High Speed Aerodynamics

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Module No. # 01

Lecture No. # 06

One-dimensional Gas Dynamics (Contd.)

We will continue our discussion on one dimensional gas dynamics and we will try to derive some very important relationship which are quite useful and may be used in many situations. Let us say the energy equation for one dimensional compressible steady flow is h plus half 2 square h constant. If the gas is thermally and calorically perfect the enthalpy can be written as C p T and the equation becomes C p T plus half u square equal to constant and this constant can be written as C p T naught.

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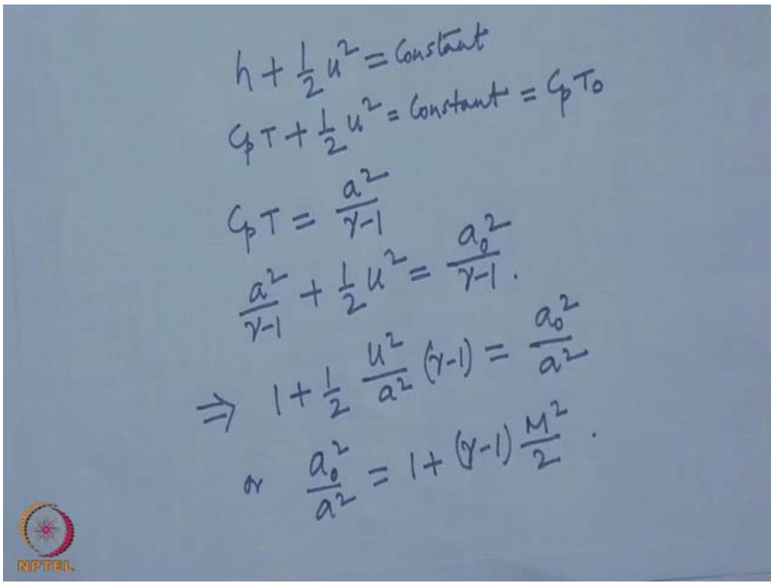

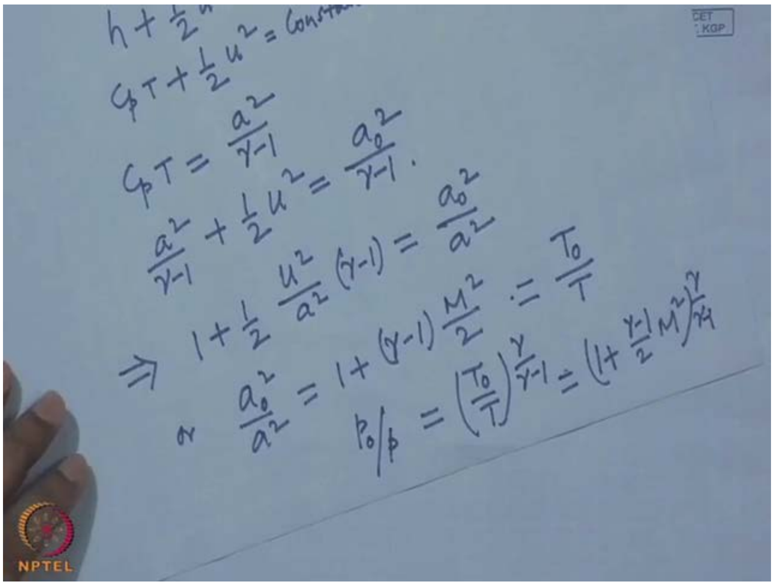

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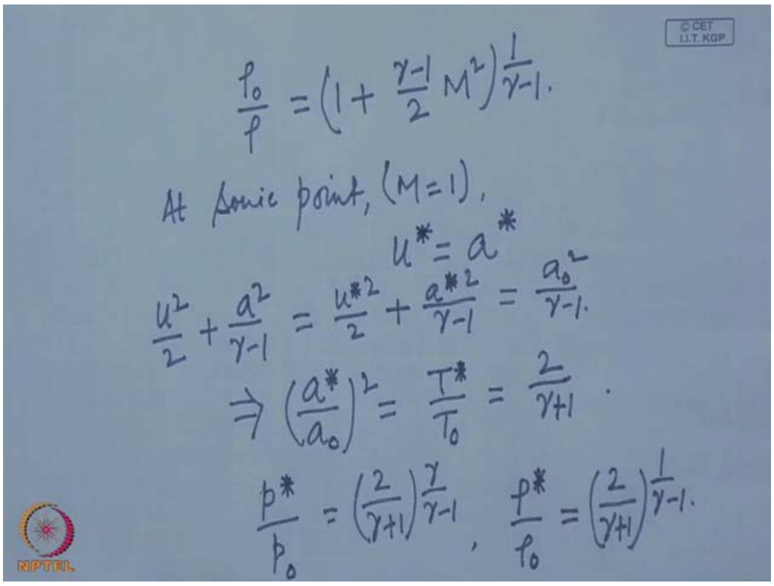

Now, C p can be expressed as C p T can be written as a square by gamma minus 1 or a is the local speed of sound. So, for calorically perfect gas the energy equation can also be written as a square by gamma minus 1 plus half u square. So, this equation can be converted to or a g 0 square by a square is m is the local mach number since a 0 square and by a square can be written as T 0 by T. So, this gives a relationship of the stagnation temperature ratio of the stagnation temperature to local temperature in terms of local mach number and specific gas ration further using isentropic relationship the stagnation pressure by local pressure is and (Refer Slide Time: 04:29). Similarly, the density can also be written as 1 plus gamma minus 1 by 2 m square to the power 1 by gamma minus 1.

