High Speed Aerodynamics

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Lecture No. # 40

Transonic Flow (Contd.)

So, we will discuss estimation of critical Mach number to be precise the lower critical Mach number of an airfoil and our estimation will be on the basis of small perturbation theory. That is it will be applicable to thin airfoils, but that is quite acceptable since the airfoils used in aircraft are always thin.

Now, we will first assume that, we know the let us say the incompressible pressure distribution or the incompressible flow about the airfoil.

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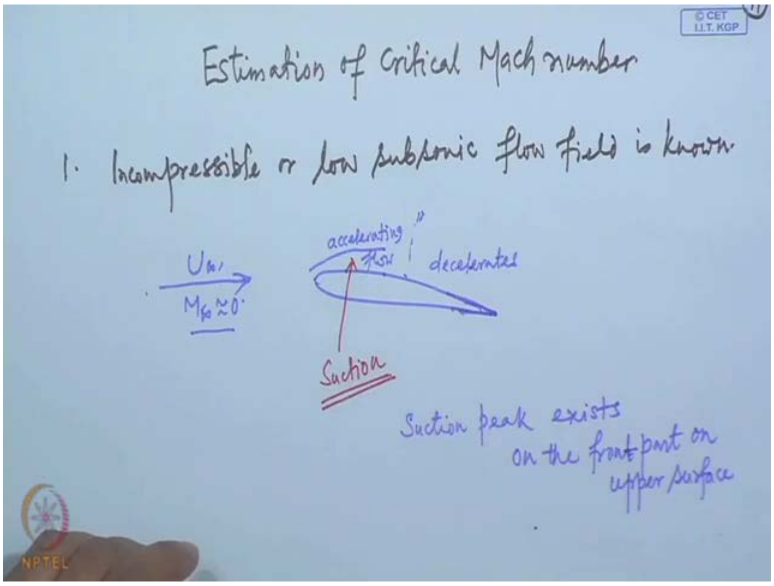

So, estimation of critical Mach number and our assumption is that, incompressible or say a low subsonic flow field is known. We know for an airfoil at incompressible or very low speed flow, we call it U infinity and M infinity is nearly 0 or 0. We know that, on the upper surface, the flow accelerates up to some distance and then, decelerates let us say in this part the flow accelerates and then, it decelerates in this part. Consequently, there is higher suction on this front part.

So, in this part there is suction the pressure falls compared to the undisturbed stream pressure and that, there is a point at which the maximum suction or the maximum flow velocity occurs. So, one suction peak exists on the front part on the front part on upper surface of course, this suction peak point depends on the airfoil or the type of airfoil, but usually it is somewhat downstream of the leading edge, for our conventional airfoil that, 4 digit naca series, the suction peak is quite close to the leading edge and since, we are assuming that the incompressible flow solution is known, the suction peak is known.

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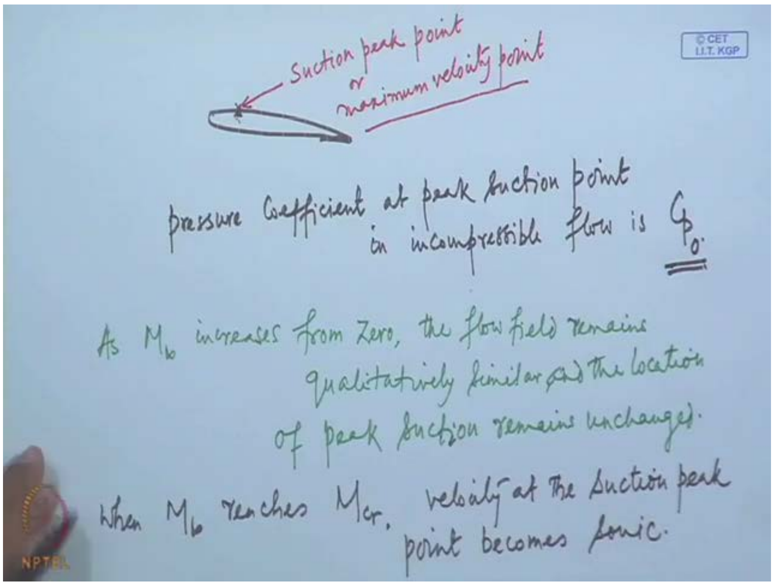

So, let us say that we have the suction peak known, let us say this is the suction peak point or maximum velocity point. That is at this point the maximum velocity occurs. Let us define that, the pressure suction peak pressure coefficient. So, the pressure coefficient at that point, at peak suction point let us see incompressible flow at peak suction point in incompressible flow, we denote it say lets C p 0. This is of course, the largest negative pressure coefficient, on this airfoil and let us says with increasing Mach number the flow pattern remains similar and the location of the suction peak point remains the same.

