High Speed Aerodynamics
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Module No: # 01
Lecture No: # 08
One-dimensional waves

We have seen that when the shock wave moves through a feed, it sets the fluid behind it into a motion. In other way, that when a disturbance moves through the fluid, it sets the fluid into a motion. Now, we would like to see that, what will be the general form of this disturbance and what would be the subsequent motion in a general sense because, a moving body moves through a fluid that also creates a disturbance in the fluid in which we are interested and the disturbance is created by a moving body or also propagated to other parts of the fluid and also to the other parts of the body. In general, the disturbance related to the fluid is a wave motion and the speed of propagation of this disturbance is called the wave speed. Through this mechanism of wave propagation, various parts of the body interact with the fluid and also with each other and the forces on the body are developed. Of course, the general problem is unsteady and to study this problem we need to use the appropriate equations and solve them.

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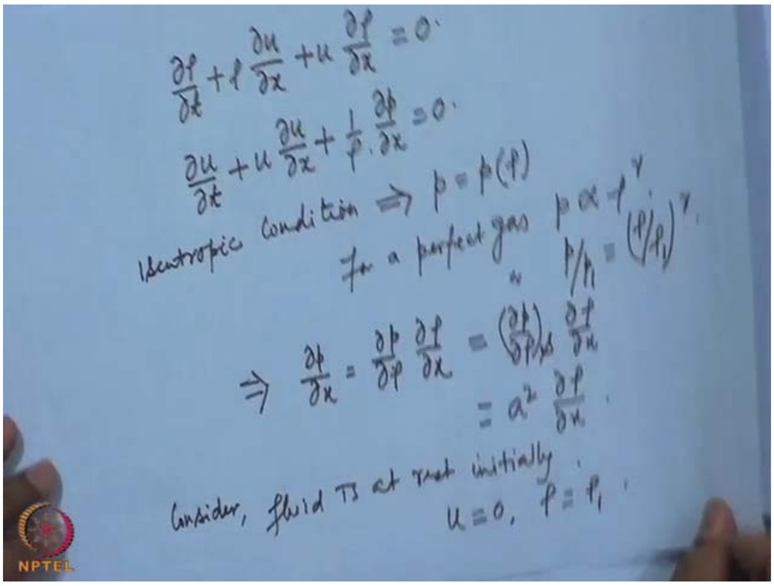

Now, considering an adiabatic non viscous motion in the constant area duct we have the continuity equation d rho d t plus rho d u d x plus u d rho d x equal to 0 and the Euler's equations or momentum equations as d u d t plus u d u d x plus 1 by rho d p d x equal to 0.

Since you are considering an adiabatic non viscous motion the flow is isentropic and consequently, the energy equation can simply be written as either h 0 that is, the total enthalpy equal to constant or entropy equal to constant. Since you are considering a non viscous motion and adiabatic motion, the flow is isentropic and the isentropic conditions exist and as we have known that in an isentropic flow in isentropic condition pressure is just a function of density alone and for a perfect gas this becomes p is proportional to rho to the power gamma. Or which you can write as p by p 1 equal to rho by rho 1 to the power gamma; this implies d p d x to be d p d rho and d rho d x. Since it is an isentropic process d p d rho or a d p d rho at a constant entropy is the square of speed of sound and this of course, you can substitute in the Euler momentum equation and eliminate p with the help of this relation.

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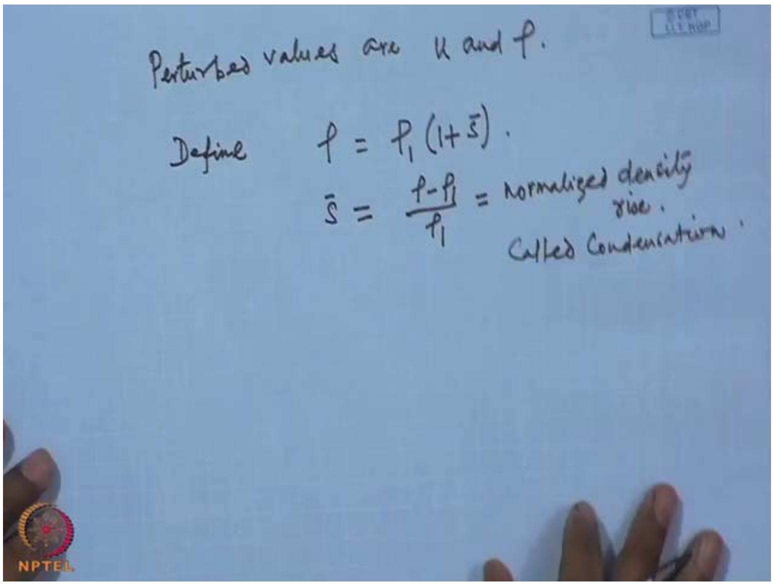

