High Speed Aerodynamics

Prof. K. P. Sinhamahaptra
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

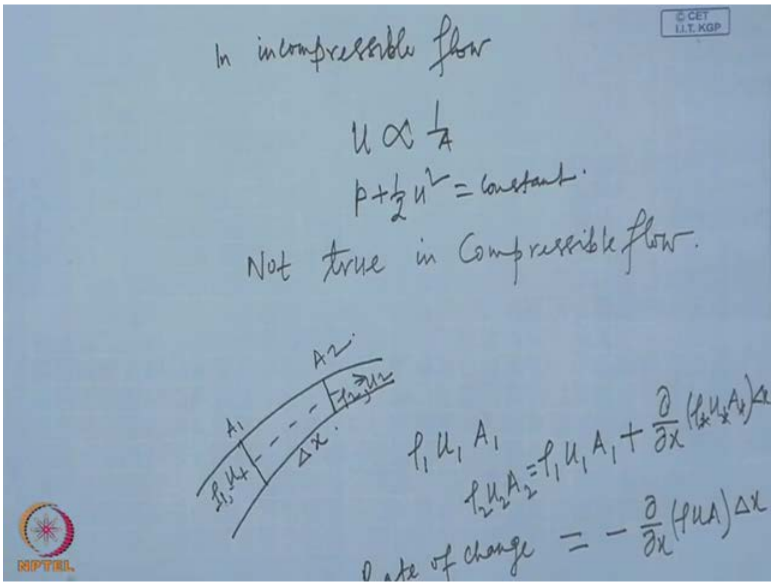
Module No. # 01

Lecture No. # 05

One-dimensional gas dynamics

We will discuss one-dimensional gas dynamics today. This one-dimensional gas dynamics or one-dimensional compressible flow can be used to analyze the flow through tube, flow through channel forever. The geometry can be expressed by the variation of cross section along its axis. So, this can also describe the flow through a wind tunnel, flow through nozzle, flow through various other ducts and where the flow properties at each cross section are considered as uniform. That is, the pressure density temperature velocity at any actual location over the entire cross section is given as a function of the actual coordinate x. The flow quantities of course can be time dependent. We do not exclude the possibility of unsteady or time dependent flow and the results that you obtain they can be applied to flow where there are non uniform conditions at certain cross sections, provided that there are some conditions. Cross sections where the flow properties are uniform, where the properties are non uniform at any particular stations or at particular cross sections, these results applied to certain average quantities. And, even though these consideration is for one-dimensional flow, the result that we obtain can even be used for individual stream tubes in a three dimensional flow. So, the results are quite general.

(Refer Slide Time: 02:49)



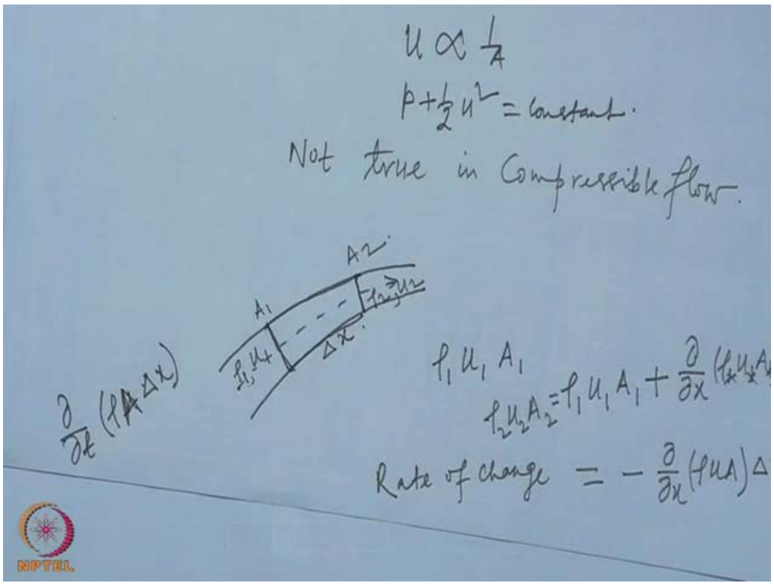
Now, if the flow were incompressible then, the complete one-dimensional flow results or obtained from the kinematic relationship where the flow velocity is inversely proportional to the cross sectional area a. That is, an incompressible flow in incompressible flow the complete relationship is obtained from this relation or A u equal to constant or rather that the volume flow rate at any cross section each constant and then the pressure can be obtained from the incompressible Bernoulli’s equation that p plus half over u v square at any cross section is constant and we get the velocity field as well as the pressure field the only 2 unknown in an incompressible flow. However, in a compressible flow the relationship are little more complex and due to the variation of density along the axis the relationship changes. So, these are not true incompressible flow.

Now, to obtain these flow solutions in incompressible flow, we will first derive the basic conservation laws for compressible flow which includes the mass conservation or the continuity equation the momentum equations or the Euler’s equation for income in visit flow and the energy equation. Now, of course, all these equations can be obtained from the general mass conservation or momentum conservation or energy conservation law that are derived in basic courses in feed mechanics. However, in this case we will derive those equations from the first principles once again applicable for one-dimensional flow.

