High Speed Aerodynamics

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Module No. # 01
Lecture No. # 07
One-Dimensional Gas Dynamics (Contd.)

So, continuing our discussion on flow through a uniform duct we have seen that the flow velocity at the 2 stations 1 and 2, satisfy the relationship u 1 u 2; a, a star square or which can be written as M 1 star equal to 1 by M 2 star. That is, the speed ratio at the 2 stations satisfy this reciprocal relationship and which clearly shows that if M 1 star is greater than 1 then, M 2 star will be less than 1 and if M 1 star is less than 1 M 2 star will be greater than 1 and vice versa. We have already seen that M 1 star become less than 1 when the flow is subsonic that is when M 1 is less than 1.

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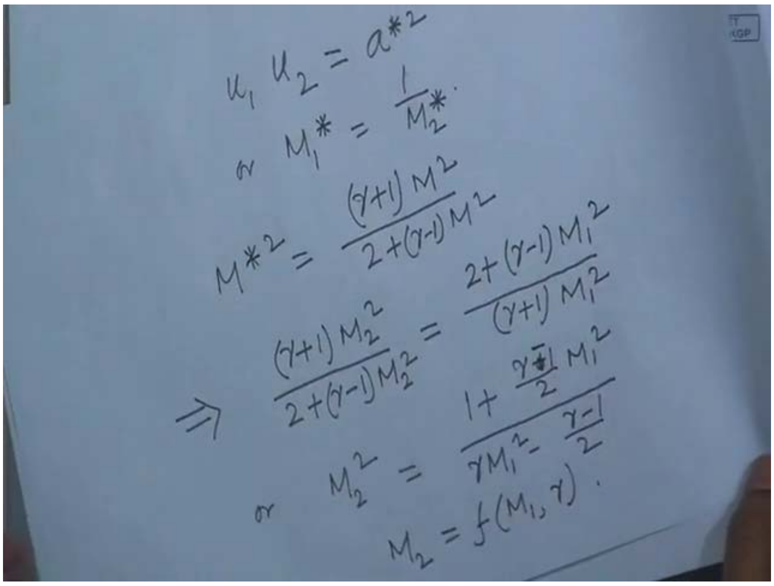

So, what we can see from here that when the flow is subsonic at station 1, the flow is likely to be supersonic at station 2. Similarly, if the flow is supersonic at station 1 it will be subsonic at station 2. However, there is nothing in this relation that excludes this one of these possibilities. However, as I mentioned that based on physical ground that if there is some sort of non equilibrium region between station 1 and station 2 where the viscous effects or the heat transfer is present then, through this dissipative processes acceleration flow is physically not possible and the jump from subsonic to supersonic is most probably will not occur in a real flow. However, we will later on show it using second law of thermodynamics that the deceleration from subsonic, supersonic to subsonic flow is physically possible. However, the alternative subsonic to supersonic acceleration through this jump will not be possible using the relationship between the speed ratio and the local Mach number which you have already derived and is given as M star square is gamma plus 1 M square by 2 plus gamma minus 1 M square. So, substitute substituting this speed ratio at station 1 and station 2 in terms of the Mach number at station 1 and station 2 we get the relationship as gamma plus 1 into square by 2 plus gamma minus 1 which can be written in the form 1 plus gamma minus 1 by 2 M 1 square by gamma M 1 square minus gamma minus 1 by 2 that is the Mach number at station 2 is simply a function of Mach number at the station 1 and of course, from the properties of the gas that is we can write that M 2 is simply a function of Mach number and gamma you can see here that if M 1 is 1 then, M 2 is also 1.

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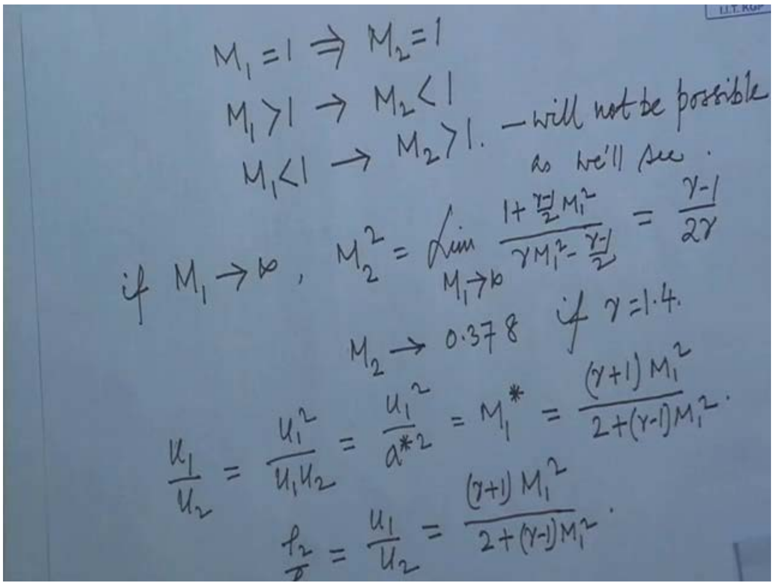

