}{\partial v} \right)_T dv$$

Now, we will express these specific heats, using entropy. The specific heats we will discuss using entropy, in terms of entropy. We have the specific heat as, delta Q by delta T, and this is delta Q can be expressed as T delta S, reversible. If we consider this heat is added reversibly, then delta Q is T delta S by delta T.

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$$\Rightarrow \left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial S}{\partial T}\right)_v + \left(\frac{\partial S}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

$$\text{or } \frac{C_p}{T} = \frac{C_v}{T} + \left(\frac{\partial S}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

$$\text{or } C_p = C_v + T \left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p$$

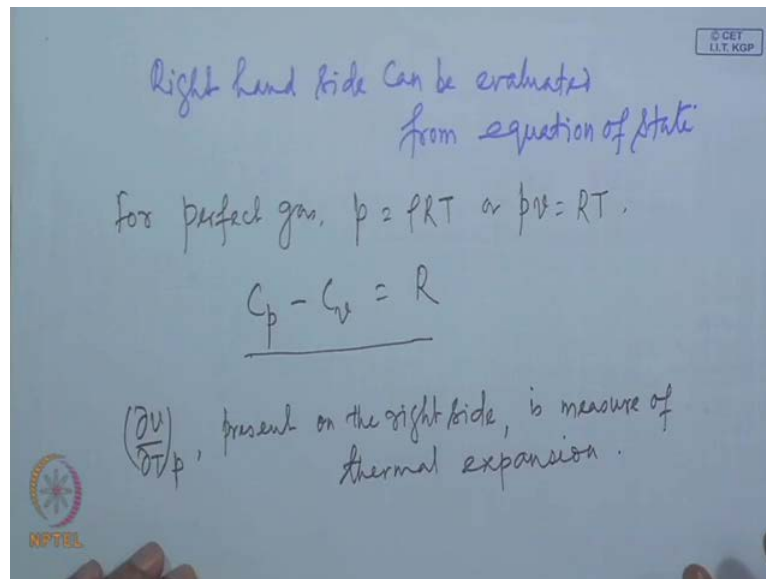
$$\text{or } C_p - C_v = T \left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p$$

Using Maxwell relation
 $\left(\frac{\partial S}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$

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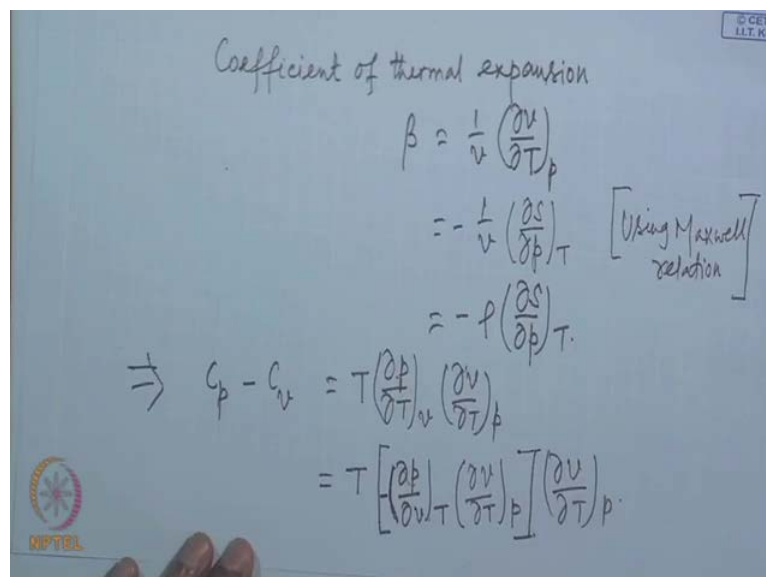
Or we can write that T, dS, dT . Hence the two principle specific heats C_p can be written as T, dS, dT at constant pressure, and the other one C_v is T, dS, dT at constant specific volume. Now, let us take entropy as a function of temperature and specific volume. We can do this since we have, mentioned repeatedly that for a pure substance, any state function can be expressed as function of two other parameters. So in this case you have the expressed entropy as function of temperature and specific volume. And this then gives us that, the change in entropy dS is, $dS = dT$ at constant volume into dT plus $dS = dV$ at constant temperature into dV . We then can write, $dS = dT$ at constant pressure, is $dS = dT$ at constant volume, plus $dS = dC_V$ at constant temperature, into $dV = dT$ at constant pressure. Now, from the definition of C_p , this is what is C_p by T , and this is C_V by T . So we have ((no audio 05:33 to 06:04)) this of course, multiplying by T is C_p equal to C_v plus T into. Now, from Maxwell's relation, using one of the Maxwell's relation dS, dV at constant temperature is $dP = dT$ at constant volume. We can write $T dP = dT$ at constant volume into $dV = dT$ at constant pressure. For you have used, using Maxwell relation dS, dV at constant temperature equal to $dP = dT$ at constant volume. So we get the difference between the two specific heat, C_p minus C_v equal to T into $dP = dT$. The right hand side can be really obtained, if the thermal equation of state is known.