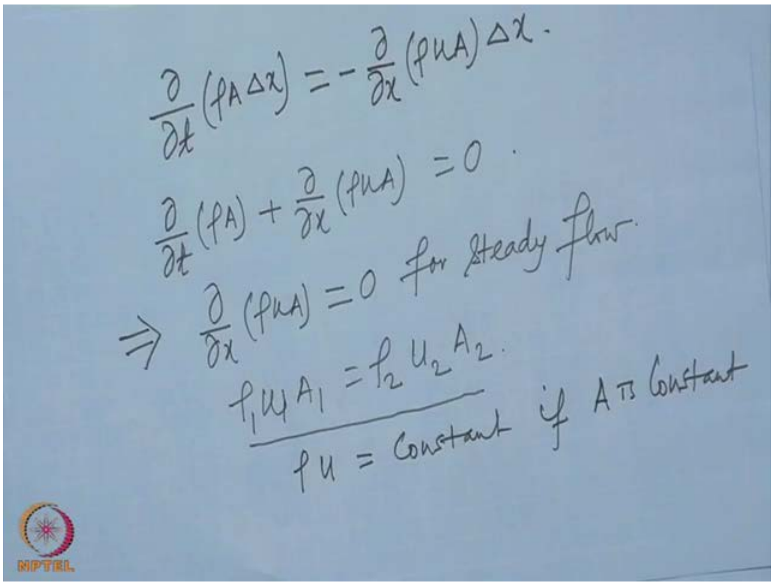
Let us say that we have a tube which we have, a tube with cross section is varying along its axis and you consider this is the x direction. Let us consider 2 sections where the cross sectional area are A 1 and A 2 and the flow properties that is, density velocity are given as rho 1, u 1 and rho 2, u 2. Let this length be delta x; then, the mass flux that enters through the cross section at A 1 is given by rho 1 u 1 A 1 and that at cross section 2 this will be rho 1 u 1 A 1 plus at A 1 rho 1 u 1 A 1 at 2 it is rho 1 u 1 A 1 plus rate of change of this mass flux over the length into the length and this is equal to the mass flux at station 2 rho 2 u 2 A 2.

So, the rate of change of mass flow over this 2 sections is given as minus d d x of rho u A sorry, not rho 1 u 1 rho u A into delta x. Now, this rate of change of mass within this control volume that is enclosed between these 2 cross section A 1 and a 2 and by the wall of this tubes is d d t of d d t of rho u sorry, rho a delta x. That is the rate of change of mass within the control volume bounded by the cross section A 1 A 2 and the side walls.

(Refer Slide Time: 08:06)

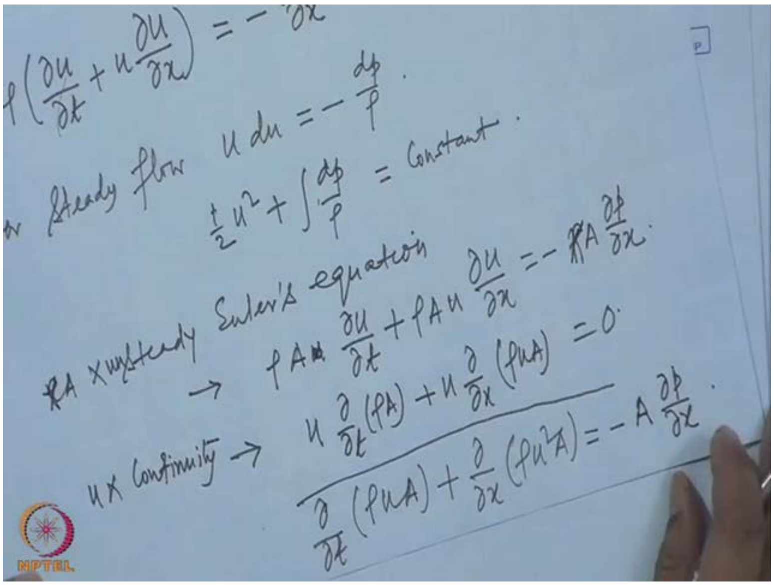


(Refer Slide Time: 08:48)



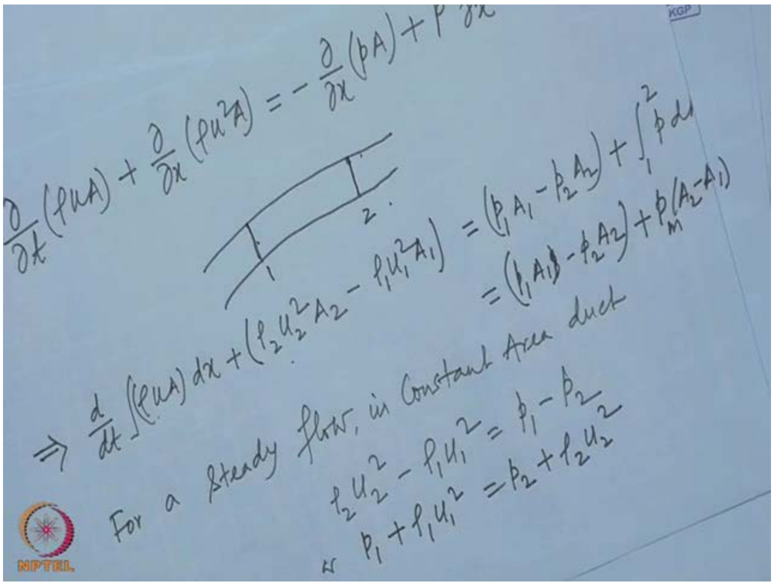
So, what in a sense we get is that, d d t of rho A delta x is minus d d x of rho u A delta x or d d t of rho A plus d d x of rho u A integer this is the mass conversation or continuity equation for one-dimensional compressible flow through a tube or duct which cross sectional area is A. if the flow was steady that is that provide you do not change with time then, we get d d x of rho u A equal to 0 for steady flow which can be written as rho 1 u 1 A 1 equal to rho 2 u 2 A 2. So, this also can be taken as the continuity or mass conservation equation for steady one-dimensional compressible flow that is, the mass flow that at any cross section area remain constant. Whatever mass is crossing a particular cross section that must pass through all other stations and if the cross sectional area remains constant then, this equation changes to rho u equal to constant if A is constant.