Since considered the energy equation for when adiabatic flow or the total enthalpy or total temperature remain constant throughout. So, in these relationship T 0 and a 0 are constant throughout the flow and can be taken as the actually reservoir conditions. However, the process may not be isentropic. Hence, the stagnation temperature and stagnation density or the local reservoir values they will only be constant if the process are isentropic deriving these relationship we have used the reservoir condition as the reference condition; however, on useful reference point can be taken as the point where the fluid sonic or if we may consider that where the mach number is 1 these properties are called then sonic properties and quite often they denoted by a superscript asterisk at the sonic point the flow speed u star at sonic point that is our aim is 1 u star is same as a star and the energy equation can be written as u square by 2 plus a square by and this implies a star by a 0 square; that is T star by T 0 is 2 by gamma plus 1.

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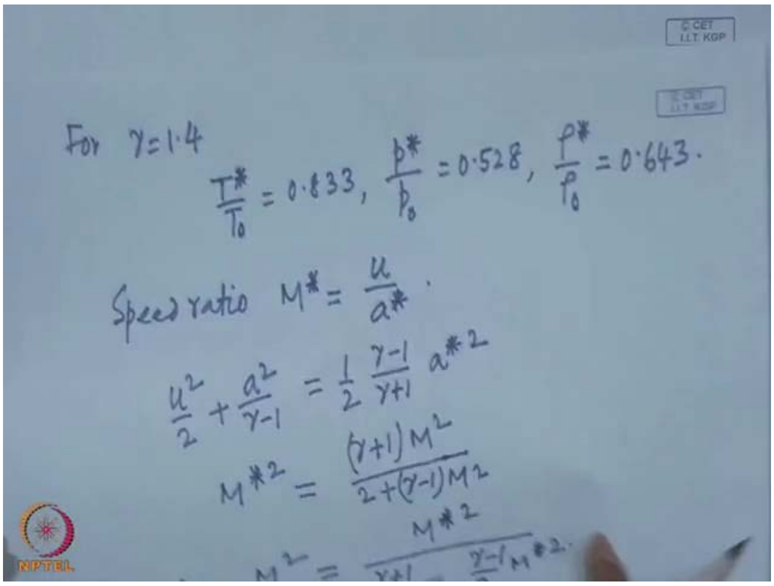

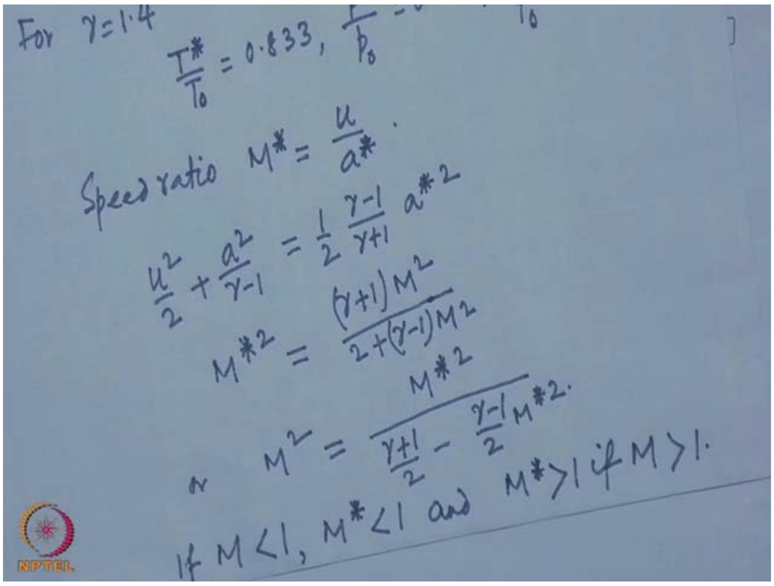

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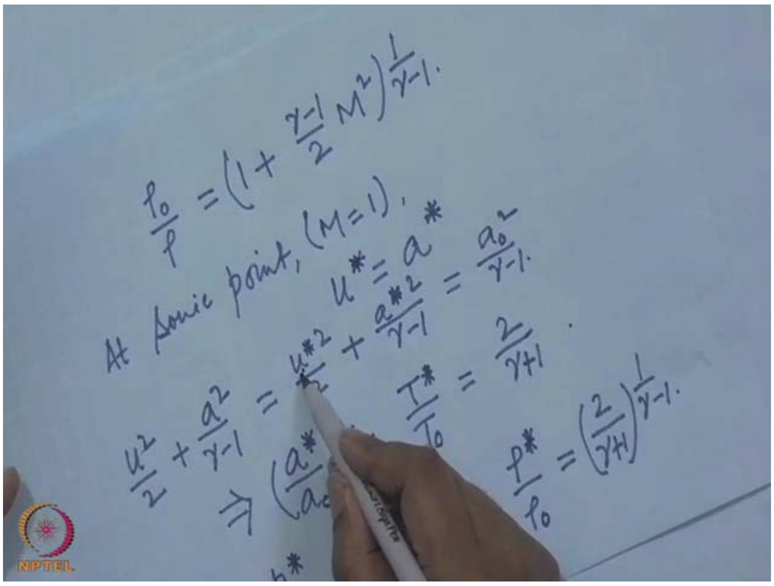

Similarly, we can have the other relationship p star by p naught as 2 by gamma plus 1 to the power gamma by gamma minus 1 and these equations can be used even if there is no point in a flow where the mach number is really unity as in case of reservoir condition that (( )) stagnation point may not be present in the flow. But, the stagnation point always refers to a local condition that if the flow at that point or brought to rest adiabatically for temperature or enthalpy and isentropically for pressure. Similarly, local sonic condition can also be ensured in the sense that we may always think that the flow at that point or brought to the sonic condition adiabatically or isentropically as the case may be. This receives take a definite value if you have a fix if you have a known value for the specific gas ratio gamma for here it is usually taken as 1 point 4 and consequently these relations these values become T star by T 0 is point 833 p star by p 0 is point 528 and point 643 the speed ratio in star which is given as u by a star is a very convenient quantity and used in many situations using the energy equation in the form this equation can very easily be obtained by substituting u star equal to a star in this equation (Refer Slide Time: 11:45). Now, this equation can be simplified to M star square equivalent to gamma plus 1 or alternatively M square is equal to this relationship. So, that is, if M is less than 1 M star is less than 1 if M is less than 1 M star is also less than 1 and M star is greater than if M is greater than 1.