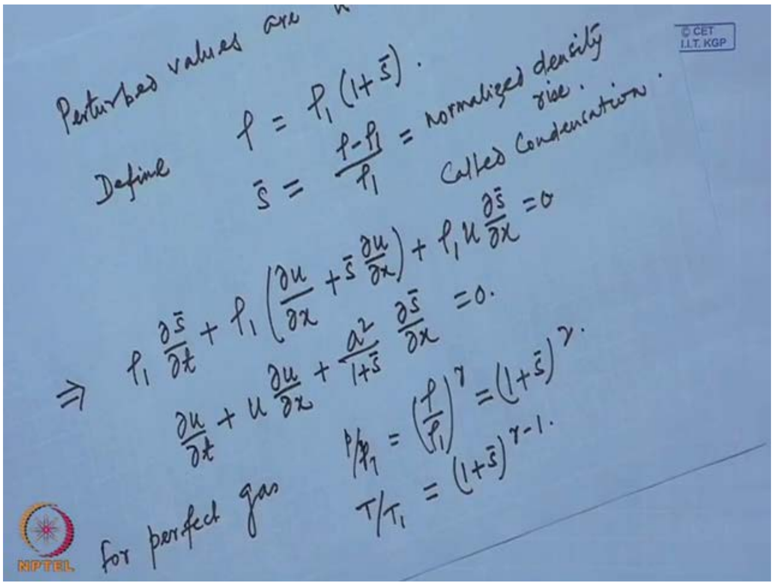

Now, the disturbances or perturbations are defined irrelative to the fluid that is at rest or at uniform motion. Let us take that the initially the fluid was at rest consider initial consider the fluid rest initially and the velocity is 0 and let us say density to be rho 1 now assume that the perturbed values are given by perturbed values are u and rho.

Now, let us define rho to be rho 1 into 1 plus s bar this s bar the dimensionless quantity s bar as defined here is simply rho minus rho 1 by rho 1 that is normalized density rise normalized density rise. Look at the use of this symbol s bar remember that it is not entropy which is the normalized density rise are called condensation when the value of this parameter is negative of course, it represents error ( ). Substituting these into the equations the continuity equation becomes rho 1 by d t plus rho 1 into and d u d t.

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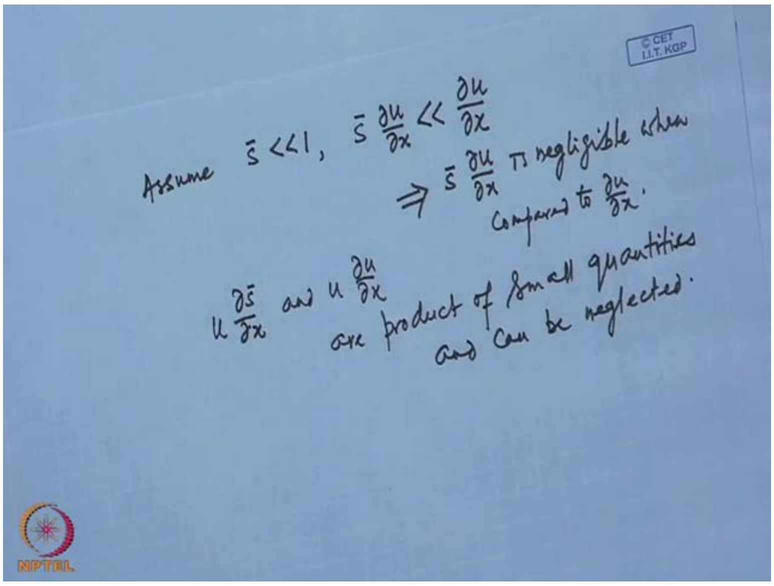