However, if M 1 is greater than 1 this shows if M 1 equal to 1 then, M 2 is also 1 if M 1 is greater than 1 then, M 2 is less than 1 and similarly M 1 is less than 1 M 2 is greater than 1 which of course, we will see later on that will not be possible ((no audio 06:04 to 06:37)) whether you see that as M 1 approaches infinity; a very large value that is M 1 approaches infinity then, which is simply gamma minus 1 by 2 gamma and thus M 2 reaches point 3 7 8 if gamma is 1 point 4.

The velocity ratio between station 1 and station 2 can also similarly be obtained as u 1 by u 2 as u 1 square by u 1 u 2 that is u 1 square by a star square that is ((no audio 08:11 to 08:43)) within the continuity equation for the present problem we all can also see that rho 2 by rho 1 is u 1 by u 2 and again can be written as gamma plus 1 M 1 square by 2 plus gamma minus 1 M 1 square the pressure difference between these 2 stations can be obtained from the continuous momentum equations written in the form p 2 minus p 1 equal to rho 1 u 1 square minus rho 2 u 2 square which can be written as rho 1 u 1 into u 1 minus u 2 where rho 2 u 2 square is written as rho 2 u 2 which is equal to rho 1 u 1 that is since rho 2 u 2 square is rho 2 u 2 into u 2 which is rho 1 u 1 u 2.

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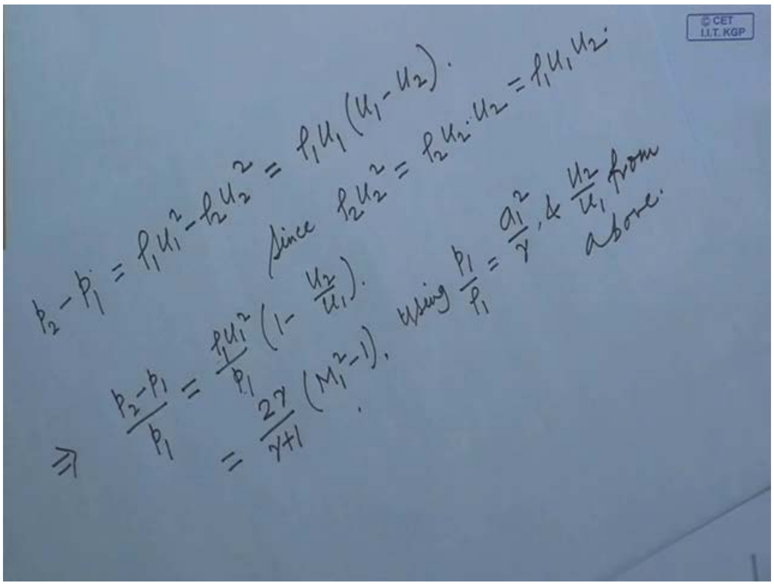