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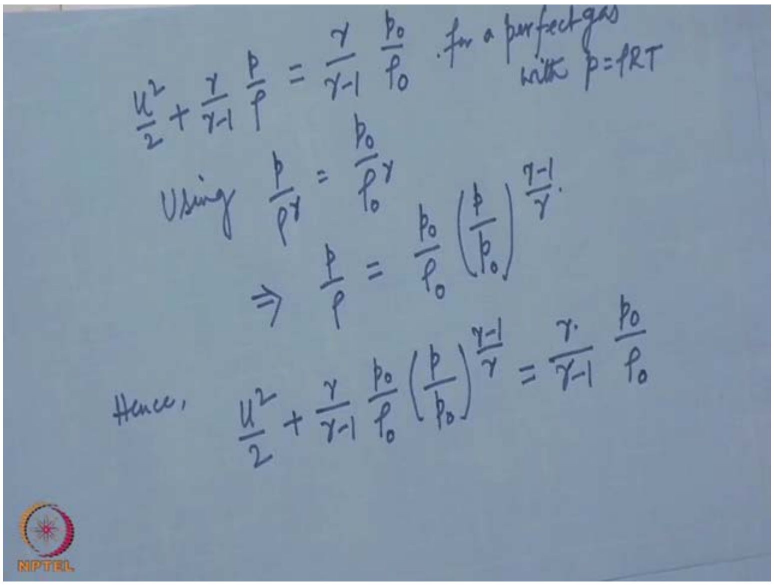

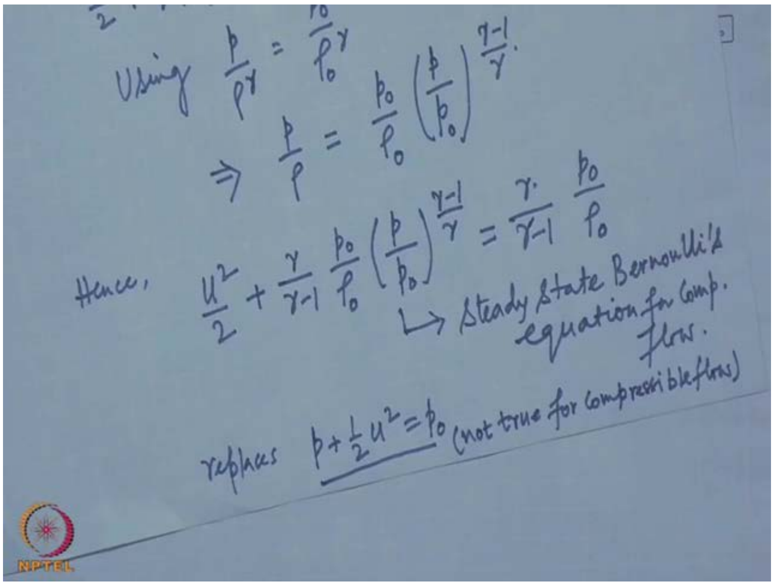

The energy equation can also be written in the form 2 square by 2 plus gamma by gamma minus 1 p by rho for a perfect gas using the isentropic condition using isentropic condition p rho to the power gamma which implies p by rho to be p 0 by rho 0 into p by p 0 to the power gamma minus 1 by gamma. So, the energy equation then becomes ((no audio 15:31 to 16:17)).

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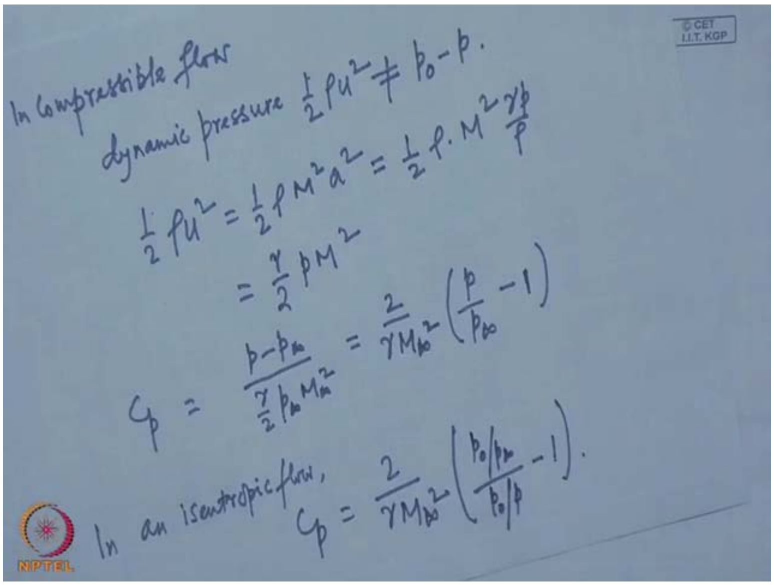

So, this now then becomes the relationship between the velocity and pressure in terms of stagnation quantities in and one dimensional isentropic compressible flow and can be thought of is the steady state Bernoulli's equation for compressible flow for compressible flow that is that this now replace the conventional incompressible Bernoulli's equation which holds for incompressible flow, but not true for compressible flow in a compressible flow the dynamics pressure. That is, half density time square of velocity which is usually used to normalize pressure and forces is not the difference between stagnation pressure and total pressure. That is, in compressible flow in compressible flow dynamic pressure (( )) the dynamic pressure can be written as in this form ((no audio 19:27 to 20:02)).