So, as M infinity increases So, as M infinity increase from 0. The flow field qualitatively remains the same qualitatively similar, with and the location of peak pressure, peak suction remains unchanges. Of course, when you say Mach number is increasing from 0, but we are implicitly implying that, Mach number remains, below the critical Mach number. That is the flow field that, we are considering is fully or purely subsonic and within that, limit the flow field remains qualitatively the same. That is the streamline patterns and everything, they are identical and the peak suction point also remains unchanged.

However, the magnitude of these suction or pressure at any other pint of course, changes and the change can be obtained using the linearized similarity rule as an example the prandtl-glauert rule. So, for all these cases we can get the pressure coefficient from prandtl-glauert rule and we can know, what is the pressure distribution? At some other Mach number within the critical Mach number range.

Now, we also have seen earlier that, as Mach number increases the flow velocity on the surface also increases. So, when the critical Mach number is reached the point that will be become sonic is obviously this point. Since, at this point the velocity is become maximum. So, this is the point that will become sonic when critical Mach number is reached. So, when M infinity reaches M critical velocity at the suction peak point becomes sonic.

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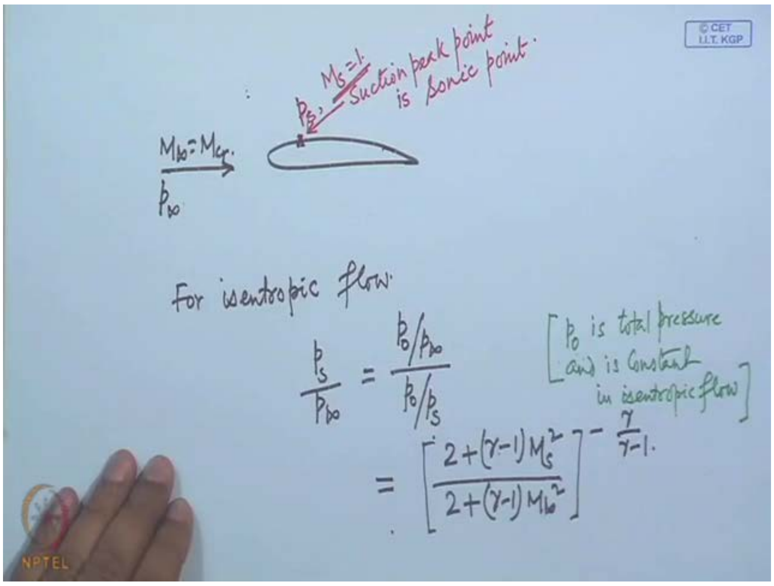

That is, let us M infinity equal to M critical, suction peak point is sonic point. Now, within this range of course, the flow is isentropic. Since, we are considering only inviscid flow and subsonic flow so there is no question of any change in entropy and we know for a for an isentropic flow ((no audio 12:54 to 13:24)) for isentropic flow, at the suction peak let us say, the pressure is p or suction peak point or sonic point. So, let us denote this pressure to be p s. Since, the flow field is isentropic we can have the relationship p s by p infinity, which can be written as the p 0 by p infinity by p 0 by p s. Now, since in an isentropic flow, p 0 is a stagnation pressure is constant, total pressure and is constant in an isentropic flow.

Now, these relations of course, you can substitute in terms of the Mach numbers p zero by p infinity in terms of the local Mach number M infinity and similarly, p 0 by p s in terms f the local Mach number M s. So, here the local Mach number is so this. Now, becomes 2 plus gamma minus 1 M s square by 2 plus gamma minus 1 M infinity square, to the power minus gamma by gamma minus 1.