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So, the right hand can be evaluated, from equation of state. We can very easily see, that if the equation of state is perfect gas. For perfect gas, and we then have C_p minus C_v equal to R , a very well known relation, where R is the specific gas constant, of the perfect gas, which we know that if we consider air, R is usually taken as 287 joule per kg Kelvin. Let us look to this equation from one more aspect. In this equation we have, on the right hand side $d v d T$ at constant pressure. Now $d v d T$ at constant pressure, that is the change in specific volume with temperature. Keeping the pressure constant, is related to coefficient of thermal expansion. So $d v d T$ of p , present on the right hand side, is measure of thermal expansion, change in volume due to change in temperature, with the pressure held constant. So this per unit volume is known as the coefficient of thermal expansion. So, $d v d T$ at constant pressure per unit volume is the coefficient of thermal expansion.

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So we have coefficient of thermal expansion, we denote it by beta, it is $1/v \cdot (dv/dT)_p$ at constant pressure, which using Maxwell's relation becomes $-1/v \cdot (dS/dp)_T$ at constant temperature. And, using the preferred fluid dynamic parameter, this is $-\rho \cdot (dS/dp)_T$ at constant temperature, and in fluid dynamics or the action of gravity, is very important, this term plays a major role. So the difference in specific heat, now also can be expressed in terms of this coefficient of thermal expansion, which we can see again that. So C_p is T into, that first term $(dp/dT)_v$ at constant v . Now let us write once again, $T \cdot (dp/dT)_v$ at constant volume, into $(dv/dT)_p$ at constant pressure. We write this as $T \cdot \beta$, using a thermodynamic identity $(dp/dT)_v$ at constant v is $(dp/dv)_T$ at constant temperature, into $(dv/dT)_p$ at constant pressure. This is a thermodynamic identity, which says the product of three derivative is minus one, and then. So we replace this term by this, and then keep the final term $(dv/dT)_p$ at constant pressure.

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The whiteboard shows the following derivation:

$$= -\beta v T \left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial v}{\partial T} \right)_p$$

$$= R \quad \text{if } pv = RT$$

Change in entropy

Consider $S = S(T, p)$.

$$\Rightarrow ds = \left(\frac{\partial S}{\partial T} \right)_p dT + \left(\frac{\partial S}{\partial p} \right)_T dp$$

$$= \frac{C_p}{T} dT - \left(\frac{\partial v}{\partial T} \right)_p dp$$

So this then becomes, $C_p - C_v$ equal to $T \cdot (dp/dv)_T$ at constant temperature, into $(dv/dT)_p$ at constant pressure square. Or in terms of, that is thermal expansion, this becomes $T \cdot \beta$ into $(dv/dT)_p$ at constant pressure, or one of the $(dv/dT)_p$ I have expressed using β , and which I have said equal to R , if we see that $C_p - C_v$ equal to R . The relation very well known to you is not applicable for other gases. It is applicable only for perfect gas. Now, the changing entropy is very important, because as we have seen from our discussion in second law of thermodynamics, that the change in

entropy is decided, whether a particular process is possible or not. We have seen that in a reversible adiabatic change, there will be no change in entropy.