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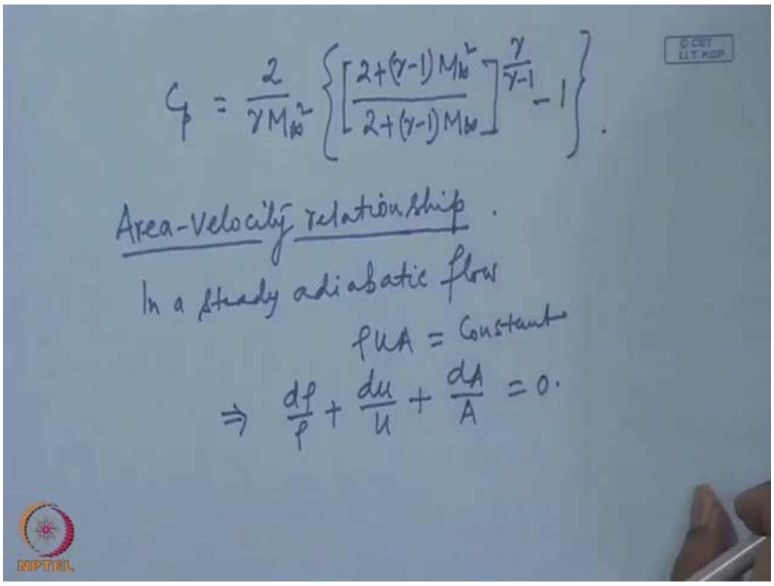

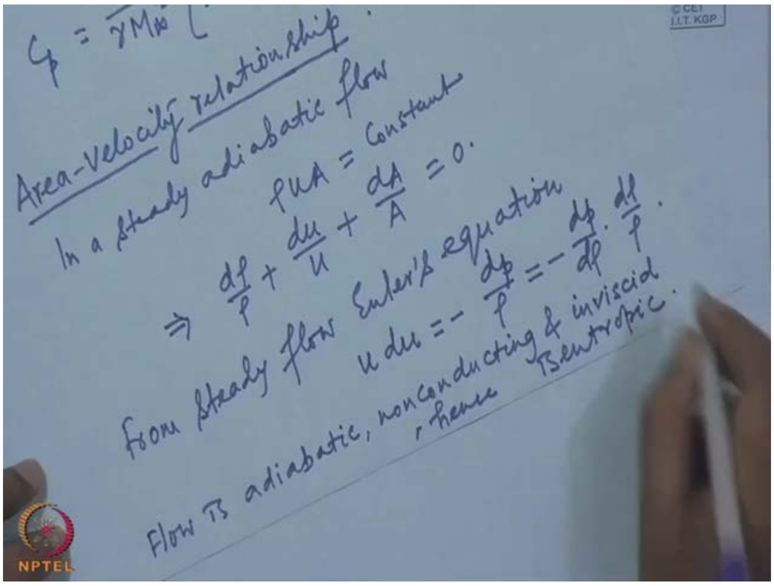

With this the pressure coefficient can be written as ((no audio 20:07 to 20:51)) in an isentropic flow this can this becomes with the help of stagnation pressure which remain constant throughout in an isentropic flow this ratio p by p infinity can be replaced as and using the local mach number this can further we written ((no audio 22:06 to 22:52)).

Through what we see here is that in a compressible flow that are certain special feature that is the Bernoulli's equation takes a completely different form. Then, its incompressible flow counterpart and the dynamic pressure here is not simply the difference between total pressure and static pressure with these additional important relationship which are mostly derive from the consideration of the energy equation.

We look to now a very important relationship in compressible flow which is known as the area velocity relationship area-velocity relationship we know that in incompressible flow the conservation of volume flow rate gives us the direct relationship that flow velocity increases with the decrease in area and the velocity is inversely proportional to the constructional area in a tube ; however, in a compressible flow the relationship become different because of change in density and if we consider a steady adiabatic flow in a steam tube varying area in a steady adiabatic flow the continuity equation rho u a equal to constant the mass flow rate is constant or in the differential form this equation can be written as d rho by rho plus d u by u plus d A by A 0 you know incompressible flow the first term that is d rho by rho becomes 0 and we get our usual well known relationship that increase or decrease of velocity is proportional to decrease or increase of area; however, the change in density we use modify these relationship using steady flow Euler's equation (( )) much…

Using steady flow Euler's equation we get u d u equal to minus d p by rho which can be written as minus d p d since the flow that you are considering we have all ready assumed steady adiabatic and in inviscid flow then this flows are isentropic and this ratio d p by d rho here ((no audio 27:37 to 28:15)).