(Refer Slide Time: 11:44)



Now, we will consider the Euler's equations or momentum equation for momentum conservation equation for in viscid compressible flow. The one-dimensional form of the Euler's equation or the momentum conservation equation for compressible flow can be written as rho d u d t this equation can be easily obtained from by substituting y and by dropping out the y and z derivative terms from the three dimensional or the general Euler's equation. For in viscid flow, if the flow is steady then the first term that is the d u d t becomes 0 and the equation can be written for steady flow the partial derivatives in this case changes to ordinary derivatives since, the properties p and u are now just function of x only and not of t.

(Refer Slide Time: 17:29)

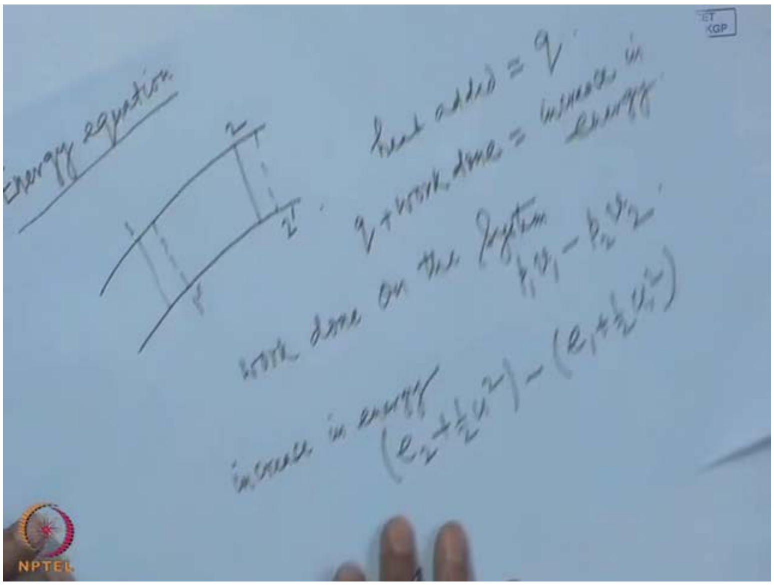


This equation can be integrated to the form half u square plus d p by rho equal to constant the integration can be completed if a relationship between pressure and density is known this Euler's equation can also be converted with the help of the continuity equation to a more useful form which can be obtained in the following manner let us consider the steady flow Euler's equation and lets multiply it by rho A. So, steady rho A multiplied by steady Euler's equation and the result is the result is rho A u or let us say multiply rho A to unsteady Euler's equation. So, that we get rho A d u d t plus rho A c d u d x we also multiply the continuity equation by u which gives u into continuity that gives us u d d t of rho A if these 2 are added we have d d t of rho u A plus d d x of rho u square A ((no audio 16:03 to 17:25)) the equation can further be change to by the right changing the right hand side to a different form as d d t of rho u A plus d d x of rho u square A to if this is this equation is integrated between 2 stations 1 and 2 that is let us say between 2 stations 1 and 2 the result becomes ((no audio 18:31 to 19:26)). This first integral is the rate of change of momentum of the fluid enclosed between station 1 and station 2 and the second term is the momentum flux in the space through the sections 1 and 2 and the right hand side implies the force that is acting in the x direction due to the pressure difference at the 2 end station 1 and 2 and also on the wall.

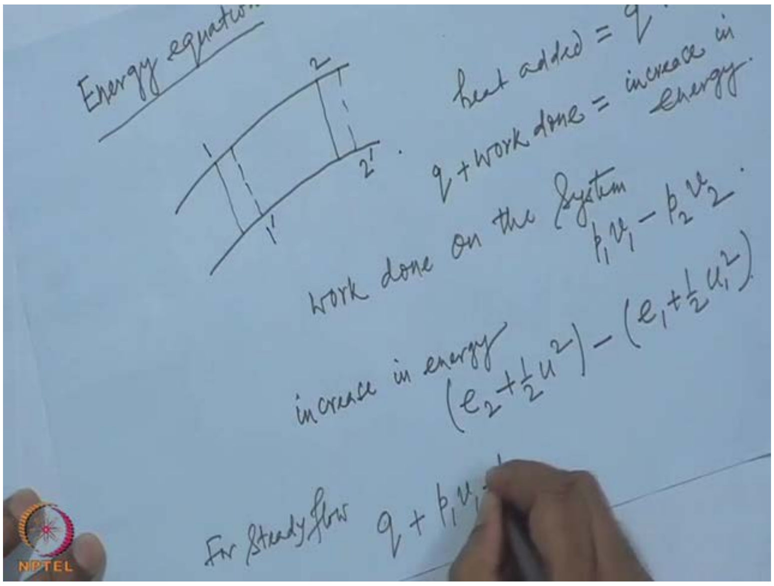
The last term defining out, defining an average pressure over the region the last term can be written as if the flow is steady then the first term vanishes and also, if the duct is of constant area. So, for a steady flow in a constant area duct the equation becomes rho 2 u 2 square minus rho 1 u 1 square equal to p 1 minus p 2 or we have.