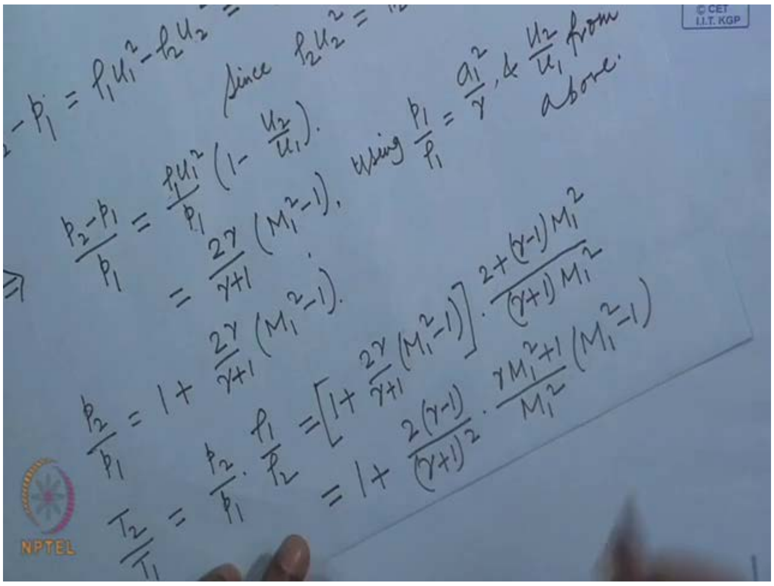

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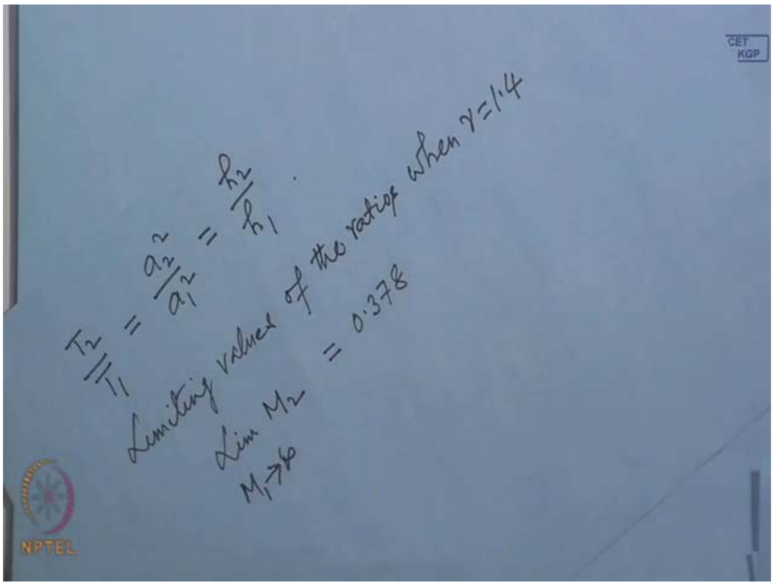

Hence, we get the pressure rise p 2 minus p 1 by p 1 as rho 1 u 1 square by p 1 into 1 minus u 2 by u 1 then, using the isentropic relation p 1 by rho 1 we can write this to be 2 gamma by gamma plus 1 ((no audio 11:30 to 12:06)) the above velocity ratio as you obtained earlier substituting that velocity ratio and using rho 1 p 1 by rho 1 as a 1 square by gamma this is the relationship that we obtained which gives the normalized pressure rise also the pressure ratio p 2 by p 1 then, can be written as 1 plus 2 gamma by gamma plus 1 into M 1 square minus 1 the ratio delta p or p 2 minus p 1 by p 1 is often referred to as the shock strength the temperature ratio can be obtained using the perfect gas relationship p rho p equal to rho r T that is p 2 by p 1 into rho 1 by rho 2 and this gives p

2 by p 1 is 1 plus 2 gamma by gamma plus 1 into M 1 square minus 1 into rho rho 1 by rho 2 which is rho 1 by rho 2 is 2 plus gamma minus 1 M 1 square by gamma plus 1 M 1 square which can be written as 1 plus 2 into gamma minus 1 by gamma plus 1 square into gamma M 1 square plus 1 by M 1 square into M 1 square minus 1 further from the temperature ratio we can obtain the ratio of speed of sound at the 2 stations as well as a ratio of the enthalpy at the 2 stations which are given as T 2 by T 1 for perfect gas.(Refer Slide Time: 15:14)

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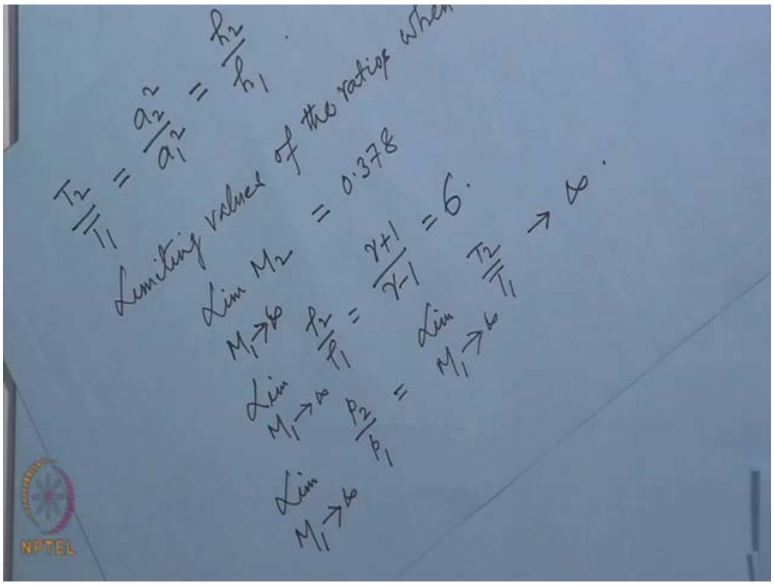

The limiting values for these ratios for air with gamma equal to 1 point 4 limiting values of the ratios when gamma is a fixed quantity given by 1 point 4 as is usually used for air we have already seen that into as M 1 approaches infinity is point 3 7 8 and the density ratio becomes ((no audio 16:38 to 17:20)) approaches to a very high value as the upstream Mach number reaches to very high value to the pressure ratio and temperature ratio also reaches a very high value and looking back to all these relations what we have seen is that for a thermo lean calorically perfect gas all the parameters are function of upstream Mach number and the specific gas ratio gamma.

