**High Speed Aerodynamics**
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**Module No. # 01**
**Lecture No. # 09**
**One-Dimensional Waves (Contd.)**

From our discussion of simple acoustic wave we have seen that that the simple acoustic wave propagates with a constant wave velocity and the shape of the wave is permanent; that is, it propagates without any (( )). However, these properties that we have found for a simple acoustic wave comes because of the linearization of the equations that we have used and that linearization n as you have see is coming from the assumption of implausibly amplitudes and gradients. That is, we have assumed that the disturbance are implausibly small and their gradients are also very small which has helped us to linearize the equation and because of that linearization incorporated in the equations we found that the waves move with a constant speed. The acoustic speed and the wave shapes remain undistorted.

However, in many practical situations these assumptions are not possible. That is we cannot assume that the disturbances are very small and their gradients are also small and when these assumptions are not possible, the condition at any given point in the wave cannot be approximated by those in the undisturbed fluid under such situation the wave velocity varies from point to point and the simple wave becomes distorted as it propagates. To disturb these type of waves with finite amplitude and finite gradients we need to solve the complete non-linear equations. However complete solution of the non-linear equations? Are extremely difficult and in many cases they are not possible. So, we will try to get it in a indirect manner instead of solving the non-linear equations directly. Let us consider a plain compression wave moving to the right.

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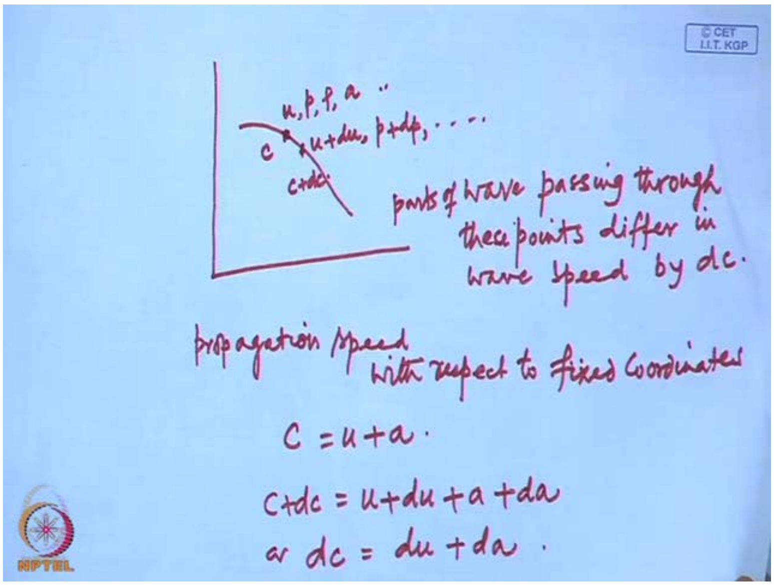


Let us say at 2 adjacent point that is let us say at 2 adjacent points the fluid properties differ in magnitude by d p for pressure d o for density d u for the particular velocity or flow velocity that is let us say the flow velocity here is u here it is u plus d u the pressure here is p pressure at this point is p plus d p and so on for all other quantities.

Now, these respective parts of the points sorry the respective parts of wave that is passing through these points also differ in the wave speed let us say by an amount of d c wave through these points differ in wave speed by d c that is if the wave speed here is c here it is c plus d c. Now, as long as the velocity and temperature gradients are moderate the viscous shot heat conduction effects are negligible and this finite wave this finite wave can be thought of as a succession of (( )) pluses.

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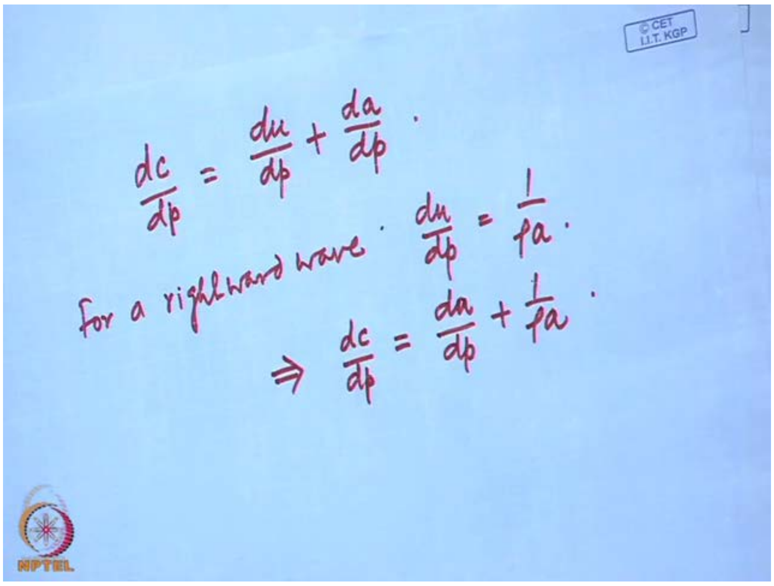