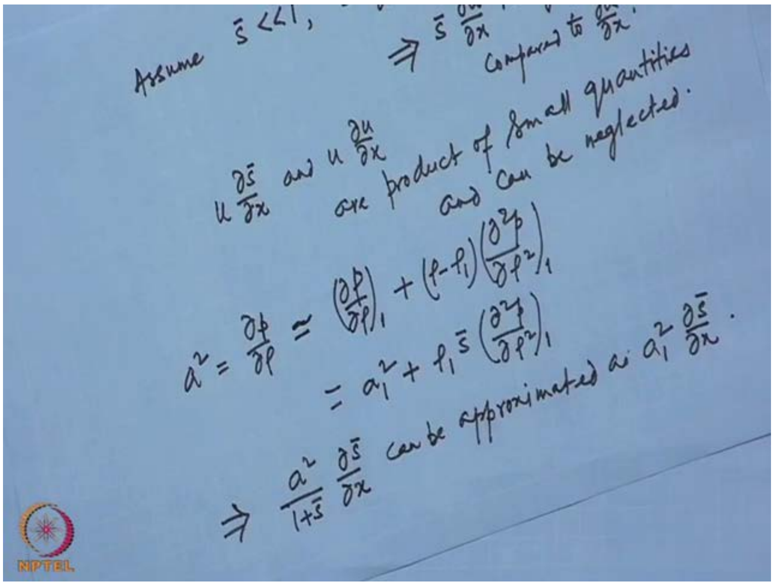

For a perfect gas we have p by p 1 equal to rho by rho 1 to the power gamma which becomes 1 plus s bar to the power gamma and t by t 1 is 1 plus s bar comma minus 1 all these equations and relations are exact for frictionless non conducting motion. However, the equations are not easily integrable as they are non-linear. So, to solve these equations analytically we need to linearise them and these equations can be linearised by a small disturbance assumptions that is, we now assume that the disturbances are small that is we will assume in s bar d u d x is much smaller than d u d x. That is negligible when compared to d u d x. Similarly, the term u d s bar d x and u d u d x are product of small quantities. Hence, they are negligible now the local speed of sound a square which is d p d rho. It can be expanded in a Taylor series about it can be expanded in Taylor series about the mean state; that is undisturbed state in this case why which is d p d rho at 1 plus rho minus rho 1.

A Taylor series (Refer Slide Time: 13:00) expansion about the undisturbed state and which is this gives that a square by 1 plus s bar into this that is the last term in the Euler's equation can be approximated can be approximated as a 1 square d s bar d x.

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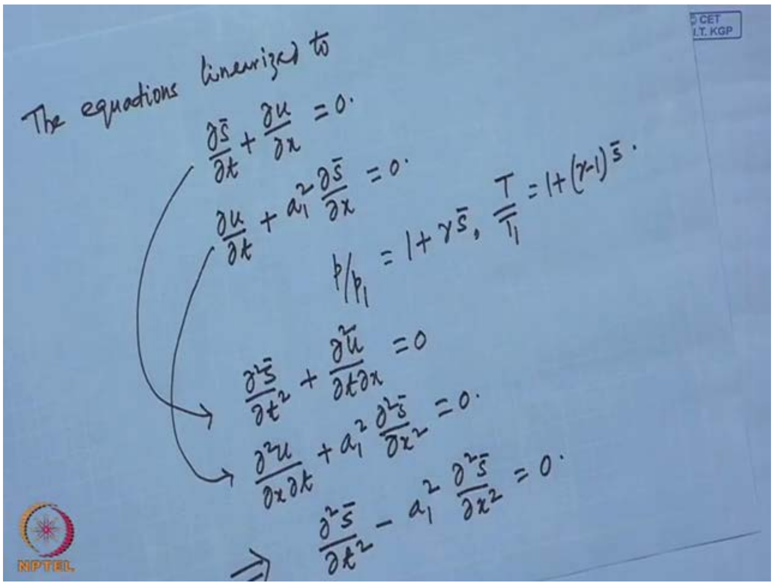

So, with these assumptions the equations can now be linearised. So, the equations now linearised to the first term d s d t and the momentum equation become these equations are known as acoustic equations because, they are very small disturbances or sound waves. That is, the sound waves produce a very small disturbance a situation like this and hence the equations are called acoustic equations. Also, that perfect gas equations can also be approximated as p by p 1 equal to 1 plus gamma s bar and t by t 1 is 1 plus gamma minus 1.

Now, to solve this equation we differentiate the continuity and momentum equations with respect to t and x differentiating the continuity equation with respect to t we have ((no audio 16:53 to 17:36)) and since the order up derivative in these two terms are immaterial these results into (Refer Slide Time: 18:23).

Similarly, if we differentiate the continuity equation with respect to x and the Euler's equation with respect to t we will get an equation in terms of the perturbation velocity which is again these are the well known wave equations. So, see that the density disturbance as well as the velocity disturbance both satisfy the wave equation and the disturbance propagate with a definite wave speed which in this case is a 1 or the speed of sound. Now, the general solution for these wave equations are given as x minus a 1 t plus G of x plus a 1 t and u equal to F of x minus a 1 t plus G of x plus a 1 t where F equal to a 1 F and G equal to minus a 1 minus G.