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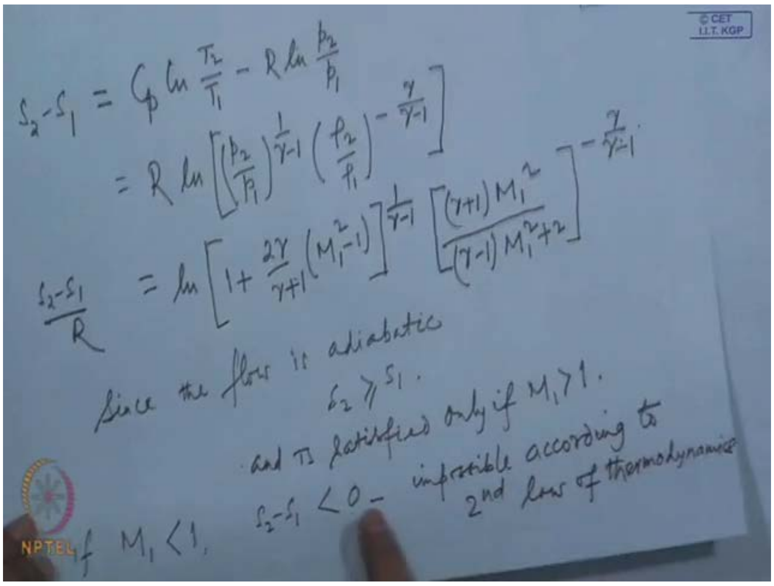

Now, computing the change in entropy between the 2 stations you can write the change in entropy between the 2 station is 2 minus s 1 is C p log T 2 by T 1 minus R log p 2 by p 1 which can be written as R log and substituting this pressure and pressure ratio and density ratio in this relation we have s 2 minus s 1 by R equal to log 2 gamma by gamma plus 1 M 1 square minus 1 ((no audio 19:31 to 20:11)) since the flow process that we have considered here is is adiabatic. So, s 2 must be greater than or equal to s 1 since the flow is adiabatic and the relation it shows here that if M 1 is less than 1 then, s 2 minus s 1 by r is negative and this condition is satisfied only if M 1 is greater than 1 and if M 1 is less than 1 s 2 minus s 1 by r is less than 0 if M 1 is less than 1 s 2 minus s 1 will become less than 0 which is ((no audio 21:50 to 22:24)) that is if M 1 is less than 1 then, we will have a decrease in entropy in the downstream which is of course, not possible as far as second law of thermodynamics is concerned and hence we see a mathematical proof that a jump from subsonic to supersonic flow is not possible in this case or that is through a shock a jump from subsonic to supersonic flow is not possible and the only possibility is from supersonic to subsonic jump.

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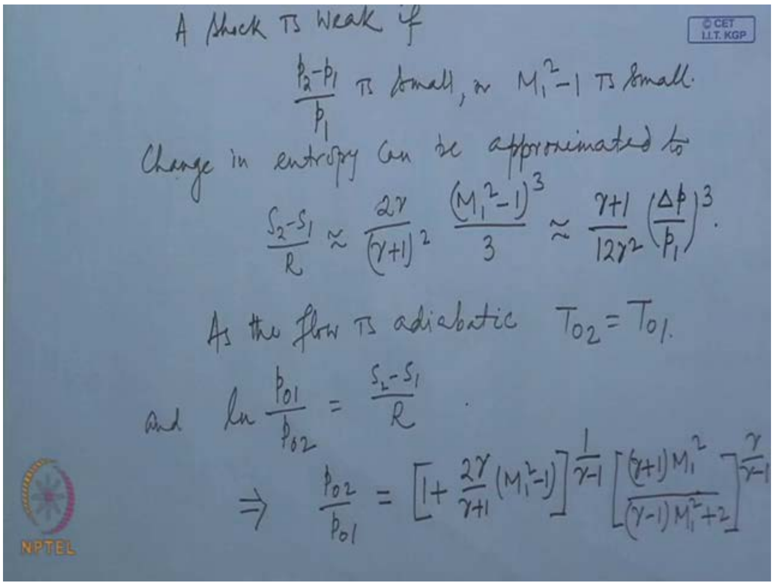