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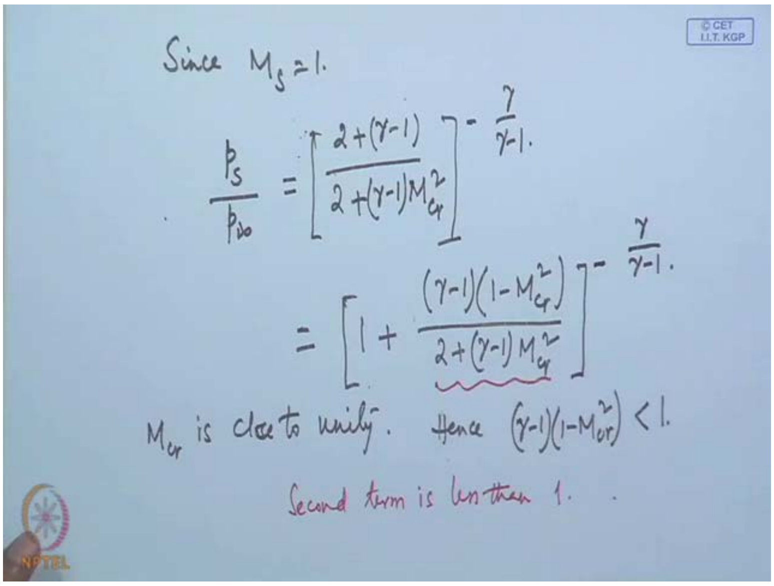

This substituting that, Mach number at the sonic point that is 1, we get this relation to since M s equal to 1 we have 2 plus gamma minus 1 divided by 2 plus gamma minus 1 and the M infinity in this case is now M critical and this can be written as 1 plus gamma minus 1 into 1 minus M critical square by 2 plus gamma minus 1. We also know that, the M critical is close to 1 and consequently this 1 minus M critical is less than 1 and gamma minus 1 into 1 minus M critical square is less than 1. So, M critical is close to 1, close to unity. Hence, gamma minus 1 into 1 minus M c r square is less than 1.

So, this second term in this expression, the second term is less than 1. Considerably less than 1 and we can expand this in binomial series in terms of power of 1 minus M c squared, neglecting the higher power.

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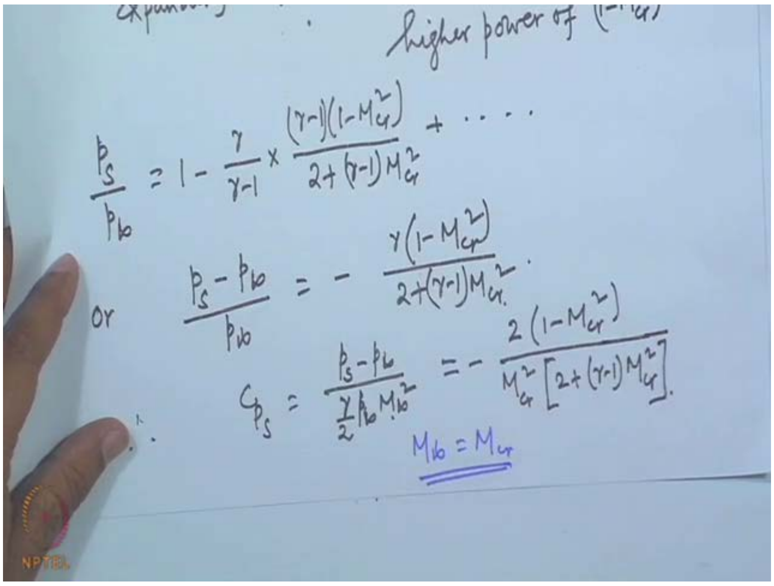

And this then, we get expanding in series and neglecting terms in higher power of what we get is p s by p infinity, this becomes 1 minus the first term becomes gamma by gamma minus 1 into gamma minus 1, 1 minus M critical square by 2 plus gamma minus 1, M critical square and these are all neglected or we have p s minus p infinity by p infinity is minus gamma into 1 minus M critical square by 2 plus gamma minus 1. Now, one of the pressure coefficient C p, at the suction point ((no audio 22:28 to 23:02)) if we remember this is what is half rho infinity, u infinity square and this then become 2 into 1 minus M critical square that is 2 by gamma M infinity square gamma get cancel by M infinity square and M infinity is M critical ((no audio 23:34 to 24:04)) we should remember, that in this case M infinity is M critical.