Now, this integral form of this momentum equation is more general than the differential form because this integral form is applicable if and when there are dissipative processes within the control process within the control volume provided that the reference station 1 and 2 are in equilibrium states. So, if there are non equilibrium region between one, the station 1 and 2 but, the station 1 and 2 themselves are in equilibrium the integral form of the equation is applicable; however, in that situation the differential form cannot be applied.

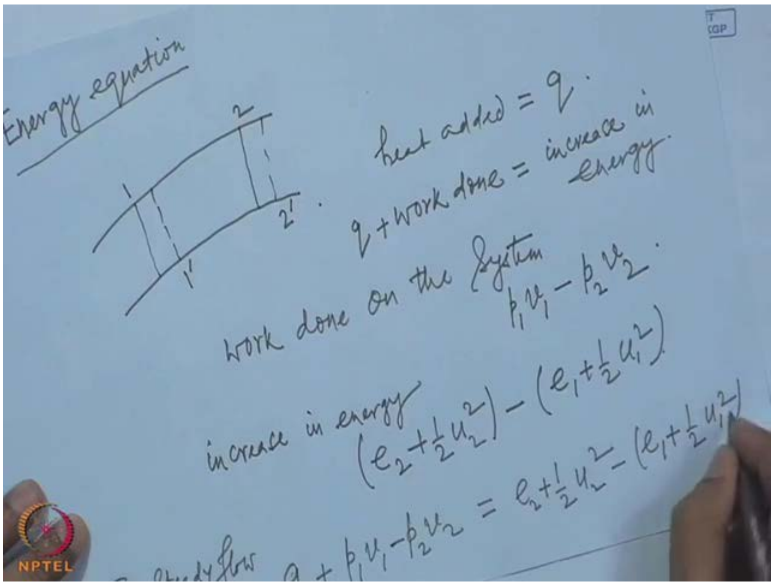
(Refer Slide Time: 23:11)



(Refer Slide Time: 24:00)

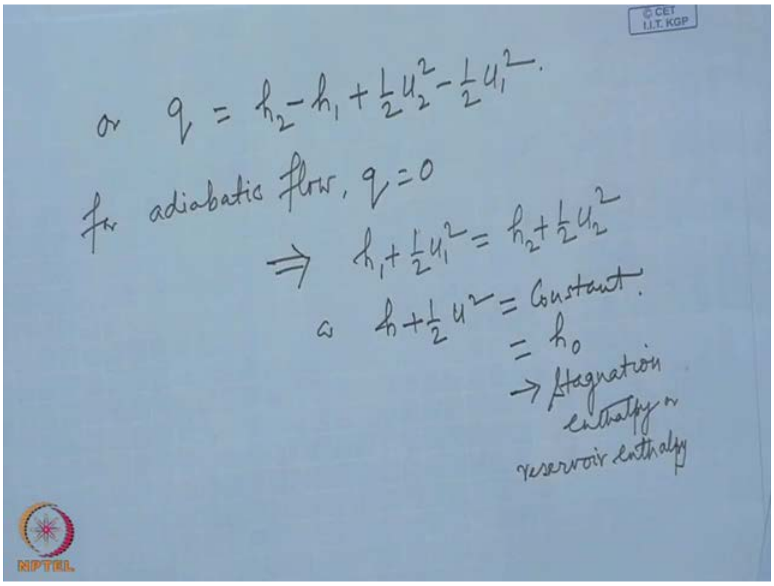


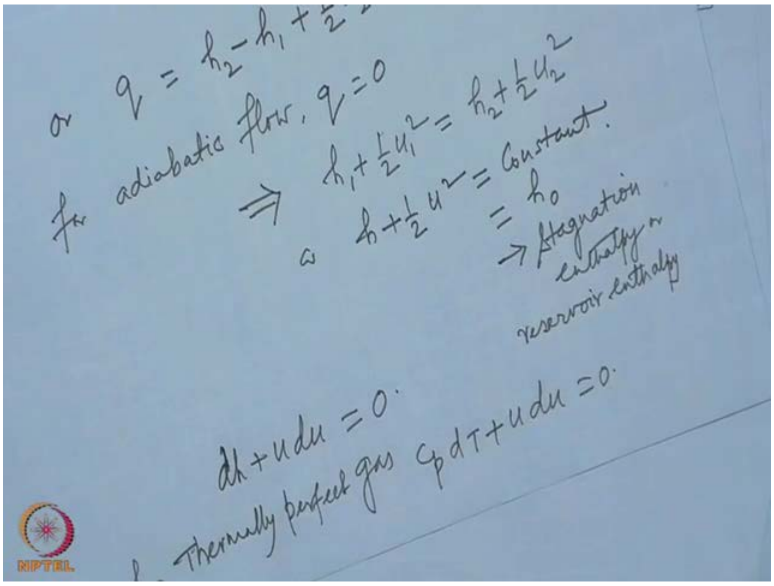
(Refer Slide Time: 25:12)



We will now consider the energy equation for one-dimensional compressible flow energy equation for the energy equation for one-dimensional compressible flow now for a fluid flow problem as we have discussed earlier the basic thermodynamic quantity is a enthalpy instead of the internal energy because of the presence of the flow work now let us consider a definite question of the flowing fluid which is enclosed between station 1 and 2 station 1 and 2 and we call this is the system. Now, consider a small time interval in which the fluid contained between these 2 station 1 and 2 is displaced 2 another region bounded by is in 1 prime and 2 prime and let us say that during this time period a quantity of heat is added if we denote by q. Now, if we apply first law of thermodynamics then we get that the heat added to the system q plus work done on the system is the gain in energy or increase in energy or the system.