A shock is called weak when the pressure rise across the shock is very small and looking to the pressure ratio relationship we can see that happens when M square minus 1 is small that is a shock is weak ((no audio 23:22 to 23:58 and 24:21 to 25:12)) and when this shock is weak that is when M 1 square minus 1 is small the change in entropy can be approximated the change in entropy then, can be approximated or in terms of pressure ratio gamma plus 1 by twelve gamma square or see that for a weak shock the change in entropy is third order of shock strength that is and in the limit of a very weak shock then, we can say that change in entropy is almost negligible and it is the process is almost isentropic or nearly isentropic.

Since the flow across the shock the shock or the flow in this situation that you considered is adiabatic we had T 0 2 equal to T 0 1 as the flow is adiabatic T 0 2 is T 0 1 and the pressure ratio the stagnation pressure ratio can be obtained in terms of the entropy difference and which can be written that p 0 2 by p 0 1 to be 1 plus 2 gamma by gamma plus 1 M 1 square minus 1 of the power 1 by gamma minus 1 into gamma plus 1 M 1 square by gamma minus 1.

since change in entropy is directly proportional to change in total pressure hence the total pressure is also change in total pressure is also of third order in shock strength here this shock strength or shock example relations we have expressed in terms of upstream Mach number. However, often they are expressed in terms of other parameters particularly in terms of pressure jump instead of Mach numbers and in general the shock jump relations are known as Rankine Hugoniot relationship.

So, considering the flow through a uniform duct we have seen that for a compressible flow through a uniform duct if we have a supersonic flow at station 1 then, we can have a subsonic flow at station 2 and since you have used integral relationship at station 1 and station 2 any non equilibrium region is possible between station 1 and 2 only required mainly that flow at station 1 and station 2 must be in equilibrium and equilibrium, non equilibrium region may exist between station 1 and station 2 since the station 1 and station 2 and then, region between them has no restriction in size. That is, the non equilibrium region can be of infinitely thin extent we can consider that station 1 and station 2 may be just the 2 sides of 1 thin line and in that case, that thin line is called as the shock wave. As you have mentioned earlier that in real flow this shock thickness cannot be 0 there will be a finite thickness even though very small and within that it is there will be very large and rapid changes of the flow parameters velocity and temperature pressure and density and this rapid changes in velocity gradient and temperature gradient give rise to this dissipative forces and which causes increase in entropy across the shock.

Since, we have considered 1 dimensional flow and the shock is sending across it that is a normal shock and in 1 dimensional flow that is the only possibility of a shock wave a normal shock and these normal shock relations that we have derived here are based on or assuming that the shock is stationary and the fluid is flowing through it with a speed u 1 upstream and u 2 on the downstream . However, in all practical many practical problems, the shock is found to be propagating through the fluid as you can think about that if the on an aircraft moves the shock system of this air craft or the wing also moves with it and in that case the problem becomes a moving shock and we would like to revisit the shock relationship for a moving shock that is when the shock is propagating through the fluid through a speed u s that is.

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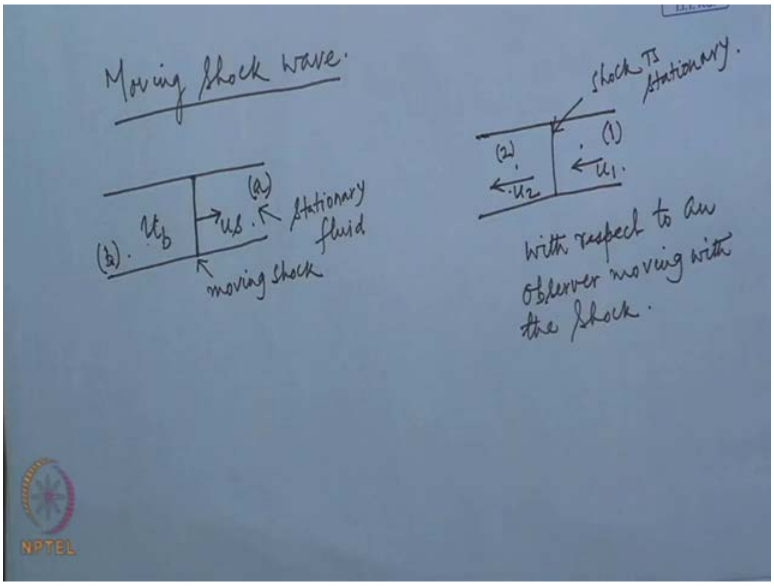

Let us consider now a moving shock wave problem moving shock wave problem let us say we have a duct and we have the shock moving with speed u s and as the shock moves with the speed u s the fluid behind it that is the fluid in this region is also being dragged with it. And, let us say that the fluid behind it is moving at a speed u b and let us say that fluid in front of it is stationary. That is, the shock is moving through a stationary fluid and we would like to see in this case the relationship of the flow parameters or fluid properties in station a and station b.