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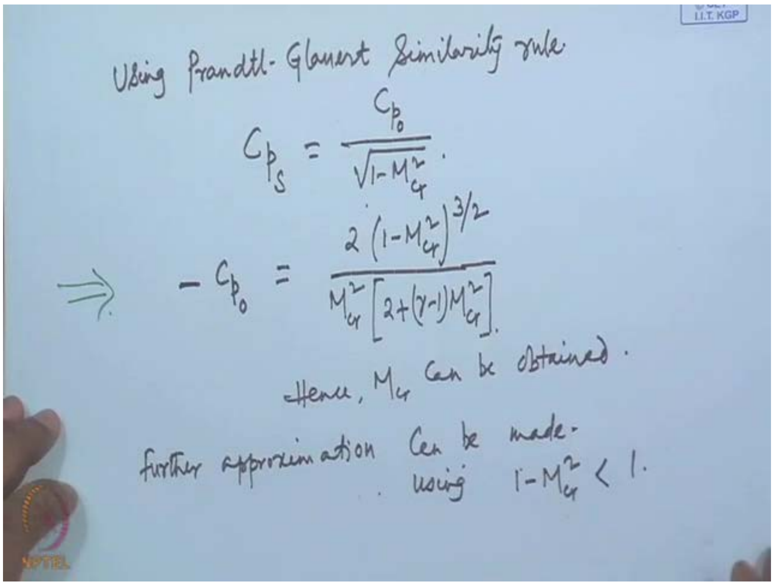

Now, if we now apply prandtl-glauert similarity rule, ((no audio 24:26 to 24:56)) we know this pressure at the suction point at the critical Mach number, can be related to the incompressible flow pressure. The incompressible flow pressure, I think we denoted by C p 0 by root over 1 minus M critical square. This then, we substitute in the earlier relation, we have minus C p 0 equal to 2 into 1 minus M critical square to the power 3 by 2 by M critical square into 2 plus gamma minus 1, M critical square.

In principle this can be solved, since C p 0 is known, M critical can be solved from this equation. However, this is an implicit equation and finding the value is not straightforward, but it can be solved quite easily. However, even further simplification can be made, which needs some sort of approximation of this equation. So, if we approximate it. So, we can say further approximation can be made; using that 1 minus M critical square is considerably less than 1.

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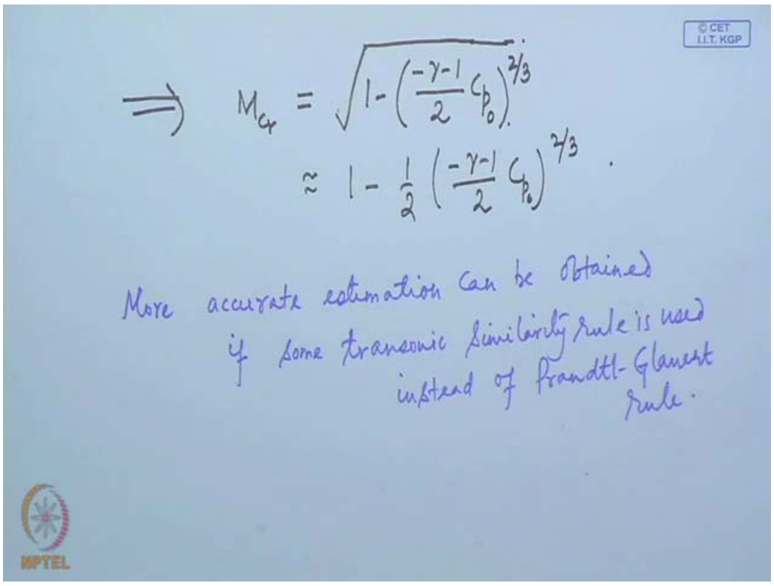

And this then, can be solved that 1 minus, minus gamma minus 1 by 2 C p 0 to the power 2 by 3 z is equal to 1 minus half into minus gamma minus 1 by 2 C p 0 to the power 2 by 3. So, this is of course, an approximate form and little more accuracy can be obtained if we use the earlier relation that is, this relation more accurate estimate can also be obtained if we use transonic similarity rule instead of prandtl-glauert rule so can say that for more accurate estimation more accurate estimation, can be obtained it from transonic similarity rule is used, instead of prandtl-glauert rule.

That is, if we replace this by some transonic similarity rule, this is replaced by some transonic similarity rule we can get a better estimate of critical Mach number. However, this is even the prandtl-glauert similarity rule is quite acceptable. So, we have discussed how quite easily can an estimate of the lower critical Mach number or the critical Mach number can be obtained knowing the incompressible flow pressure distribution or in particularly the suction peak pressure and as we have mentioned earlier that, estimating the lower Mach number is very important, because that is the most preferred operation operational point at least for commercial airliners, where it has the advantage of the higher speed without paying the penalty of larger drag and hence it is attempted to increase the critical Mach number by careful designing of airfoil.