(Refer Slide Time: 27:53)

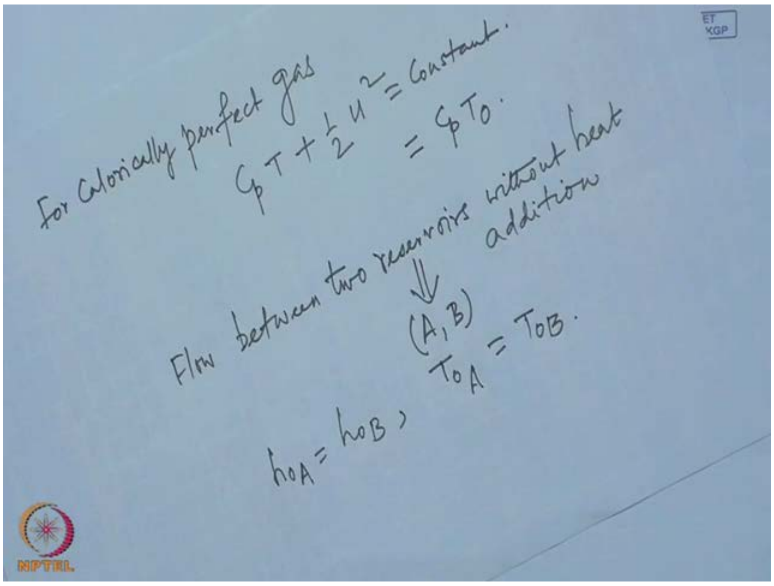


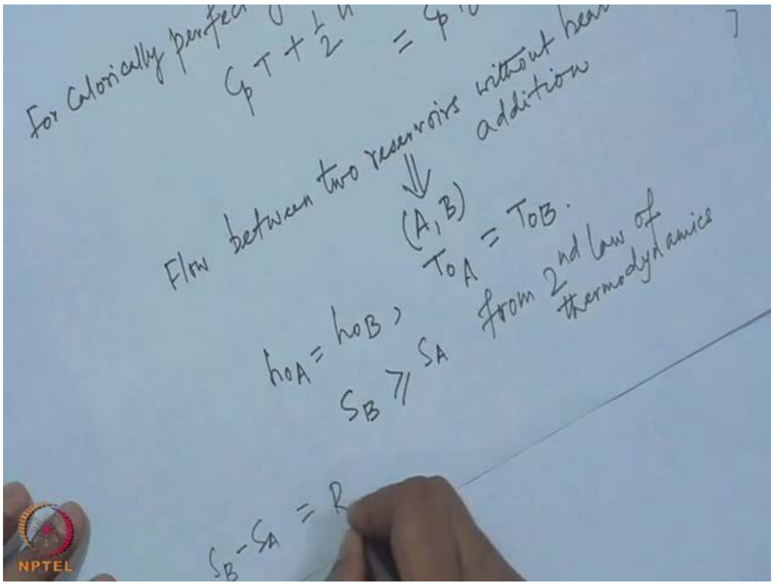


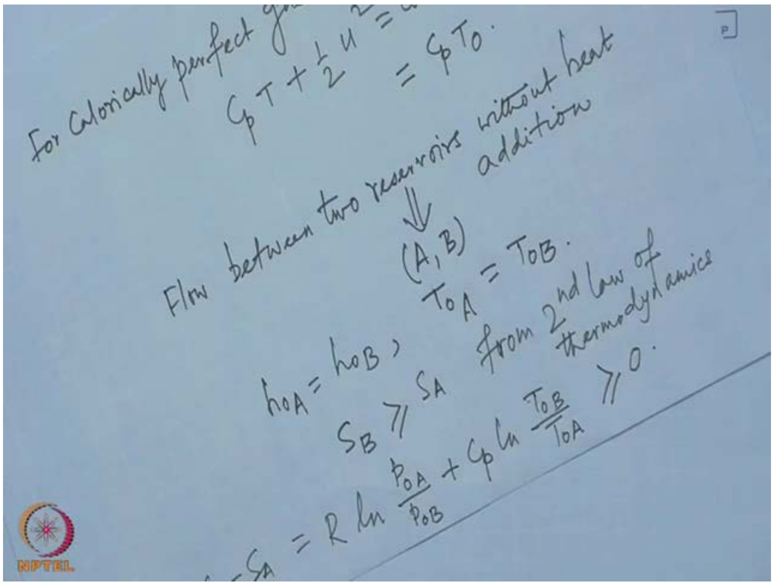
Now, let us estimate the work done on the system let us assume that at station 1 the volume displaced is the specific volume v 1 corresponding to a unit mass then the if the condition is steady then the displacement at 2 is also for unit mass of with specific volume now v 2. So, the work done on the system is work done on the system is simply given by p 1 v minus p 2 v 2 the increasing energy of the system is energy at station 2 minus energy at station 1 is energy at station 2 is the internal energy plus the kinetic energy. So, this is the energy at station 2 minus the energy at station 1 that is e 1 plus half u 1 square. So, for a steady flow we can now write the energy equation we have q plus ((no audio 27:03 to 27:35)). So, this is the energy equation for steady flow through a duct we can further rewrite this equation in the form using the definition of energy we can write q equal to h 2 or h 2 is p 2 plus p 2 v 2 minus h 1 plus the difference in kinetic energy if the flow are adiabatic then q is 0. That is, no heat is added to the system nor taken away from the system that is some of the enthalpy and the kinetic energy remain constant through a duct that is in a adiabatic flow the energy equation simply gives that the sum of enthalpy plus kinetic energy remain constant and this gives rise to the definition of a new quantity known as the stagnation enthalpy, or stagnation enthalpy or reservoir enthalpy or reservoir enthalpy that is the enthalpy when the flow velocity of an adiabatic flow becomes 0. That is, if the flow is brought to rest adiabatically then the total energy of the system or total enthalpy of the system is known as stagnation enthalpy or reservoir enthalpy.