Let us now analyse the character of these solutions and for this purpose let us take G equal to 0. So, the density disturbance at time t is given as simply F of x minus a 1 t this represents a disturbance for wave which at time t equal to 0 had the shape. That is, at t equal to 0 the disturbance had the shape and we can see that at time t the disturbance is exactly the same speed, but with the corresponding displace points displaced the distance a 1 t to the right.

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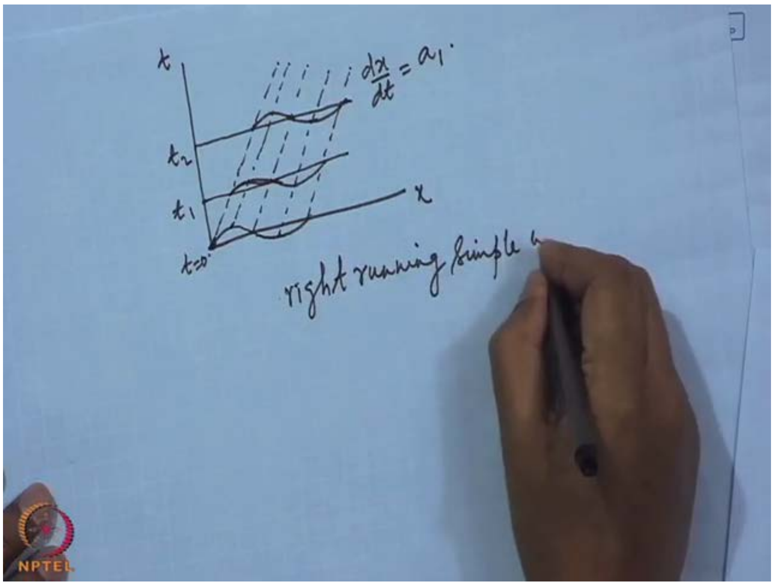

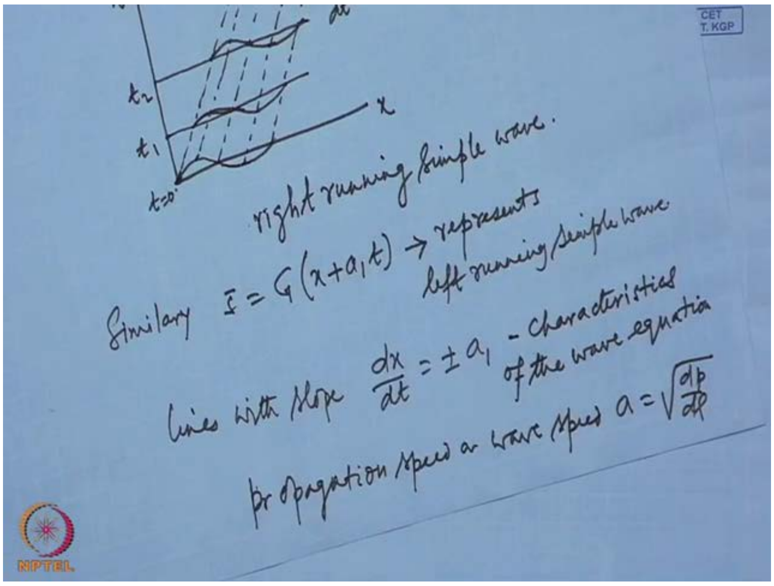

That is the velocity of each point in the wave and hence the wave is a 1 which you can see that let us say that this is the shape of the disturbance at time t equal to 0 and what you see that the disturbance retain its same shape, but ((no audio 22:36 to 23:34)) at all time the disturbance is retains its original undisturbed shape and the points displaced a distance by a 1 t to the right and we can say that each point on the disturbance has a velocity speed of a 1 and hence the wave speed is a 1.

A wave in which the propagation velocity is in 1 direction it is called a simple wave that is if the wave propagates only in 1 direction it is called a simple wave. So, in this case this is the wave which is propagating to the right is called a right running wave. Similarly, the wave described by this is right running simple wave similarly the other part of the solution G of x plus a 1 t represents left running simple wave.

That is in this case, the wave is propagating to the left with speed a 1 the lines in the next plane which traces the progress of the wave there is a lines with slope d x d t equal to plus or minus a 1 are called the characteristics of the wave equations that is lines with slope lines with slope d x d t equal to plus minus a 1 are characteristics of the wave equation. The disturbance propagates through the fluid with a speed propagation speed or wave speed or wave speed is square root of d t d rho and this is called the acoustic speed or sound speed.