So, we can see that any airfoil where the acceleration is very rapid as in case of say the four digit naca series that the acceleration downstream of the leading edge is very rapid and the critical Mach number or the suction peak point is reach very quickly. When, the acceleration become even faster is if angle of attack is increased. So, if we change the shape of the airfoil. So, that the acceleration is milder and little then, we can have an airfoil in which the critical Mach number is still increased. Eventually, this is the practice followed for designing the supercritical airfoils. So, this is all about estimating the critical Mach number.

Now, we will mention a few words about the solution of the transonic flow equations. We have earlier mentioned that, the transonic flow equations even in small perturbation case are non-linear and analytic closed-form solutions are, not readily available. However, in the earlier days some classical solutions are obtained, using a particular approach known as hodograph transformation. Of course, we will not try to solve the problem, using this hodograph formulation, but for the completeness of our discussion. We will mention, what this hodograph transformation is and how the hodograph transformation is applied to solve some of the classical problems and one such classical problems is flow past, wedge with after body or flow past a wedge.

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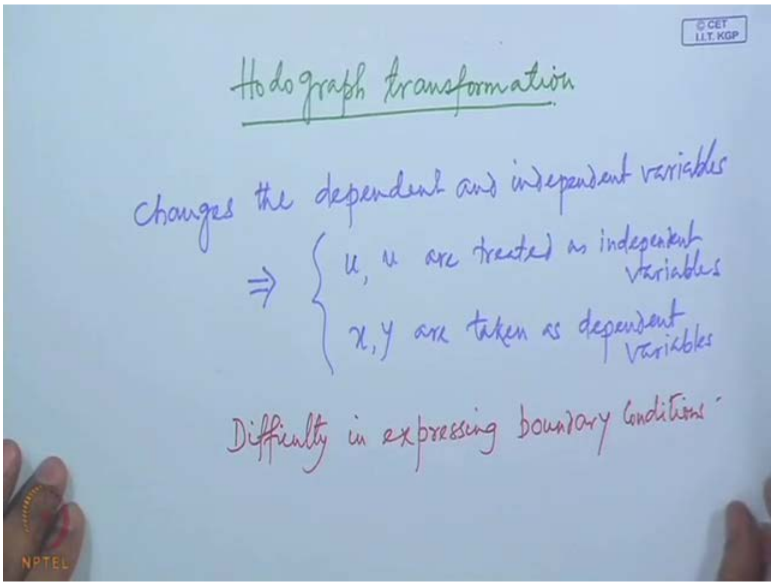

So, in hodograph transformation, so the transformation is hodograph this transformation changes the role of dependent and independent variables. To this changes the role of dependent and independent variables, that is in this context u v are treated as independent variables and x y are taken as dependent variables. Now, immediately the point that comes to our mind that, if we change or geometric coordinates or the independent variables to be the dependent variable in this case and make the velocity components as our dependent, independent variables then, our boundary condition must be expressed in terms of these new dependent variables on in terms of the new geometric the geometric coordinates, but usually the boundary conditions are known in the physical plane. That is at given x and y we have the u and v known or the some condition is known.

But the reverse is usually, not true and is usually not even expressible and they are (( )) the main difficulty of this hodograph transformation technique. So, we say difficulty faced in using boundary condition, that is in the hodograph plane it is usually not possible to express the boundary conditions, accepts for some special cases and that wedge with an after body is such a special case. In which the boundary condition perhaps can usually be expressed in the hodograph plane.