Now, what we did earlier that we satisfied the conservation laws that is namely continuity momentum and energy equation between the 2 region at 1 ahead and 1 behind the shock wave now see that we cannot do that in this problem we cannot satisfy the continuity equation between station a and in a region a and region b simply because that the fluid in region a is not part of the fluid that is flowing in flowing through region b it is always separated by this shock.

However, what we can do in this case is, we can consider a change in reference system in which let us say you consider an observer who is moving with the shock. Let us say in this case, the an observer who is moving with the shock wave then, with respect to this observer then, with respect to this observer the fluid in region a which will now designate as region 1 is moving with a speed u 1 to this reduction and fluid in region 2 that is b is moving at a speed u 2 ((no audio 36:03 to 36:35)) this is with respect to an observer moving with the shock.

Now, in this configuration of course, we can and the shock is stationary the shock is stationary. Now, in this configuration of course, we can satisfy the conservation laws between a station in region 1 and region 2 because, now the fluid in region 1 and region 2 are part of the same flow.

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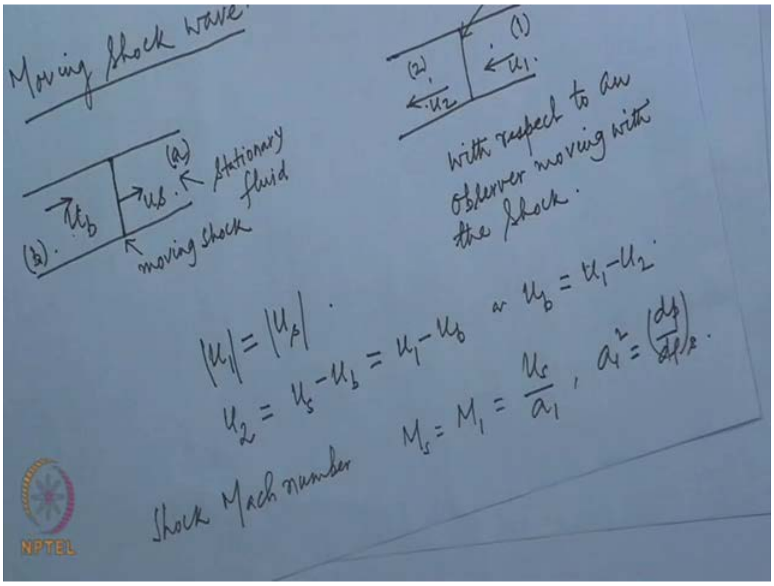

Now, what you find here that u 1 equal to u s and u 2 equal to u s minus u b which is u 1 now the static density is pressure and temperatures on either side of the shock they are of course, not affected by these transformation the shock jump relation for these quantities that is density pressure and temperature remains same as we have obtained earlier the jump relation the jump relation in this case are may be written in terms of shock speed and the speed of the fluid behind the shock and using the transformation the shock Mach number we can defined as ((no audio 40:13 to 40:57)).

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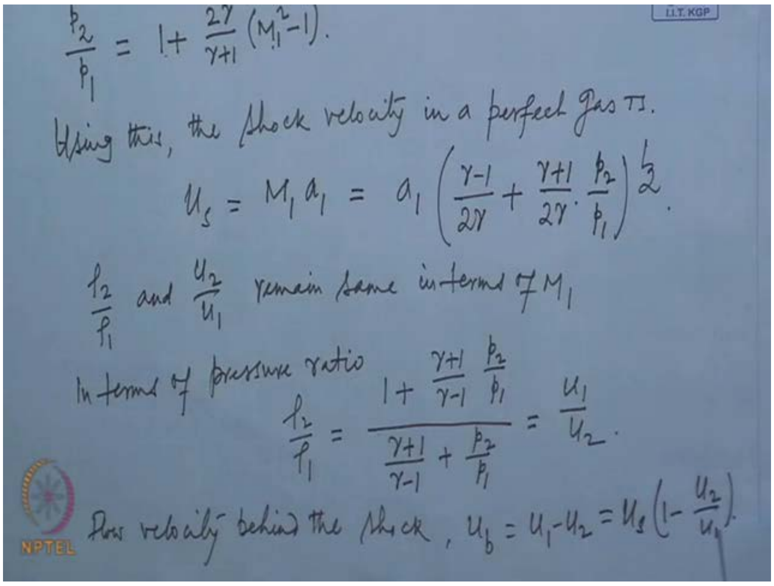