So in a reversible process, entropy cannot decrease. So now let us try to find the change in entropy, in terms of other measurable quantities. And let us say we want to see the change in entropy, in terms of temperature and pressure. Of course we can express, as you have already done, entropy is a function of temperature and volume. Entropy is a function of temperature and pressure. Entropy is a function of pressure and volume and so on. Now, this gives $dS = \frac{C_p}{T} dT + \left(\frac{\partial S}{\partial p}\right)_T dp$ at constant temperature into $dS = \frac{C_p}{T} dT + \left(\frac{\partial S}{\partial p}\right)_T dp$. Now this is, by definition is C_p by T , so we have C_p by T dT , and this $dS = \frac{C_p}{T} dT + \left(\frac{\partial S}{\partial p}\right)_T dp$, we can express using Maxwell's law $dS = \frac{C_p}{T} dT - \alpha dp$ to be minus dV dT p into p .

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Handwritten notes on a whiteboard:

$dE = p dv$

Compare the contributions to entropy change

$$-\frac{C_p dT}{\beta V T dp} = \frac{C_p}{C_p - C_v} \left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \frac{\partial T}{\partial p}$$

$$= \frac{\gamma}{\gamma - 1} \frac{\left(\frac{\partial v}{\partial T}\right)_p dT}{\left(\frac{\partial v}{\partial p}\right)_T \delta p}$$

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So, we write $dS = \frac{C_p}{T} dT - \alpha dp$ at constant pressure. The dV dT at constant pressure can be again expressed in terms of that coefficient of thermal expansion β , which is β into v . So, this can be expressed as minus βv dP . Also or in terms of preferred fluid dynamic parameter, minus β dP by ρ . Multiply this by T , we have $T dS = C_p dT - \beta T dP$ by ρ , or $C_p dT - \beta T dP$. And from combine first and second law, this becomes $dE + p dV$. Now often, it is necessary to know the dominance of these two term, which of these term is dominant, that is which contributes more to the change in entropy, and that can be expressed. Compare contributions to, you see that the entropy changed, because of this term as well as this

term, and you can compare their relative contribution, and that is the contribution is $C_p dT$, and the other term contribution is $\beta v T dp$, of course this term is negative.

The $\beta v T$, using this relationship we can write $\beta v T$ in terms of C_p minus C_v . So this can be written as C_p by C_p minus C_v , into $d p d v$ at temperature, into $d v d T$ at constant pressure, into $d T d p$. Dividing this by C_v , this becomes γ by γ minus one, where γ is C_p by C_v , the ratio of specific heats, and you can also write this expression $d v d T$ at constant pressure into $d T$ or. So we see that the contribution depends on, the ratio of the volume change due to temperature, and volume change due to pressure. So the ratio of the volume change due to temperature, and the volume change due to pressure. So it is again, so the dominance of the contribution depends on.

So, the change in specific volume due to temperature, and change in specific volume due to pressure, that is the thermal effect, and for is known as the compressibility effect. So in a particular process we can check, which of these two terms are, more important than the other as far as the change in entropy is concerned. We have until now expressed the equation of state, in terms of say the pressure as a function of specific volume and temperature or internal energy, as a function of volume and temperature. Now, since any two property can also be expressed in terms of any other two functions, so the equation of state can also be expressed in terms of other variables, and particularly if the equation of states are expressed in terms of entropy, then they are called canonical equation of state.

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Canonical equations of state

$$E = E(S, v)$$

$$H = H(S, p)$$

Change in entropy in perfect gas

$$S = S(u, T)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

$$= C_v \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dv = C_v \frac{dT}{T} + \frac{R}{v} dv$$

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So these are called the canonical equation of states. E expressed as function of entropy and specific volume or density, and H expressed as a function of entropy and pressure. Now this change in entropy will, now further extend the relations, that is some of these derivatives are can be obtained, if the equation of state is known, and we can express the entropy change in or more measurable form, if you know the equation of state. See as an example for perfect gas. So we will consider now change in entropy in perfect gas. Once again let us write entropy is a function of volume and temperature, so that this is by definition C_v by T , so this is $C_v dT$ by T . And dS $d v$ by T using Maxwell's relation this becomes $d p$ $d T$ at constant volume $d v$. Now for a perfect gas, $d p$ $d T$ at constant volume can very easily be evaluated, which is simply R by v . So this becomes $C_v d T$ by T , and $d p$ $d T$ by v is simply R by $v d v$.