The result is applicable to disturbances in which the velocity temperature velocity and temperature gradients are very small and u y a 1 is much smaller than 1. So, these are as you have assumed for the purpose of linearization that the disturbance speed or the velocity u is quite small which now comes to that u by a 1 is much smaller than 1 and in which the gradients of the disturbances that is velocity temperature pressure are all very small and consequently the dissipating forces will have no considerable effect.

In that situation, the results are applicable that is these acoustic waves solutions or acoustic waves are applicable where the disturbance is small and the gradients are also very small. So, that dissipating process has no considerable effect. That is, the motion is isentropic and as we have already discussed that the sound wave is isentropic. So, the result applies to propagation of sound wave as well with the sound speed as given by a square equal to d p d rho at constant entropy.

The amplitude of the ordinary audible sound is quite small and the local production of entropy is negligible friction and local entropy production is negligible for computing the speed of ordinary sound. But, the cumulative effect on the amplitude is not negligible; the quantity square of the speed of sound provides a pressure density relation and eliminates pressure from the momentum equation which you have already seen that d p d rho d p d x was replaced by a square into d rho d x. Of course, a non isentropic process are present pressure also depends on entropy and the relation is not correct, it has to be augmented by change in entropy change term.

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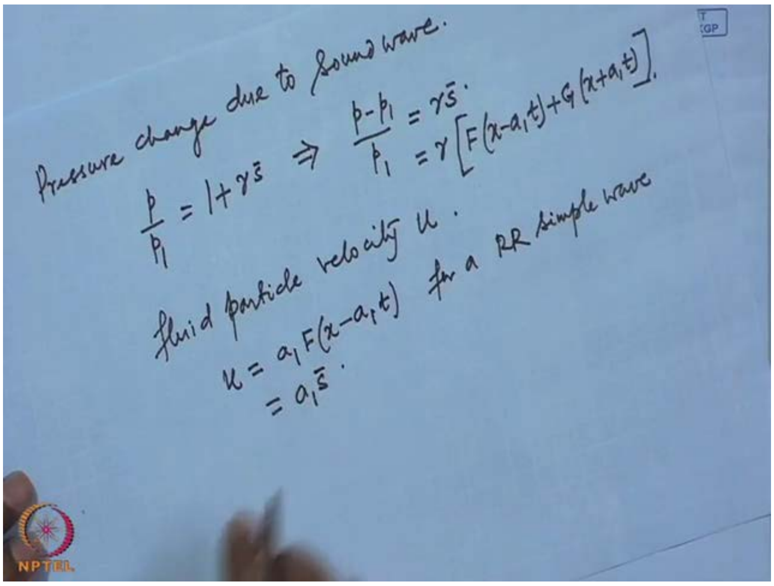

Now, a square can be evaluated from the equation of state that is for a perfect gas we know a square become gamma p by rho or gamma r t now let us consider what happens to the velocity of the particles that makes the wave that make up the wave and also how does pressure changes. Now, the pressure and the pressure disturbance accompanying so, pressure change in a sound wave due to sound wave you have seen that p by p 1 is for small disturbance 1 plus gamma s bar or this implies the pressure change is ((no audio 31:13 to 31:46)).

That is, the pressure wave has the same shape as the density wave. However, its amplitude is differed by a constant factor gamma now as the wave progresses through the fluid as the wave progresses through the fluid the pressure disturbance sets the fluid into a motion and the fluid particle also undergoes the motion with this velocity being u which is called the particle velocity u. And, in general is much smaller fluid particle velocity u u and it is usually much smaller than the speed of sound during the propagation of a sound wave. For a simple wave the velocity disturbance is as we have already seen u equal to a 1 F x minus a 1 t for a right running for a right running simple wave which of course, is equal to a 1 s. Similarly, for a left running simple wave simple wave u equal to a 1 the various parts of the wave are called condensation and redefection depending on whether this condensation parameter s bar is positive or negative that is whether the density is higher or lower than the undisturbed density rho 1 in combining the two we can write that u equal to either plus minus a 1 s bar.

They the effect the wave produces on the fluids of course, depends on the gradient of this density and pressure distribution and on the direction of the motion on the wave that is the portion of the wave where density is increasing as it passes it is called a compression and that which decreases the density is called an expansion. So, the corresponding disturbance of particle velocity are given by u equal to plus minus a 1 s for the left and right propagating waves respectively it may be seen that a compression accelerates the fluid in the direction of wave motion where as an expansion discinerate it is the non simple wave is a super position of two simple wave that is F and G and the relation between particle velocity and density is for general non simple wave for general non simple acoustic wave acoustic wave 2 by a 1 F G.