Now, using the pressure ratio p 2 by p 1 as 1 plus 2 gamma by gamma plus 1 M 1 square minus 1 we can obtain the shock velocity the shock velocity in a perfect gas ((no audio 42:38 to 43:40)) the density and temperature ratio similarly can obtained by the Rankine Hugoniot relationship that is rho 2 by rho 1 and u 2 by u 1 also remains same in terms of M 1 and using the (( )) M 1 as obtained from here this in terms of pressure ratio we obtain them ((no audio 44:25 to 45:08)).

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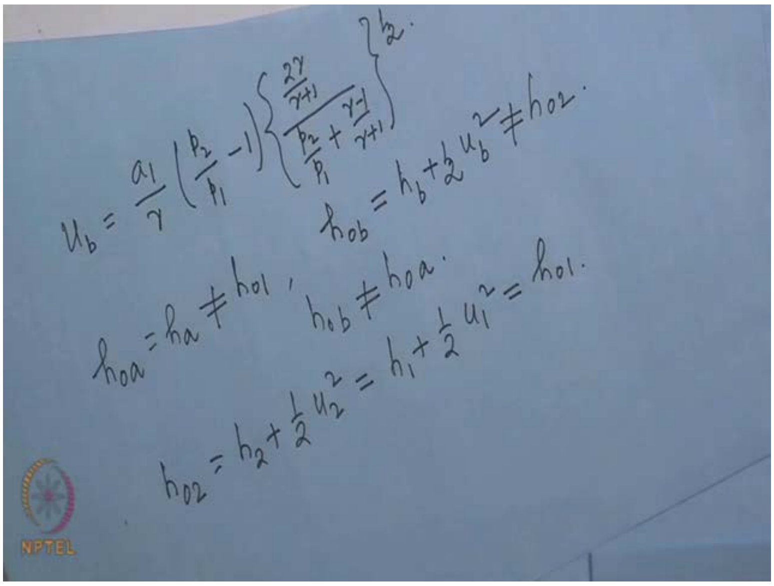

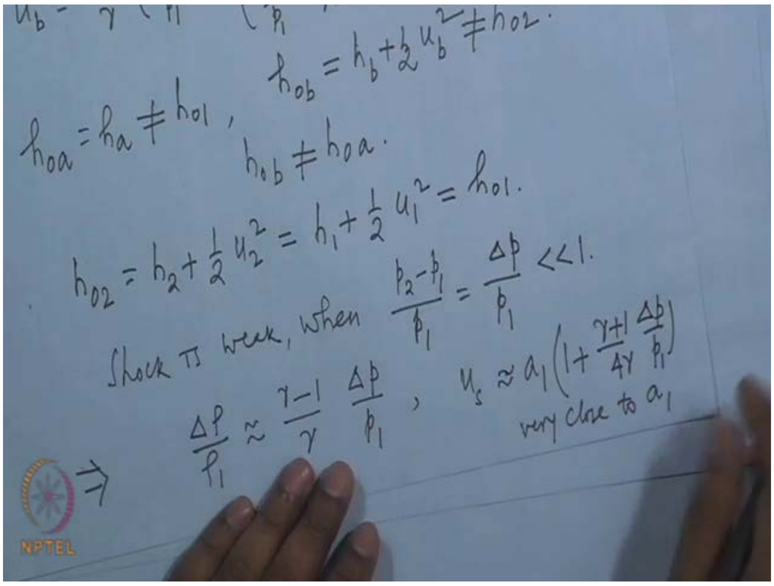

Similarly, we can direct the ratio of the temperature the flow velocity behind the shock flow velocity behind the shock that is u b can be written as u 1 minus u 2 which can be written as u s into 1 minus u 2 by u 1 and in terms of the pressure ratio this becomes a 1 by gamma ((no audio 46:06 to 46:38)) the static quantity as we have mentioned that in same for both the system. However, when you come to total quantities they are they are not same in the 2 system and (Refer Slide Time: 38:03) say it should be noted that in coming back to that configuration 1 and a and b and 1 and 2 it can see the that the total quantities at region a are same as the static quantities at region a because, the fluid they are stationary that is h 0 a is same as h a. However, this is not same as h 0 1 h 0 b that is a stagnation enthalpy in region b is simply the static enthalpy at region b plus the kinetic energy of the fluid in region b that is half u b square. However, this is not same as h 0 2 total enthalpy at 2.