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Integrating between initial state (1) to final state (2)

$$S_2 - S_1 = \int_1^2 C_v \frac{dT}{T} + \int_1^2 \frac{R}{v} dv$$

For a calorically perfect gas C_p, C_v, R are constant

$$\Rightarrow S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$= C_v \ln \frac{T_2}{T_1} + R \ln \frac{P_1}{P_2}$$

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Now this can be integrated between, integrating between initial state one. We have s_2 minus s_1 , from 1 to 2 $C_v dT$ by T . To complete this integration we have to know, the relationship between the specific volume and specific gas constant as a function of temperature, and volume. However, for a clerical perfect gas, we know that this C_p, C_v, R are all constant, and then we can compute this integration. ((no audio 33:06 to 34:06))

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Similarly, $S = S(T, P)$

$$\therefore ds = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

$$= C_p \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_P dP$$

$$= C_p \frac{dT}{T} - \frac{R}{P} dP$$

Integrating $S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

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Similarly, we consider entropy function of temperature and pressure, and this is by definition is C_p by T , so this is C_p into dT by T . And again using a Maxwell's relation, this becomes minus dv dT at constant pressure into dP , where this term is replaced by

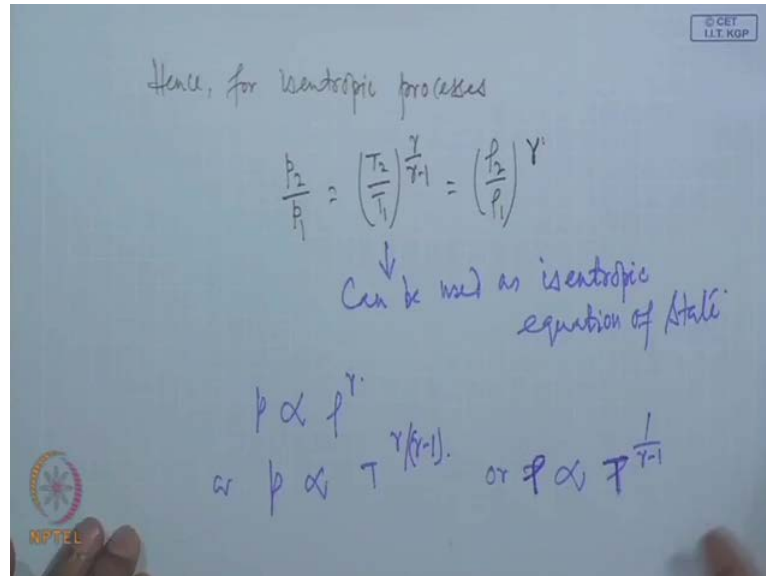
this using Maxwell's relation, and then for perfect gas this becomes, and once again integrating similarly, ((no audio 35:27 to 36:00)).

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The image shows handwritten mathematical derivations on a whiteboard. The first equation is $C_p \ln \frac{T_2}{T_1} = R \ln \frac{p_2}{p_1}$. This is rearranged to $\Rightarrow \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{C_p/R} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$. The second part starts with "Also," followed by $R \ln \frac{v_2}{v_1} = -C_v \ln \frac{T_2}{T_1}$. This is rearranged to $\Rightarrow \frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{-C_v/R} = \left(\frac{T_2}{T_1}\right)^{-\frac{1}{\gamma-1}}$. Finally, it is noted that $\Rightarrow \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

Now, let us consider the process is isentropic. ((no audio 36:02 to 36:37)) We have $C_p \log$, if we consider this relation $C_p \log T_2$ by T_1 equal to $R \log p_2$ by p_1 . And this gives p_2 by p_1 equal to T_2 by T_1 to the power C_p by R , and if we consider this relationship, if we consider the earlier relationships, then we have $R \log v_2$ by v_1 equal to minus $C_v \log T_2$ by T_1 , ((no audio 38:19 to 39:03)) or in terms of. ((no audio 39:06 to 39:40)).

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So for isentropic processes we can, hence for isentropic processes we have. We have $p_2/p_1 = T_2/T_1$ to the power γ by γ minus one, equal to ρ_2/ρ_1 to the power γ . And this can be used as isentropic equation of state. ((No audio 40:43 to 41:25)) Any further relation if we need, we will discuss at appropriate moment, but with this we conclude our brief review of thermodynamics. So, let us see what we have done in our introductory lectures first of all, we have considered what are the essential features of high speed aerodynamics, and there, where we discussed that, when speeds are large the change in pressure will also be large, which will cause change in density. And as a result there will be many new phenomena, and even there will be some phenomena which will need relook into, which will now have new interpretation, and there will be some results which are, completely contrary to the results, that you have come across in incompressible flow, all because that the pressure changes here are very large, and density can no longer be considered independent of pressure. Also we discussed that at very high temperature, at very high speed.