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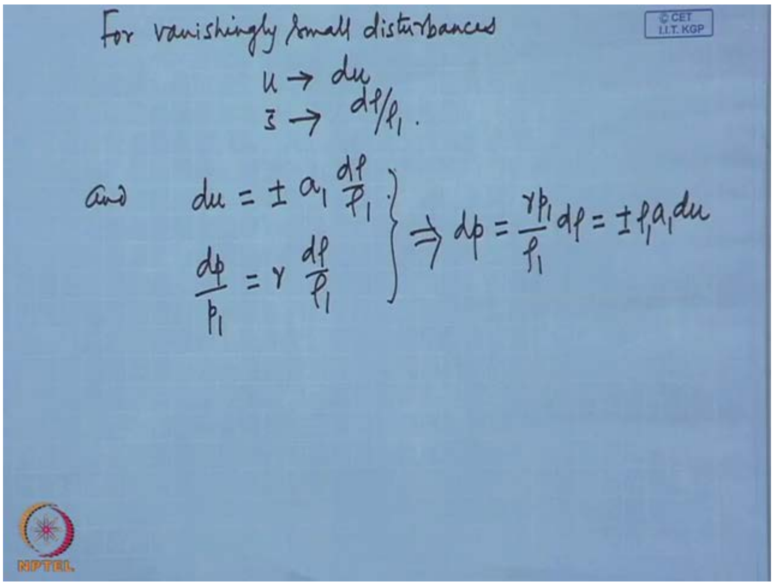

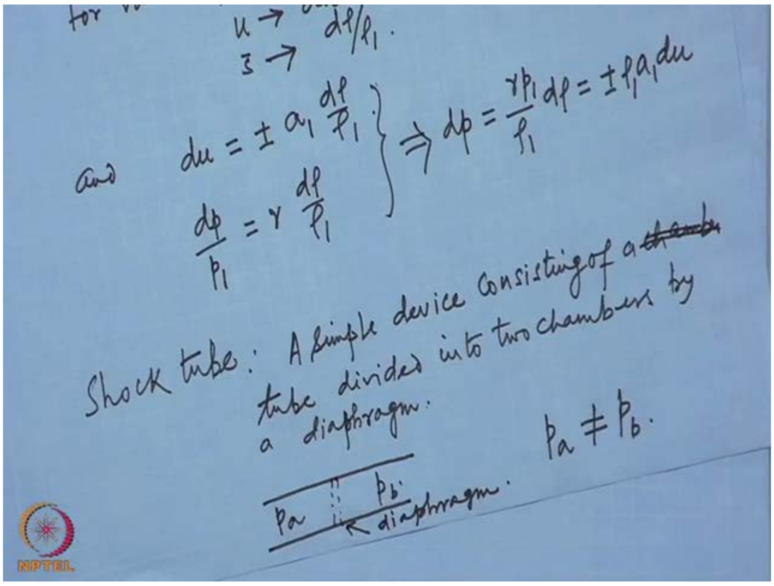

When the disturbances is vanishingly small, we can express these perturbation quantities in terms of differential where u that is the particle velocity u and the condensation parameter s bar can be replaced by d u and d rho by rho 1 for vanishingly small disturbance ((no audio 37:11 to 37: 44)) and we have d u equal to plus minus a 1 s which can be written as plus minus a 1 d rho by rho 1 similarly the pressure changes is d p by p 1 can be written as gamma d rho by rho 1 and combining the two, these together also imply that d p equal to gamma p 1 by rho 1 d rho ( ). That is, a square d rho and it becomes plus minus rho 1 a 1 d u and these relations holds for infinitesimally small disturbances - very small disturbances. Now, this solution of work motion can be used to study very important, some of the very important problems and once such problem that we will be handling here is the shock tube problem. Now, shock tube is a very simple device. Shock tube is a simple devices simple device consisting of a tube that is divided into 2 parts by a membrane or a diaphragm in which pressures are different that is a simple device.