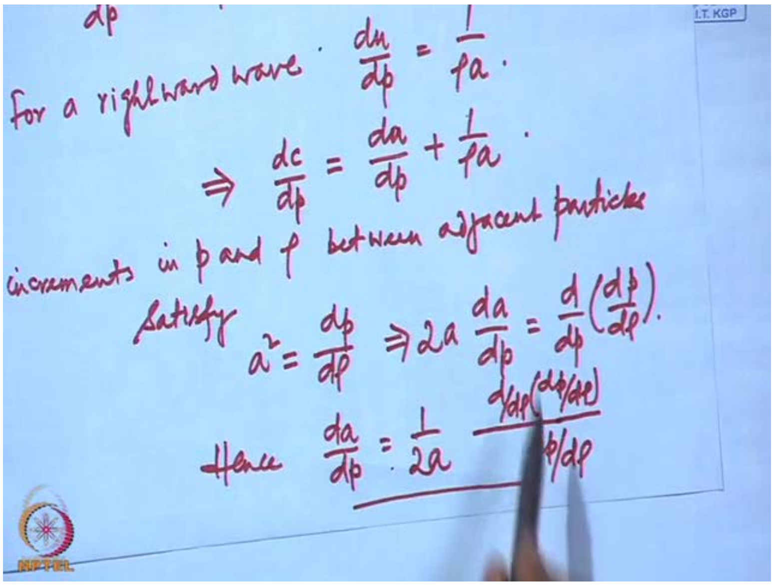


Now, each part of this wave travel at local speed of sound with respect to the fluid in which it is propagating. Consequently, the propagation velocity of a part of the wave with respect to fix coordinates is then… propagation speed with respect to fixed coordinate with respect to fixed coordinate with C equal to u plus a. Now the propagation velocity of the adjacent parts of the wave is C plus d c is u plus (( )) du plus a plus da or dc is d u plus d a differentiating these with respect to p we have dc dp now for a rightward wave we have already seen for a rightward wave we have already seen that du dp equal to 1 by rho a and hence we have dc dp da dp plus 1 by rho a.

Now, considering that the inter fluid was originally at rest with uniform pressure and temperature then each particular fluid undergoes asymptotic changes and consequently the increment in pressure and density between adjacent fluid particles obey. Now, increments in p and rho between adjacent particles satisfy which gives 2a da dp and hence, if we change this derivative with respect to rho it becomes dd rho of dp d rho by d p d rho.

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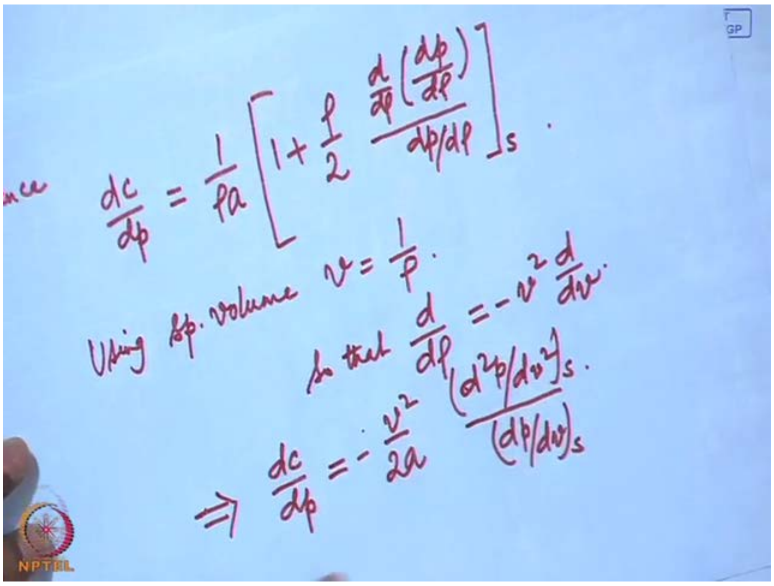