Also, the total enthalpy in region b is not same as the total enthalpy at region a. However, the total enthalpy in region 2 which is simply h 2 plus half u 2 square that is equal to h 1 plus half u 1 square equal to h 0 1 and using this relationship the total quantities or the stagnation quantities can be found behind and across the shock. Shock will be called weak shock if the normalized pressure jump is very small that is a shock is weak shock is weak when p 2 minus p 1 by p 1 that is the normalized shock jump is very small the disturbances are also then, very small and they can be obtained by expanding these relationship in term when series of delta p by p 1 and retaining only the first order terms and this gives the density rise to be gamma minus 1 by gamma into delta p by p 1 the shock speed will be approximately u 1 into 1 plus gamma plus 1 by 4 gamma that is the speed of a very weak shock is very close to 1 very close to 1 sorry very close to a 1.

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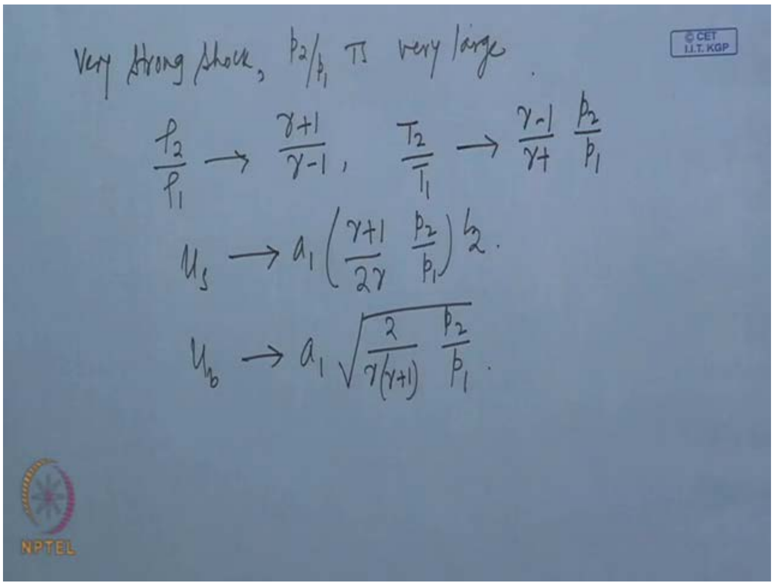

For a very strong shock this p 2 by p 1 is very large that is very strong shock that is p 2 by p 1 is very large and in that case rho 2 by rho 1 approaches as before gamma plus 1 by gamma minus 1 T 2 by T 1 approaching gamma minus 1 by gamma plus 1 into p 2 by p 1 a shock speed approaches u 1 into gamma plus 1 by 2 gamma into p 2 by p 1 to the power half and u b that is the flow velocity behind the shock is a 1 into.

So, in our discussion of flow through a uniform duct or flow through a normal shock we have seen that when a supersonic flow crosses a normal shock its pressure increases very rapidly or pressure increase by jump and in the limiting case of a perfect gas with gamma equal to 1 point 4 the pressure jump can approach very high value similarly the density also undergoes a very rapid or very jump and the limiting value of the density jump can be 6 for perfect gas with gamma equal to 1 point 4 which is usually chosen for air the downstream Mach number become subsonic and across a very strong shock the downstream Mach number approaches a limiting value for air with gamma equal to 1 point 4 the value become point 3 7 8 you have also seen that the second law of thermodynamics excludes the possibility of a jump from subsonic to supersonic condition meaning that shock cannot occur in a subsonic flow and through a shock flow cannot accelerate accelerating shock is impossible as dictated by second law of thermodynamics.

We have also considered a moving shock problem. However, the shock is moving at a certain speed through a stationary fluid and we have seen that the shock then, induces a velocity to the fluid which is behind it that is the shock sets the fluid behind it into a motion and raises its pressure and density temperature the ratios of the temperature density and pressure that is all the static quantities remain unaltered as in case of a stationary normal shock. However, we have expressed this relation now in terms of pressure further we have seen that the stagnation quantities are in the 2 configurations are different and we have given how the stagnation properties in the 2 regions can be calculate computed.

We have also found the limiting values for the density pressure and temperature rises as well as the shock speed and the flow velocity behind the shock what we have seen particularly today that a moving shock or a disturbance the sets the fluid in motion. However, how this motion develops? That is what we will try to see next in our next lecture; thank you.