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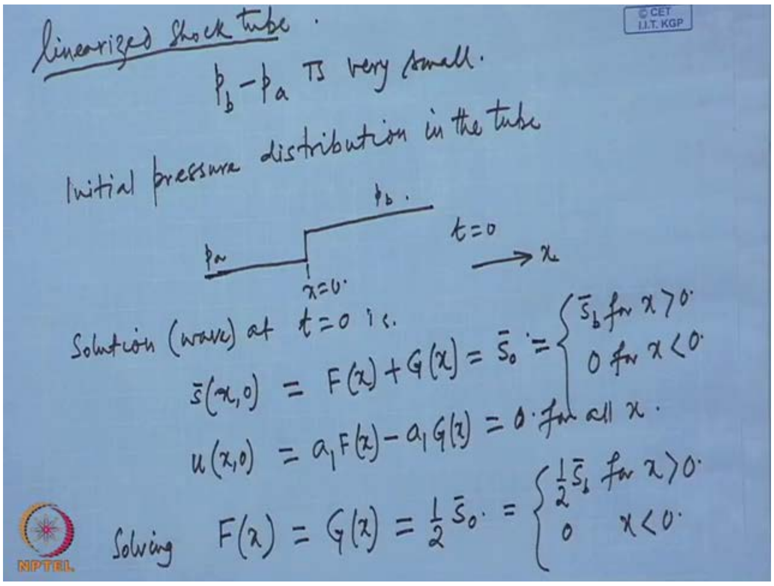

A simple device consisting of a chamber of a consisting of a sorry consisting of a tube divided into two chambers by a diaphragm and the pressure is different in the 2 chambers. That is, we can have a shock tube is simply this a diaphragm here diaphragm and the pressure here are different say p a p b; usually 1 side is a very high pressure and the other side is low pressure. Now, since you have discussed about linearised wave motion we will consider this as a linearised shock tube. We will consider a linearised shock tube that is in which linearised shock tube linearised shock tube that is that p b minus p a is very small.

That is, let us say that the initial pressure distribution in that tube initial pressure distribution in the tube distribution in the tube is that is at t equal to 0 there is a step distribution of pressure where the low pressure side has the pressure p a and the high pressure side has the pressure p b and at t equal to 0 the fluid particles are not in a motion that is the particle velocity 0 everywhere.

Now, let us say at that instant we suddenly remove the diaphragm or the membrane and consequently now, of fluid motion we will set in which fluid from high pressure is in we will try to flow through the low pressure region. Now, as you have considered the disturbance very small, this disturbance that is the difference in pressure is very small. So, this pressure, this disturbance that is disturbance created by the removal of the diaphragm also satisfies the linearised equations and hence the linearise solution.

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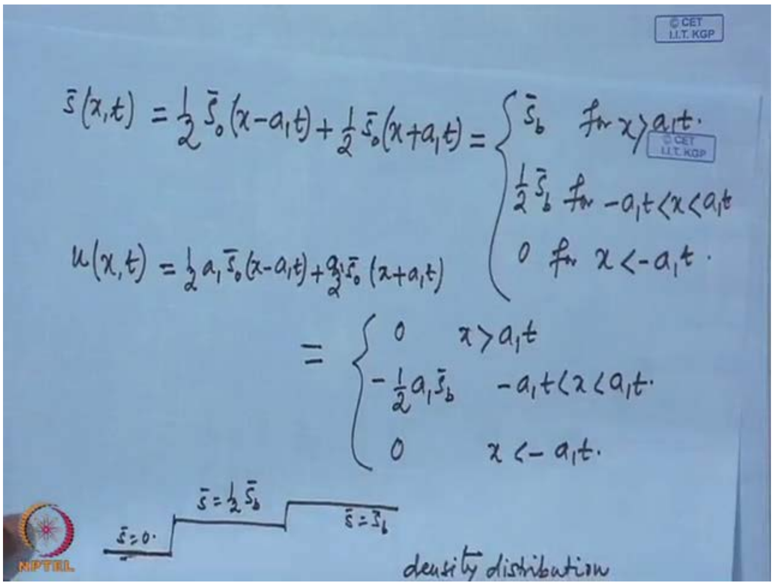