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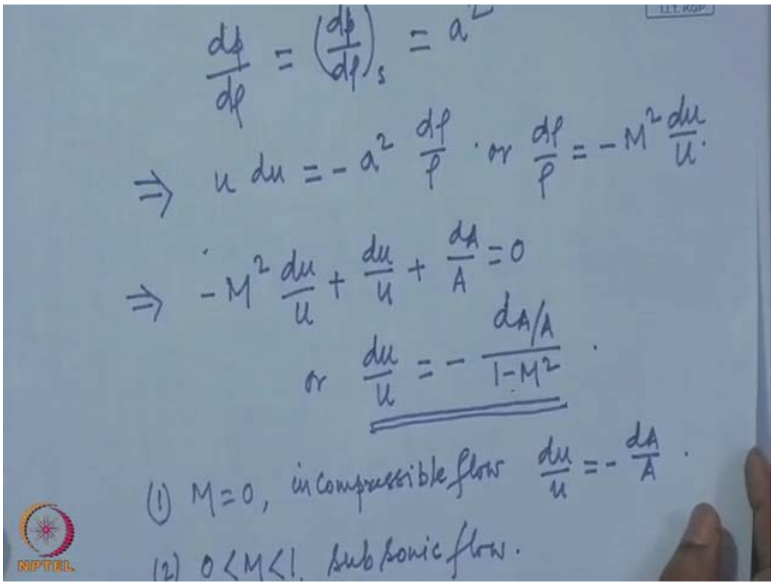

Hence consequently we have this d p d rho is d p d rho isentropic and hence it is a square y. So, the Euler's equation now then become u d u or substituting this in the differential form of the continuity equation we have ((no audio 29:38 to 30:29)).

This is the area of velocity relationship in a compressible flow

Now, let us see what happens at different values of mach number or at different flow condition when the mach number is 0 that is when the flow is incompressible we have which clearly shows that an increase in area gives a proportional decrease in velocity and a decrease in area gives a proportional increase in velocity which is quite well known to us from our incompressible flows

Now, let us consider the case where name is greater than 0 that is subsonic flow we have the denominator here 1 minus M square is positive. So, d u by A is still proportional to minus d A by A; however, there is a constant at any particular subsonic mach number. So, the relationship qualitatively remains the same as in incompressible flow that is increase in area causes a decrease in velocity or decrease in area causes an increase in velocity; however, in this case since the constant is not unity rather a number more the more than unity the effect on the velocity is relatively greater than incompressible flow that is the change in velocity occurs at a faster rate; however, qualitatively still it remains the same that is if the duct is converging the flow will accelerate and if the duct is diverging the flow will decelerate

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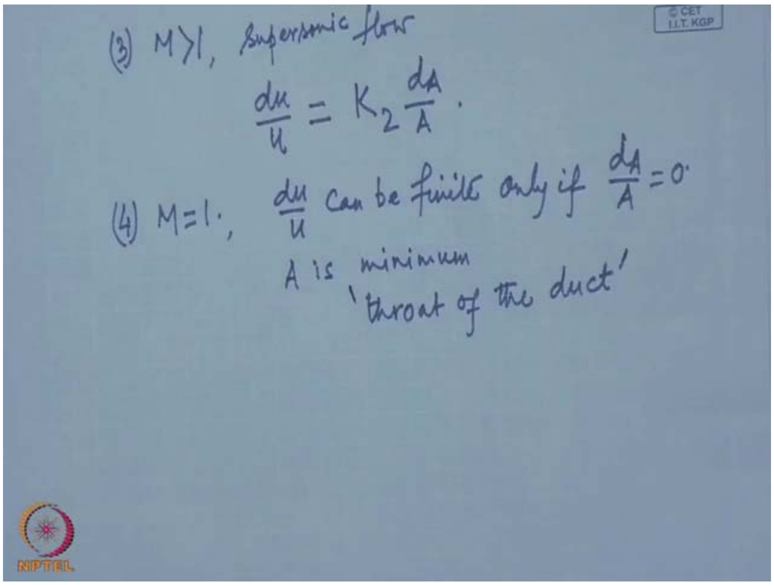

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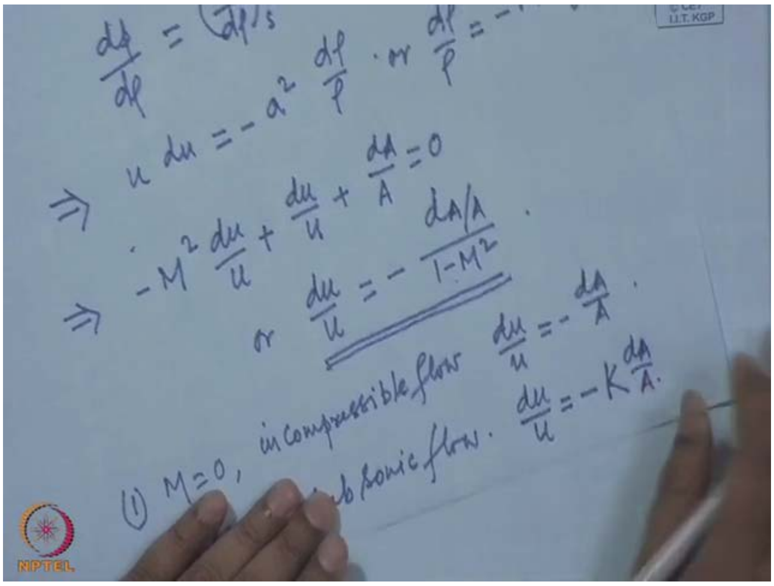

Now, let us consider the third case that is when mach number is greater than 1 or the flow is supersonic we now have (Refer Slide Time: 34:26) the right hand side 1 minus m square is now become negative and consequently the entire right hand side is now become positive. So, in this case d u by u is a positive constant let say K 2 and you see that the flow flow area changes into (( )) that is if there is an increase in area the flow speed increases and if there is a decrease in area the flow velocity decreases that is the flow supersonic flow now accelerate in a diverging duct, but decelerates in a converging duct which is completely opposite to that of incompressible flow

Now, this how can this happen that when the area is increasing the flow velocity is increasing this is due to the fact that at supersonic speed the density decreases at a much faster rate than the velocity increase and flow the area must also must increase. So, to accommodate the same quantity of flow that to compensate the decrease in density both velocity and areas must increase