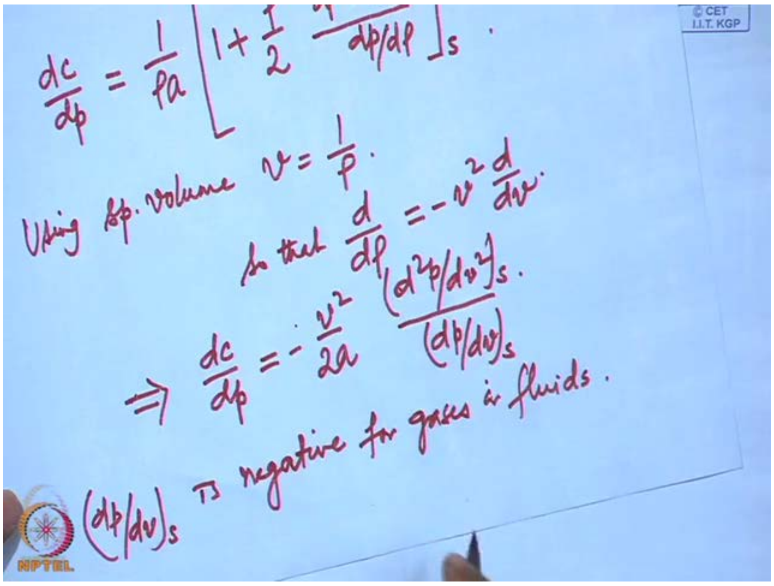

Now, substituting these substituting these relation here we get ((no audio 10:50 to 11:34)) at constant entropy. Now if using specific volume using specific volume v equal to 1 by rho. So, that d d rho becomes minus and we get ((no audio 12:20 to 13:01)). So, this relation gives how the local wave speed changes with the pressure p now for a fluid dp dv at constant entropy is negative. So, sign of these d c d p of course, depends on sign of the second derivative of dc dv dp d v. So, dp dv is negative ((no audio 13:45 to 14:17)).

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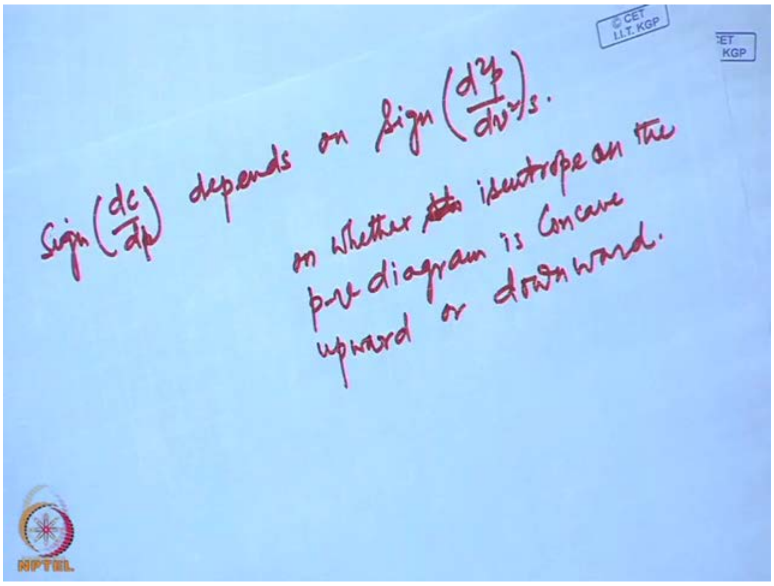


So, sign of dc dp depends on sign that is the sign of dc dp will depend on whether the isentrope is concave upward or isentrope on the p v p v diagram is concave upward or concave downward. That is on whether isentrope on the p v diagram is concave upward or downward. And we can also see that when d c d p is positive it implies that the higher pressure part of the wave will have higher speed then the lower part lower pressure part of the wave that is higher pressure parts of the wave will overtake the lower pressure parts when d c d p is positive and that will happen when d 2 p d v 2 is positive and the opposite happens if d c d p is negative. That is when d c d p is negative the lower pressure parts will overtake the higher pressure parts. Consequently, this means that a compression wave will steepen as it progresses when d 2 p d v 2 is positive; however, the compression wave will flatten if d 2 p d v 2 is negative.