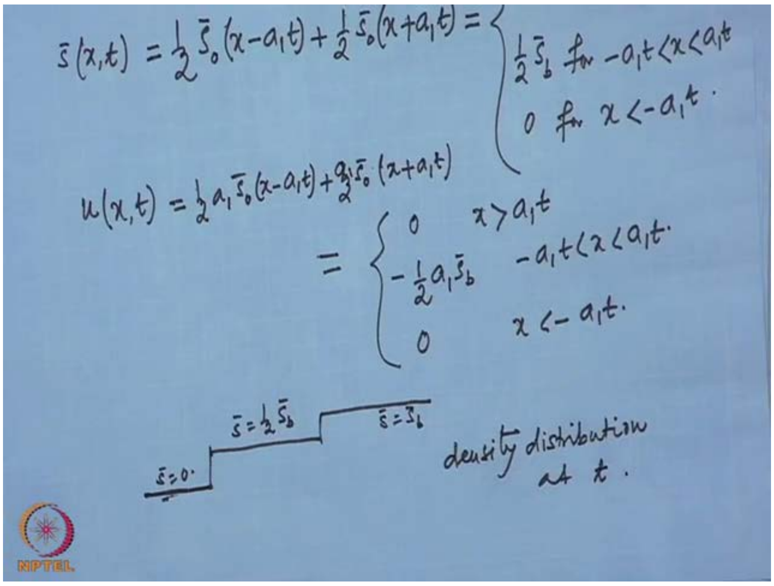

Let us say at time t equal to 0 the solution is described by solution that is a wave at t equal to 0 is sorry s bar at x 0 equal to F x plus G x we call this to be s 0 and obviously, the density will also have a step distribution corresponding to the pressure. And, let us say this is s b for x greater than 0 and it is 0 for x less than 0 the particle is velocity is 0 everywhere. So, can write a 1 F x minus a 1 G x equal to 0 for all x. Solving these two simultaneously what we get is that solving we get F x equal to G x and this is half is 0 which is half of s b for x greater than 0 and 0 for x less than 0 the motion at any subsequent time t will be given by motion at any subsequent time t will be given by s bar x t equal to half x minus a 1 t plus half and substituting to these values this will be s b for x greater than a 1 t for x greater than a 1 t is half of s b bar for minus a 1 t less than x less than a 1 t and 0 for x less than minus a 1 t.

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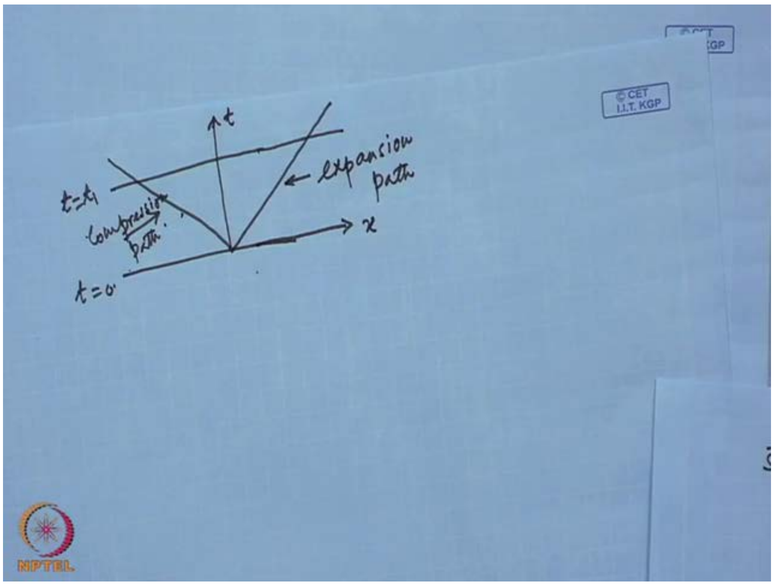

Similarly, the velocity will be half of a 1 s 0 x minus a 1 t plus half of s 0 half a 1 s 0 x of s 0 of x plus a 1 t and this will also become 0 when x is greater than a 1 t minus half a 1 s b bar for minus a 1 t less than x less than a 1 t and again 0 for x less than minus a 1 t that is we can see that at a at any time the density or ((no audio 50:46 to 51:20)) and this is a solution at time t that is density distribution and the pressure distribution and velocity distribution are also of similar.

That is, what we see that starting from the initial time and expansion is moving to the right in this case and a compression is moving to the left. So, that a density on the left what is gradually increasing and on the right it is decreasing or you can say you can have let us say at this is t equal to 0 at t equal to t 1 ((no audio 52:44 to 53:33)). That is a compression wave is propagating to the low pressure side and an expansion wave of equals strength is propagating to the high pressure side. Now, what we have covered today is basically wave motion in 1 dimension where we have first linearised the continuity equation and Euler's equation assuming that the disturbances are small and we have seen that the disturbance propagates with a constant wave speed which happens to be the speed of sound implying that the small disturbances small disturbance waves are isentropic and acoustic wave. We have also seen that for the linearised wave equation or for the linearised wave problem the wave retains it shape at all times for a simple... a wave is called simple if it moves in only 1 direction.

So, you have we can have in 1 dimension both left running and right running simple waves and a general non simple wave is a combination of the two, where wave propagates in both the direction that is to the left as well to the right. We have been applying this linearised solution to a linearised form of the shock tube or we have assumed that the pressure in the 2 chambers that is in the high pressure chamber and low pressure chamber or marginally different. So that, when the diaphragm is removed small disturbance are linearised web propagation sets in and consequently, we have what we see that are the... we have a compression wave propagating to the high low pressure side and an expansion wave propagating to high pressure side and how the state distribution of pressure and density is going to be eventually smoothed out.