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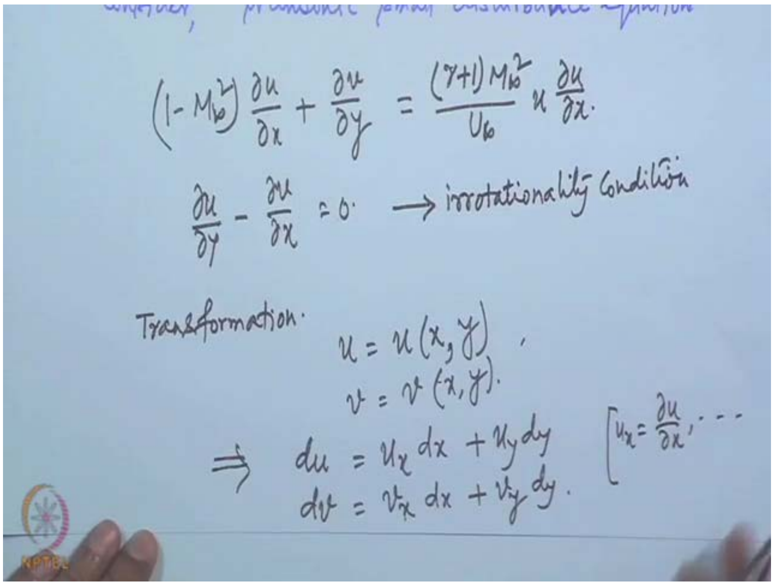

Now, let us see how this transformation is applied. first of all consider the transonic small disturbance equation, consider transonic small disturbance equation and we had that, equation 1 minus M infinity square d u d x plus d v d y and of course, we will be restricting to two-dimensional, the nonlinearity condition in addition with the irrotationality condition, now to apply the transformation let us write, ((no audio 41:07 to 41:44)) and using the chain rule then, this gives us d u equal to u x d x plus u y d y and d v is v x d x plus v y d y, where this subscripts x and y they denotes derivative with respect to x and y. That is u x equal to d u d x and so on.

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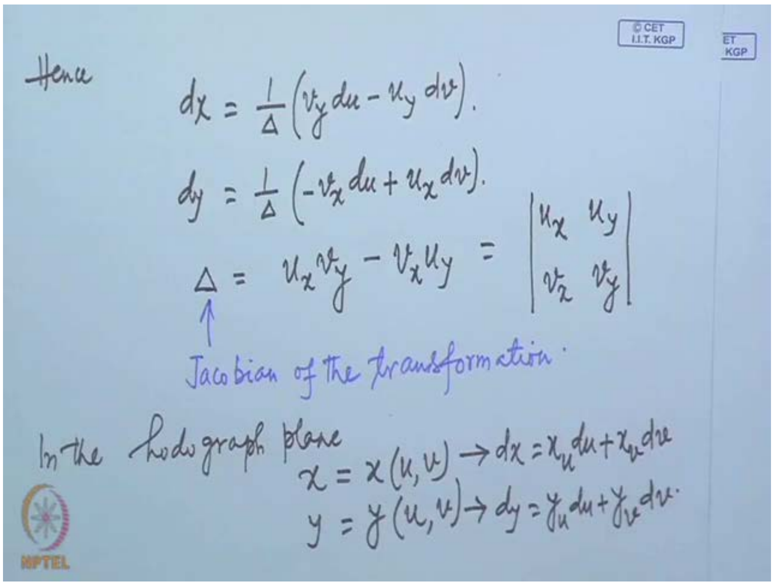

Now, taking these as two algebraic equations, we can solve for d x and d y and solving those 2 we get d x is 1 by delta v y d u minus u y d v and d y is and delta is u x v y minus v x u y. Eventually, it is the Jacobian determinant the Jacobianl of the transformation ((no audio 44:17 to 44:57)) in the hodograph plane in the hodograph plane we have x is the dependent variable, which is function of the independent variables u and v and similarly, y is also a function of the 2 independent variables u and v. This of course, gives d x equal to x u d u plus x v d v and this gives d y is y u d u plus y v d v. Now, we can equate this d x d y with the d x d y obtained here.

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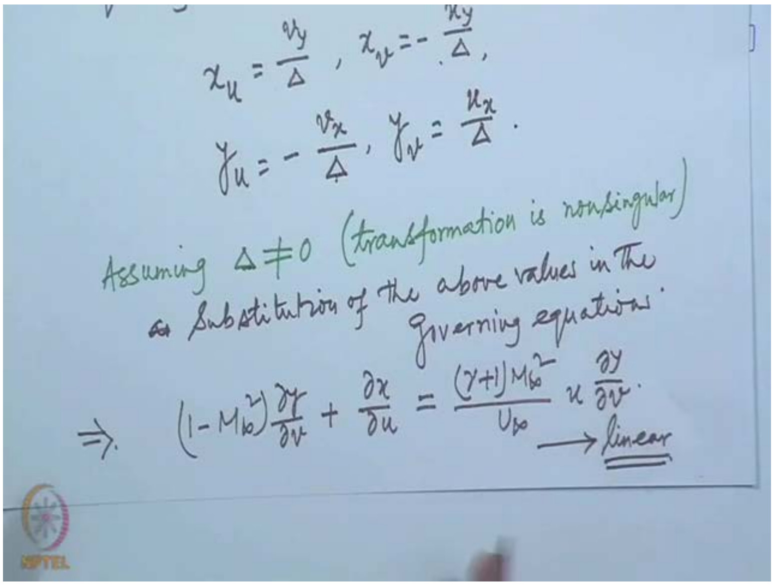