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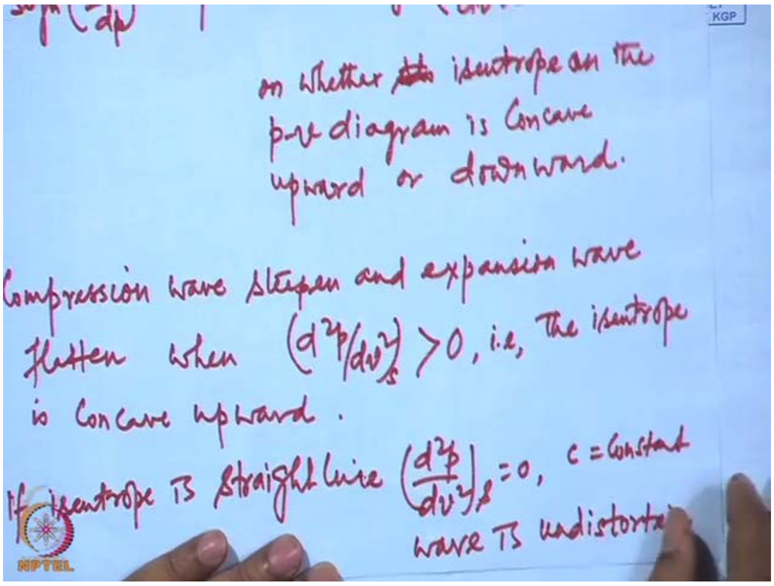


So, the results you can write that compression part of the wave or compression wave steepen and expansion of wave flatten when is positive that is the isentrope is concave upward. This is of course, the usual case for all real fluids the other alternative, but is when d2p dv2 is less than 0 then compression wave will flatten and expansion wave will steepen; however, no such real fluid exist and consequently in all cases the compression wave steepen and expansion wave flatten when a wave of finite amplitude propagates through that fluid. If the isentrope is straight line that is a second derivative d2p dv2 is 0 then of course, the wave speed remain constant. If isentrope is straight line ((no audio 19:37 to 20:12)) and wave is undistorted.

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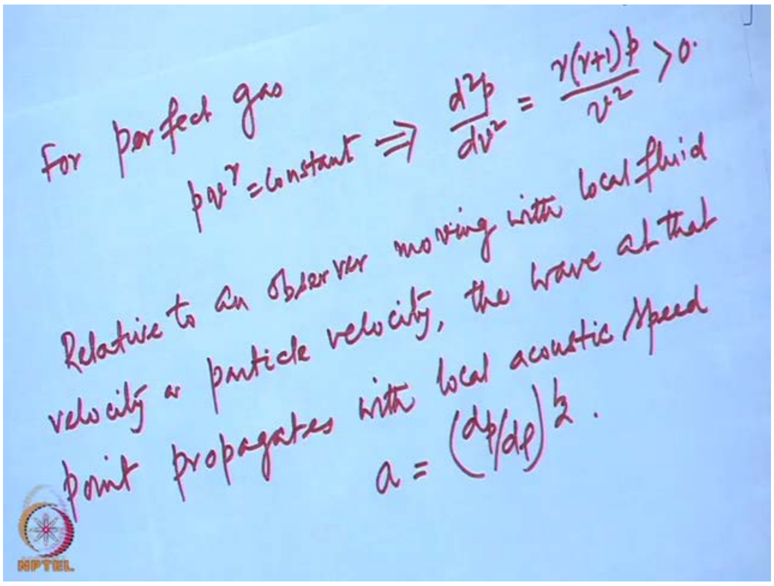