Now, let us consider the 4th case that is when m is 1 at sonic condition now in this case since the denominator becomes 0 d u by u can only be finite only if d A by A is 0 that is the area of the duct reaches the optimum of course, this optimum in this case must be minimum because we have seen that in the subsonic flow the flow will accelerate if the area decreases and supersonic flow will accelerate if the area is increases if area increases consequently at the sonic speed the area must be minimum. So, or other way that the sonic condition can only be achieved where the area is minimum in a throat that is if a flow accelerates from subsonic to supersonic then the first part of the duct must be converging where the subsonic flow will accelerate and will reach the sonic condition where the area is minimum and then the area must increase further. So, that the supersonic flow can accelerate. So, a mach number unity can only be achieved where the throat is ; however, it does not automatically imply that if there is throat in a duct the mach number there will always be unity that of course, depends on the pressure or the prevailing conditions at the downstream and upstream; however, if the mach number is reached unity it must always reach at the throat of the duct and nowhere else.

Similarly, we can also see that if we have a supersonic flow and we want to decelerate it then the supersonic flow must decelerate in a converging duct and reach the sonic condition at the throat and will again further decelerate in a diverging duct. So, a converging diverging duct is essential if we want to accelerate a subsonic flow to a supersonic flow or we want decelerate a supersonic flow to a subsonic flow of course, asymptotically.

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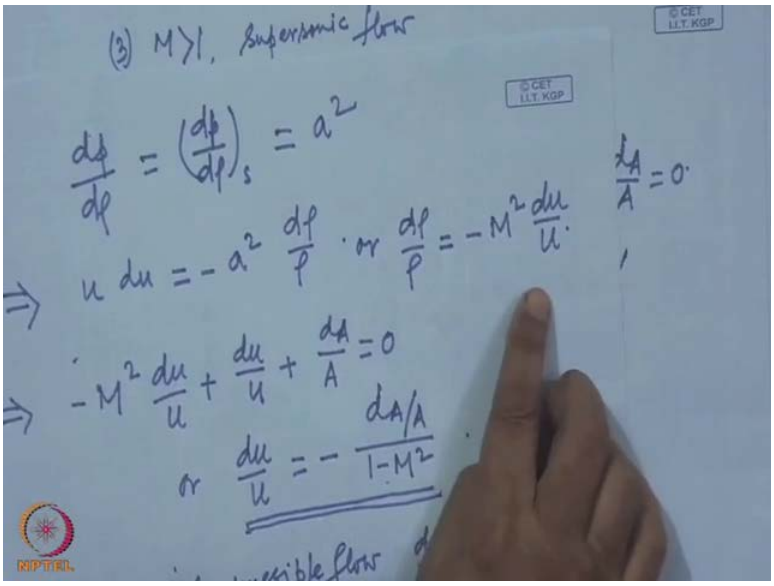

Now, before you go to another topic let say that revise some of these things that we have already discussed today we had first derived certain important relationship using the energy equation in different from where you have obtained relationship between the stagnation properties and local properties also obtain the properties of at the sonic condition most often we have considered only perfect gases; however, for other type of gases these relations relations can be obtained provided we know the equation of state explicitly. So, that we can express the enthalpy in terms of temperature and other parameters we have also seen the form that the compressible Bernoulli equation takes we have discussed that dynamic pressure in a compressible flow is not simply the difference between the stagnation pressure and static pressure and we have expressed the free stream dynamic pressure in terms of free stream total pressure and free stream mach number we have obtained a relationship for the pressure coefficient in terms of free stream properties and local mach number and then finally, we have discussed the compressible flow area velocity relationship where we have seen that we obtain the classical incompressible flow flow area velocity relationship from the compressible flow relationship where mach number is 1 in this context we also seen that when the mach number is very small that density changes are also very small and usually they can be neglected and density can be treated taken as a constant which of course, we can see from these particular relationships (Refer Slide Time: 43:05) d rho by rho is equal to minus M square d u by u which clearly shows that when M square is mach number is very small d rho by rho is negligible and density can be taken as constant

We have also seen that if the flow is subsonic the area velocity relationship is qualitatively same as that for the incompressible flow that is area increase in are is associated with decrease in velocity and vice versa; however, in this case the e ffect on the velocity is relatively greater than in incompressible flow we have seen that you know compressible flow supersonic flow the area velocity relationship changes qualitatively that is in a supersonic flow opposite happens when the area increase the velocity also increases and when the area decreases velocity also decreases that is a supersonic flow decelerate in a converging duct and will accelerate in a diverging duct opposite to that of incompressible or subsonic flow

We have also seen that the mach number unity can be achieved only where the area is minimum of course, in a real situation where that that that mach unity will be achieved or not depends on the upstream and downstream flow conditions; however, in any situation if unity mach number is achieved in a flow that will always be achieved where the area is minimum and we have seen that if we want to accelerate a flow continuously from subsonic to supersonic then we must have a converging diverging duct where the subsonic flow will accelerate to the sonic condition through the converging part and at the throat the flow will reach sonic condition and then downstream the diverging duct the flow will accelerate and this also we have discussed that that in a supersonic flow the density decreases at much faster rate. So, it both velocity and area must increase. So, as to satisfy the continuity equation that is to maintain the constant mass flow rate we should also point out 1 particular fact that when the mach number is very close to 1 then the denominator in the area velocity relationship 1 minus m square is very close to 0 and the flow near the throat is very sensitive to the changes in area with this we will now consider some specific few problems in one dimensional flow and the first case that we will consider is flow through a constant area duct