Now, equating these two sets of d x and d u d y, what we get is x u equal to v y by delta x v equal to minus u y by delta y u is minus v x by delta and y v is u x by delta. Now, assuming that delta this delta Jacobian of the transformation is nonzero, that is a transformation is nonsingular ((no audio 47:48 to 48:28)) and you should remember that, if at any point, if delta becomes 0 then, the transformation is not valid substitution of the above values, that is we now have this v y u x u y v x v y expressed in terms of x u x b y u y v and if we substitute these u x u y v x v y in the governing equation that is the transonic flow small disturbance equation and the irrotationality condition, get the governing equation becomes 1 minus M infinity square d y d v plus d x d u equal to gamma plus 1 M infinity square by u infinity u into d y d v and if, we look to this equation you see that this equation is non-linear. There are no dependent variables multiplied with any of these terms.

So, this equation is now, even though the equation looks almost the same, but this equation has changed its nature. Now, here the dependent variables x and y and there is no term in this equation, where there is multiplication of two dependent variables.

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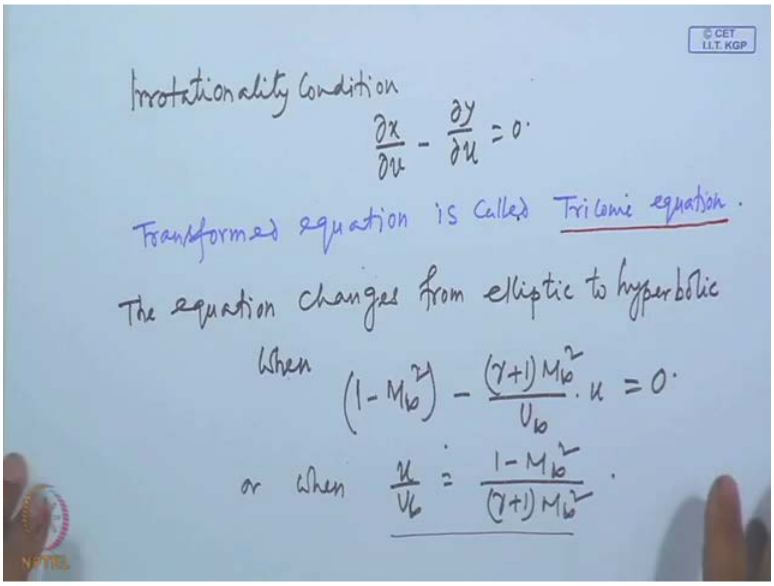

And the irrotationality condition becomes that, now d x d v minus d y d u equal to 0. This modified setup equations are known as the transformed equation is called Tricomi equation. It is called the tricomi equation so, what we see that the in the hodograph transformation, the governing equation has become linear. So, obviously the solution is much simpler, but as in the beginning itself we have mentioned that the transforming the boundary condition is now extremely difficult and in most cases it cannot be done at all and hence, these methods not very useful or not very widely used.

However, as you mentioned that in the earlier days some classical solutions are obtained by using this method. So, further the equation changes from, elliptic to hyperbolic when, 1 minus M infinity square minus gamma plus 1 M infinity square by U infinity into u equal to 0 or when u by U infinity equal to 1 minus M infinity square by gamma plus 1 M infinity squared. So, we see that even the transformed equation, has this properties that it changes its nature from elliptic to hyperbolic, which is of course, the nature of the physical problem of the transonic flow.

So consequently that, some difficulty of solving that transonic flow is still remains in the hodograph plane, because of change in nature of the flow. However, the equation is linear and solutions is comparability easy, but as you mentioned that due to the application of the boundary or transforming the boundary condition is so (( )) impossible that, there are method of hodograph plane solution is restricted to only a few classical problems. That, wedge with after body is one such example. However, we will not pursue that solution and we will discuss our discussion on transonic flow concluded at this stage.