The flow is associated with large change in kinetic energy, which is comparable to the internal energy, and internal energy and kinetic energy are coupled, and transform from one to the other will change the other considerably. And, this implied that the thermodynamics is an inherent part of high speed aerodynamics. High speed aerodynamically problem cannot be handled, without thermo dynamical consideration. And due to this, we thought brief review of the thermodynamically concepts, and certain thermodynamically relations are necessary and useful. And we have discussed, the basic thermo dynamical relationship, we have discussed about first law, we have discussed

about second law. We have discussed the Maxwell's equation of state, we have discussed the concept of internal energy entropy. The equation of state, even what change the equation of state needs in case of a real gas. Also we have defined the coefficient of compressibility and compressibility factor, using the thermo dynamical relationship.

And we have seen that the perfect gases are those gases, for which the compressibility factor is one, and we mentioned that air under ordinary conditions can be considered as a perfect gas, and many of these equations can be simplified. The equation of state can be expressed, in simple mathematical relation, given by $p \rho = R T$, sorry $p = \rho R T$ or $p v = R T$, for v the specific volume. And also we have obtained certain relationship for perfect gas. We have also discussed that this perfect gas assumption for air is, quite valid as long as the temperature does not increase considerably, where air must be considered the mixture of gases with their with chemical and thermal reactions, need to be considered. Also again this perfect gas laws are not valid, if the pressure is very high and temperature is very low. So that the gases deviates from the usual low density approximation, and in that case those real gas effects need to be considered.

Particularly that all gases liquefies that effect need to be considered. Well in high temperature this effect is not necessary, but the effect that are to be considered are, the chemical and thermal reactions, excitation of the vibration mode and many chemical reactions, but as you have mentioned at that time, and again repeating that, in this course of high speed aerodynamics. We will not consider this high temperature very high speed flows, and consequently the thermodynamics is also not discussed to consider those effects; that is we have not considered the equations for mixtures. What we have considered, is thermodynamics of a pure substance, where any state variable can be or state function can be expressed as, any other two state, as a function of any other two state variables.

Using this we have found that, for isentropic processes that equation of state can be also expressed in an alternative form, which is given here, that T as ρ to the power γ p is proportional to ρ to the power γ , or p is proportional to ρ to the power γ or p proportional to T to the power γ by γ minus one, or T proportional to ρ to the power one by γ minus one, ρ proportional to T to the power one by. In particular we have, while discussing the entropy, we discussed certain points that, that is the main role of the second law of thermodynamics, which clearly

states, whether a particular process will be possible or not. So the second law of thermodynamics is excluded possibility, in addition to defining an absolute scale of temperature, and the concept of entropy. In so discussing we have also mentioned about that, heat is a degraded form of energy, because the heat cannot entirely become converted into work, while useful work can be converted completely into heat.

And in our subsequent discussion on high speed aerodynamics, we will see that the replication of second law will come quite often, to check whether a particular process or particular result that we will be obtaining, are really possible in nature or not. As an example at this stage, we will later on see that, the conservation laws predicts, that it is possible to have a shock, which will change the flow from subsonic to supersonic, or from supersonic to subsonic. However only, the none of the conservation laws, that is the conservation of energy, momentum or mass, can definitely tell us whether the processes are feasible or not, only with the application of second law we will be seeing that, the shock really can occur, only if the flow is supersonic. So even though, second law does not give answers and governing equation as the first law.

The first law of course, gives us the most important governing equation being the conservation of energy. It supplements the conservation of mass, and conservation of momentum, which are regularly used in incompressible flow, but the second law does not give us any such, new conservation equation, but it decides whether the results of the governing equations or the conservation laws, are really possible in nature. So with this, we will conclude our review and next we will start discussion on the subject of high speed aerodynamics or gas dynamics. Of course, we will start with the simplest possible flow problem, which is of course, the one dimensional flow. So from next lecture onwards we will discuss about, one dimensional compressible flows, or one dimensional gas dynamics. Thank you.