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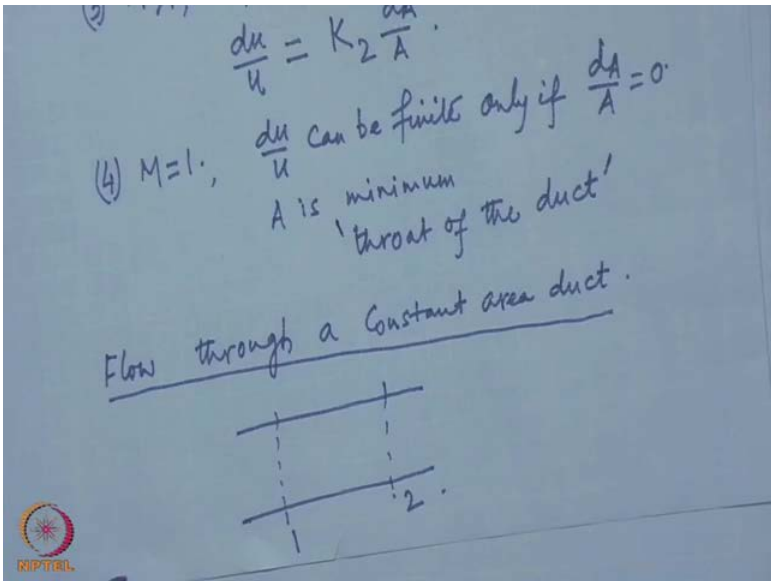

Let us say we have a duct of constant cross section. Let us consider 2 stations; in that, 1 and 2 in incompressible inviscid incompressible flow this problem has just unique solution that all flow parameters at station 1 and station 2 must be the same that is pressure and velocity at station 1 and station 2 must be same and no other solution is possible. However, in compressible flow we will see that besides this condition of uniform flow we can also have a completely different type of flow which is possible that is in compressible flow there are 2 possible solutions for this problem 1 solution is of course, the incompressible solution that is the flow is uniform throughout no change the other solution as you will see subsequently is a jump solution that is the flow parameters at station 1 jumps to at station 2.

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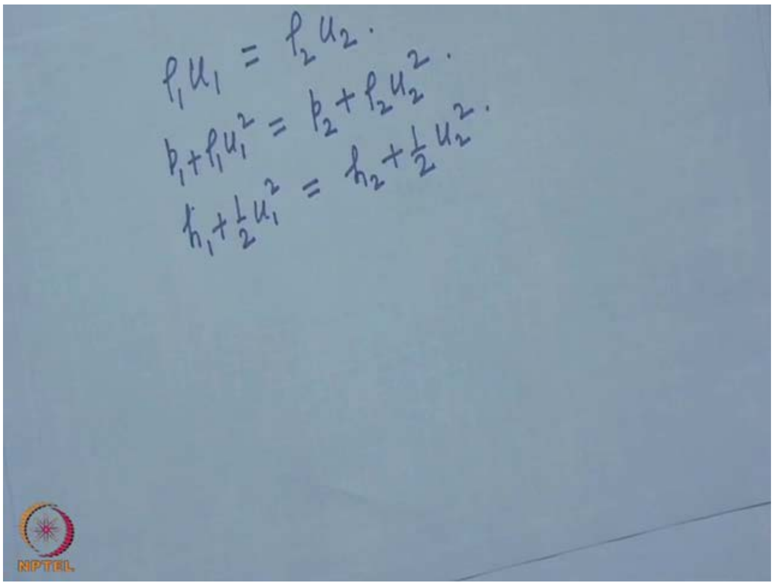

To look into this problem little closely and to find this second solution let us consider the flow problem this way let us satisfy the conservation laws at the 2 stations let us say the properties at station 1 are denoted by subscript 1 and those at station 2 are denoted at denoted by subscript 2 then for a constant area the continuity equation become rho 1 u 1 equal to rho 2 u 2 the momentum equation become p 1 plus rho 1 u 1 square into p 2 and the energy equation become you take mostly as a (( )).

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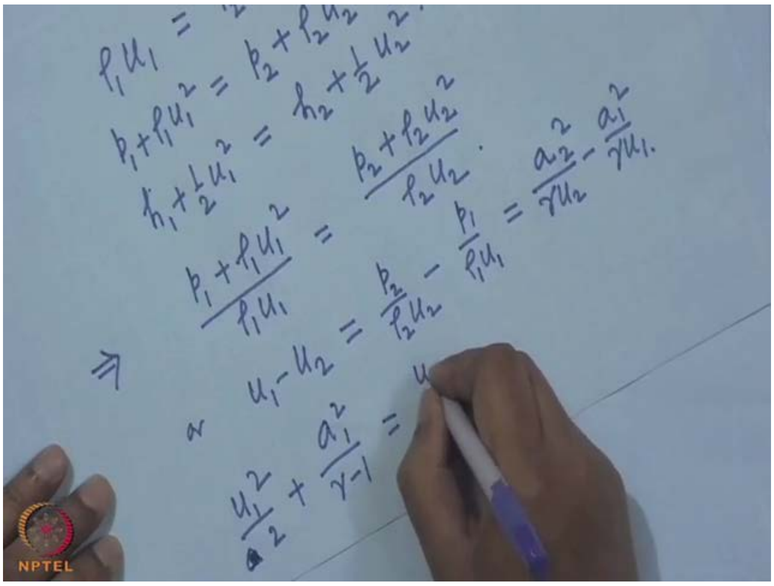