This is of course, the integral form of the energy equation and as before this integral form is valid if there is some non equilibrium or non uniform region between station 1 and station 2 that is even if there is some sort of discuss stresses heat transfer then also the equation remain valid provided at station 1 and station 2 both of them represent equilibrium steps the equation can also be written in the differential form as d h plus u d u equal to 0. This is the differential form of the adiabatic flow energy equation and if the gas where a perfect thermally perfect gas then, these equations can also be written as C p d t plus p u d u equal to 0.

(Refer Slide Time: 32:30)



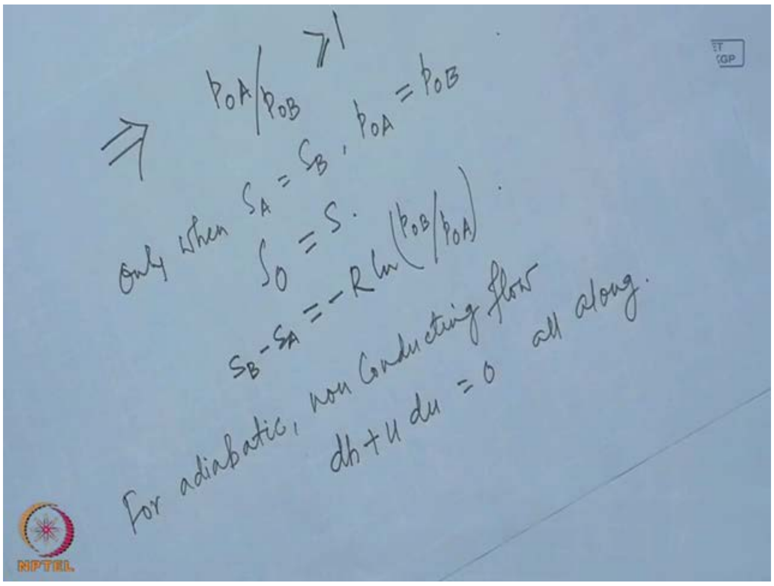


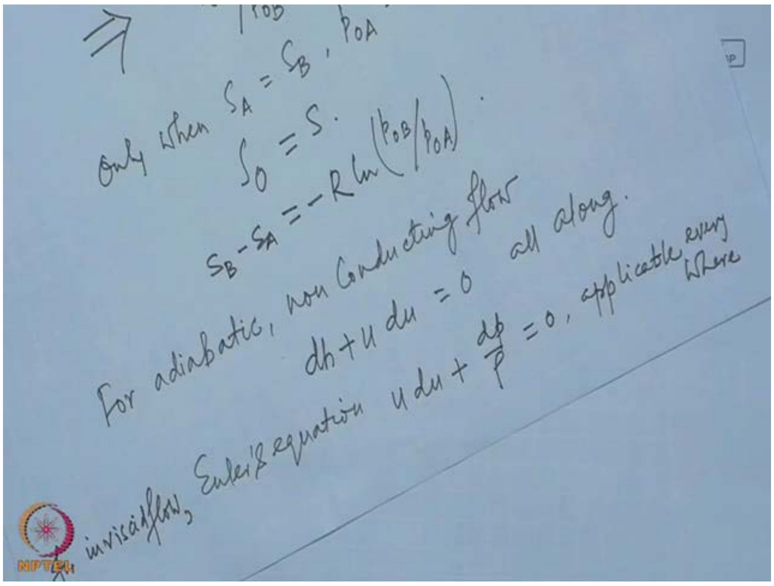


The equation can also be integrated if this specific heat at constant pressure is known as a function of temperature. So, assuming that the gas is calorically perfect where C p is independent of temperature. So, for a calorically perfect gas this can also be integrated and again if u equal to and the fluid is uniform similar to stagnation enthalpy we get stagnation temperature or reservoir temperature. So, reservoir temperature or stagnation temperature as well as reservoir enthalpy or stagnation enthalpy can also be called the enthalpy or temperature of a reservoir where the flow velocity is practically 0 and in which no heat transfer is taking place.

Now, if there are no heat edition between 2 reservoirs then the enthalpy of both the reservoir is same let us say is denoted as eight 0 and considering a calorically or thermally perfect gas the temperature is also that stagnation temperature or reservoir temperature and for 2 reservoirs flow between 2 reservoirs the 2 reservoirs we do you as say reservoirs A and reservoirs B and since the heat there is no heat is added then the enthalpy of reservoir A is same as the enthalpy of stagnation enthalpy of the reservoir B and similarly the stagnation temperature of A is same as the stagnation temperature of A. However, from second law of thermodynamics the entropy of the downstream reservoir that is s B it can either be greater than or A.