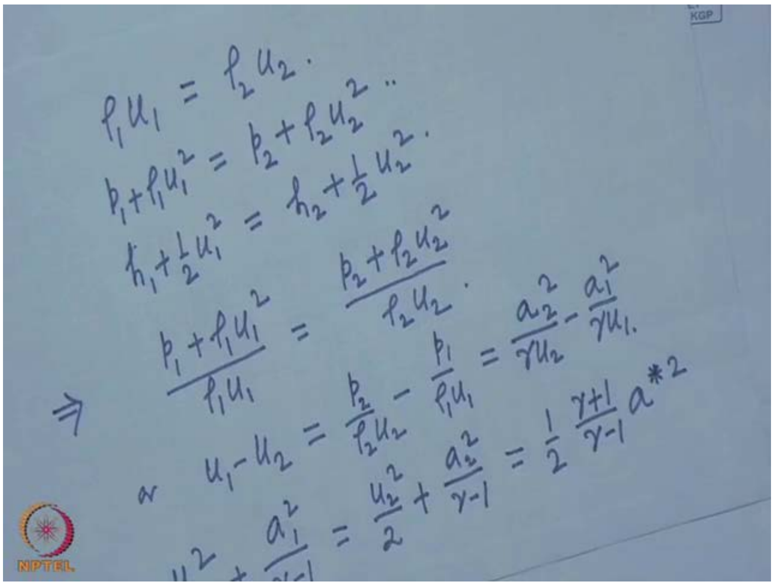

Now if we satisfy this energy equation in the integral form at station 1 and station 2 then there is no restriction on the flow between station 1 and station 2 that is we may have some region of non-equilibrium between station 1 and station 2; however, only requirement is that flow conditions at station 1 and station 2 must be at equilibrium and the equations in integral form apply so; that means, in this case we have the possibility that between station 1 and station 2 there might be a region of non-uniform non-equilibrium that is where the flow is not reversible or can be non-isentropic that there might a region of non-isentropic flow between 1 and 2; however, at region 1 and region 2 the flow is at equilibrium.

However, there is again further no restriction on the side of these non-equilibrium or this dissipative (( )) as long as the station 1 and station 2 or the different stations are outside it this non-equilibrium region may be idealized to be a very thin extremely thin region or almost a line and this station 1 and station 2 can be thought of as the 2 sides of the line and then solution jump solution is jump across a thin line and this is usually then is called this discontinuity is called a shock wave of course, a real fluid cannot have an actually actual discontinuity and this is just an idealization of the very high gradients that actually occur in a very thin region of shock wave that is shock wave in the real flow is basically very thin which is idealized in this case to be just a line and in a real flow over this thin region there is very high gradients of velocity and temperature and consequently they produce large viscous stresses and heat transfer; however, it is restricted within the shock and that is over that very thin region and in this case in the idealized case within that line and outside that line the flow conditions remain equilibrium and our conservation laws in this form are applicable.

Now, we take the momentum equation that is this equation and divide it by the continuity equation that is we write it in this way p 1 plus rho 1 u 1 square by rho 1 u 1 is that is we divide the left hand side of the momentum equation by the left hand side of the continuity equation and the right hand side of the momentum equation by the right hand side of the continuity equation this gives us u 1 minus u 2 using perfect gas relationship we can write this to be now using the energy equation of perfect gas energy equation of perfect gas in terms of speed of sound we write ((no audio 56:23 to 57:09)).

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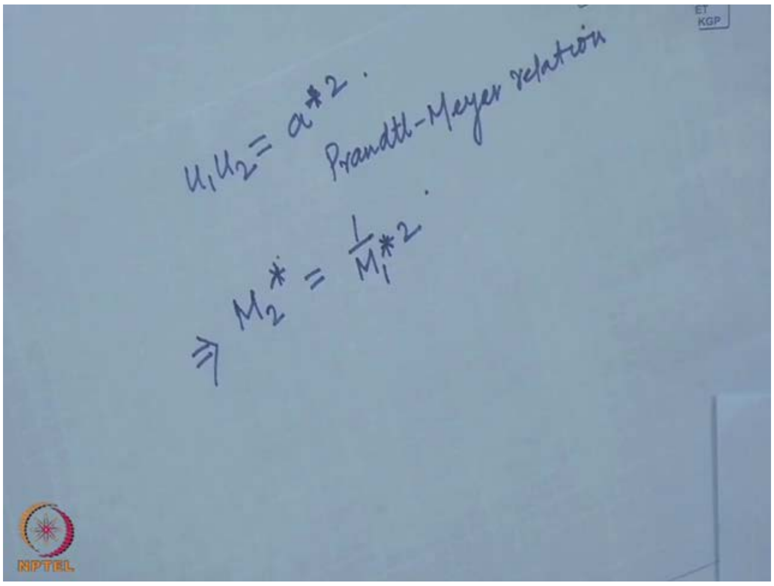

This equation now we can simplify to u 1 u 2 to be a star square which is known as Prandtl-Meyer relation we also can express this relationship in terms of speed ratio which gives M 2 star and this states that if at station 1 the speed ratio is less than 1 at station 2 the speed ratio is greater than 1 and vice versa that is now we have earlier seen that the speed ratio is less than 1 if the flow speed is subsonic and if the flow speed is supersonic the speed ratio is greater than 1. So, it shows here that if station 1 is subsonic then the station 2 becomes supersonic or alternatively if station 1 is supersonic then station 2 will become subsonic that is it is possible to have a jump through the shock from supersonic to subsonic as well as subsonic to supersonic as far as this relationship is concerned; however, there is nothing within this relationship that says whether this is a real possibility or not from the physical ground of course, we can anticipate that the perhaps the accelerating jump is not possible and increase in velocity is quite unluckily unlikely if there is some dissipative processes in action that is through a dissipative region the flow velocity is not likely to increase rather it is likely to decrease. So, physically we can say that supersonic, subsonic to supersonic jump is perhaps not possible while supersonic to subsonic jump is quite possible; however, this will further see clearly considering the second law of thermodynamics which ensures the possibility of a physical process we will see whether this is really possible or not subsequently. However, to continue further (( )).