(Refer Slide Time: 37:05)



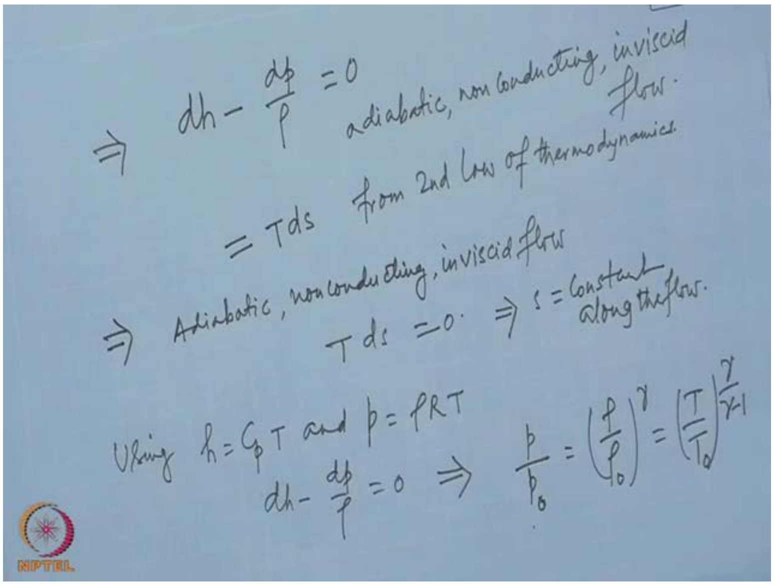


Now, for a perfect gas we have already seen that the difference in entropy is given as ((no audio 36:19 to 36:49)) and which simply gives that p 0 that is the downstream pressure can either be downstream pressure should be or must be less than the upstream pressure. This of course, holds for in a gas even though it is derived for a perfect gas,, but the result is valid for all type of gases that is an increase in entropy at constant stagnation enthalpy will be associated with a degrees of stagnation pressure.

Now, since the increase of entropy with associated decrease of stagnation pressure that represents an irreversible process where entropy is produced in the flow between the 2 reservoirs and the flow is not equilibrium throughout. However, when the flow is equilibrium throughout only then there will be no change in entropy and the entropy will remain same for both the reservoirs and the flow will be termed as isentropic and only in such isentropic processes the total pressure also remain constant. So, the reservoir conditions or stagnation conditions are also called total conditions and these terms are used to define conditions at any point in the flow. So, the total conditions for enthalpy and temperature at any point in the flow can be attained if the flow is brought to rest adiabatically. However, the condition for pressure can only be at end if the flow is brought to rest at brought to rest isentropically for stagnation condition to exist. It is not enough that the flow is brought to rest. It must be remembered that the flow must be brought to rest adiabatically for enthalpy and temperature. But, isentropically for pressure that is the equilibrium condition must exist while the flow is brought to rest or the flow is imagined to be brought to rest the equilibrium condition must always be maintained.

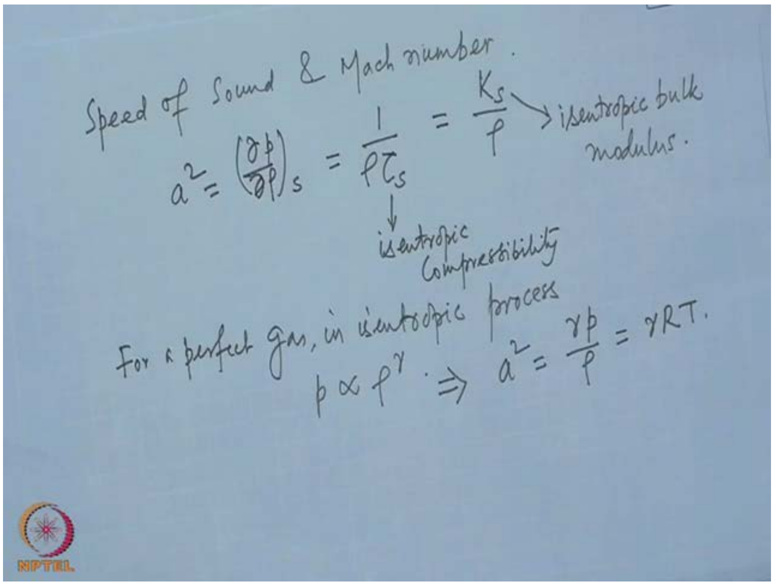
Now, since the imaginary local stagnation process is isentropic the total entropy at any point is by definition equal to the local static entropy that is stagnation entropy is same as the static entropy plus stagnation entropy is same as total entropy and in terms of total pressure the difference in entropy can be written as, what we see from these equations? That a flow which is adiabatic and is in equilibrium throughout, is also isentropic. So, for a conducting adiabatic non conducting flow the energy equation for adiabatic non conducting flow, we have the energy equation d h plus u d u equal to 0 all along the flow. Similarly, we have seen that the in the absence of viscous or fiction forces the Euler’s equation u d u plus is applicable everywhere.

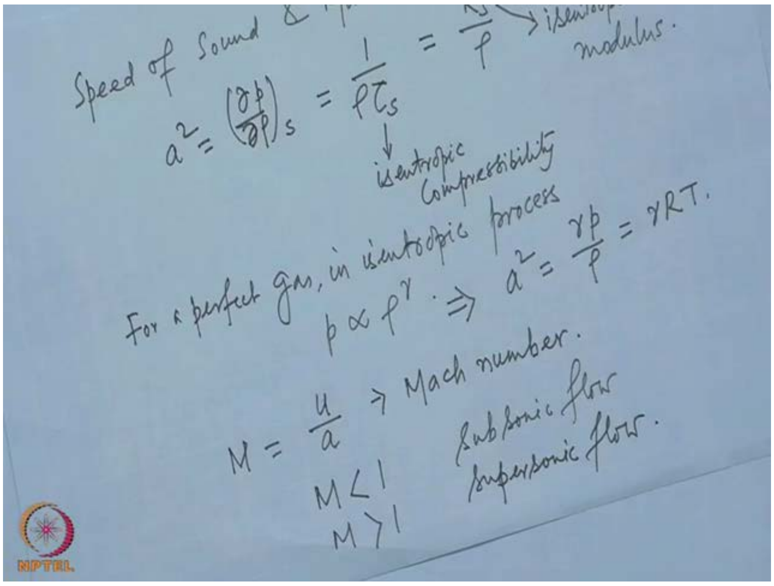
(Refer Slide Time: 43:28)



Now, combining these two equations what we get is combining the 2 d h is for is applicable for adiabatic non conducting in viscid flow that is this equation is applicable for adiabatic non conducting in viscid flow everywhere and using second law of thermodynamics this is and this is equal to T d s . So, you see that for adiabatic non conducting in viscid flow T d s equal to 0 ((no audio 44:47 to 45:35)). That is, you find that these flows are isentropic that is adiabatic non conducting in viscid flows are by definition isentropic flow. Using the perfect gas relationship that is using the perfect gas relationship this relation d h minus d p by rho equal to 0 will give us p by p 0 rho by rho 0 to the power in these relations or can also be thought of an alternative form of equation of state for a perfect gas in isentropic flow with the basic equations for one-dimensional compressible flow. That is namely, the mass conservation or continuity equation the Euler and momentum equation and the energy equation and also some associated relation.

(Refer Slide Time: 48:05)

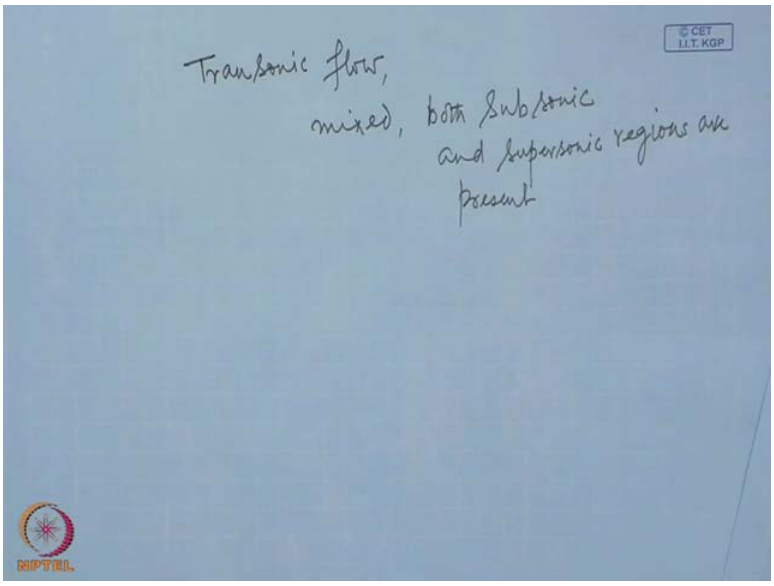




We will now pass on to some new other concepts and new definitions and first of all we will define to a very important quantity in compressible flows those are speed of sound and Mach number now speed of sound is a speed at which a small disturbances or waves propagate through a compressible fluid or in general in any elastic medium the speed at which the small disturbances or waves propagate through a elastic medium which in this case is the compressible fluid. So, it is related to the compressibility of the fluid and mathematically it is defined as the speed of square of speed of sound is d p d rho. At constant entropy that is 1 by rho tau s for tau s is the isentropic compressibility, the isentropic compressibility or can also be written as k s by rho or k s is isentropic bulk modulus.

The disturbance produces temperature and velocity gradients within the fluid; however, these gradients are. So, small that the fluid particle undergo the nearly isentropic process and using the perfect gas relationship where for a perfect gas in isentropic process as we have all seen that p is proportional to rho to the power gamma and this results now in a flowing fluid the pressure density and temperature where is continuously and fluid with the speed of sound and this speed of sound is considered as a significant measure the effect of compressibility and a dimensionless parameter is introduced to measure this effect of compressibility and this dimensionless parameter which is nothing but the ratio of flow speed by the speed of sound is called the Mach number. Since both the flow velocity and the speed of sound changes from point to point, the Mach number also changes from point to point. In a flow in an adiabatic flow and increase in speed of sound always corresponds to an increase in Mach number and the flow is divided into different regime flow is divided into different regime based on the value of the Mach number local Mach number.

(Refer Slide Time: 53:49)



When the Mach number is less than 1, the flow is called subsonic and a flow where both the situations prevail. That is, there are some part of the flow where Mach number is less than 1 and some part of the flow where the Mach number is greater than 1. Then, the flow is called as transonic flow that is a transonic flow ((no audio 53:50 to 54:47)). That is, in a flow where both that is in regimes subsonic and supersonic regimes are present simultaneously that is in some part the flow is subsonic in some part of the flow is supersonic then the flow is called transonic. So, we have discussed the one-dimensional flow and the basic governing equations of one-dimensional compressible flow; that is namely, the mass conservation or continuity equation the Euler's equation and the momentum equation and the energy equation from there we have seen what are isentropic flows. And, what happens to the equation of state in isentropic flows? We have also defined stagnation properties or stagnation quantities both total temperature and total pressure and finally, we have defined what is Mach number and the how based on Mach number based on the value of the Mach number the flow regimes are categorized.