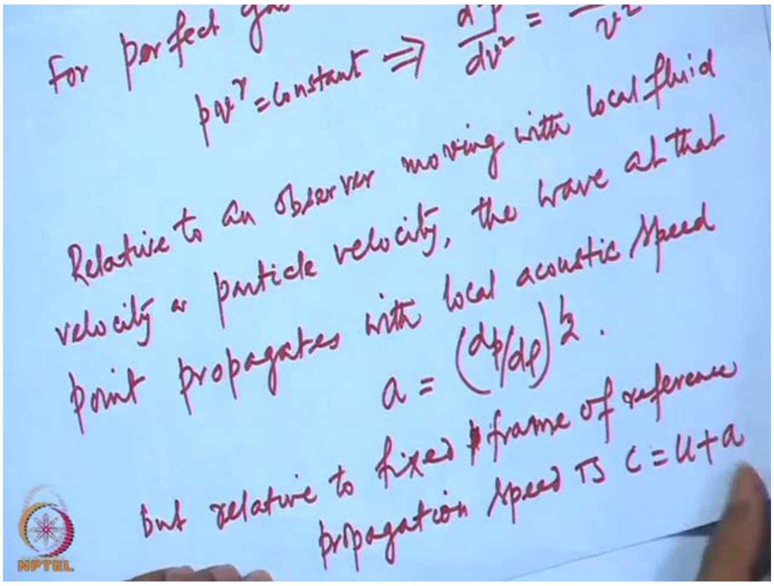


Let us consider the perfect gas for perfect gas we have ((no audio 20:39 to 21:10)) and in a perfect gas we can clearly see that the wave will distorted in such a way that the compression part of wave will steepen and the expansion part of the wave will flatten. We are looking to the problem from the point of an observer who is moving with local particle velocity the acoustic theory applies locally. Consequently, relative to observer moving with the local fluid velocity the wave at the point propagates with local acoustic speed. So, relative to an observer observer moving with local fluid velocity or particle velocity the wave at that point propagates with… however, relative to the fixed frame of reference, but relative to fixed frame of reference as we have already mentioned relative to fixed frame relative to fixed frame reference propagation speed is for the rightward wave.

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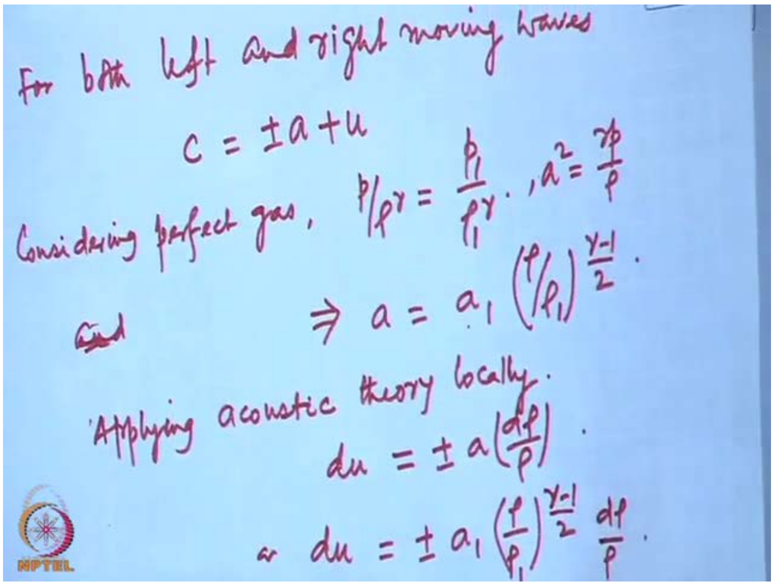


So, for both left and right moving waves both left and right moving waves C equal to plus minus a plus u or the plus is for the rightward moving waves and the minus is for the leftward moving wave. As you have seen the waves it is no longer constant as well as both a and u are also not constant and u may not be negligibly small and the gradient of u also may not be small. So, evaluate these in terms of the density; we can apply the acoustic theory locally. And considering perfect gas we have p by rho to the power gamma equal to p 1 by rho 1 to the power gamma for p 1 and rho 1 are the pressure and density of the undisturbed fluid and by rho. However, you want either speed up the acoustic wave in the undisturbed fluid; that is why the conditions are p1 and rho 1. Applying acoustic theory locally you have d u equal to plus minus a d rho by rho and substituting a here we have ((no audio 27:30 to 28:01)).

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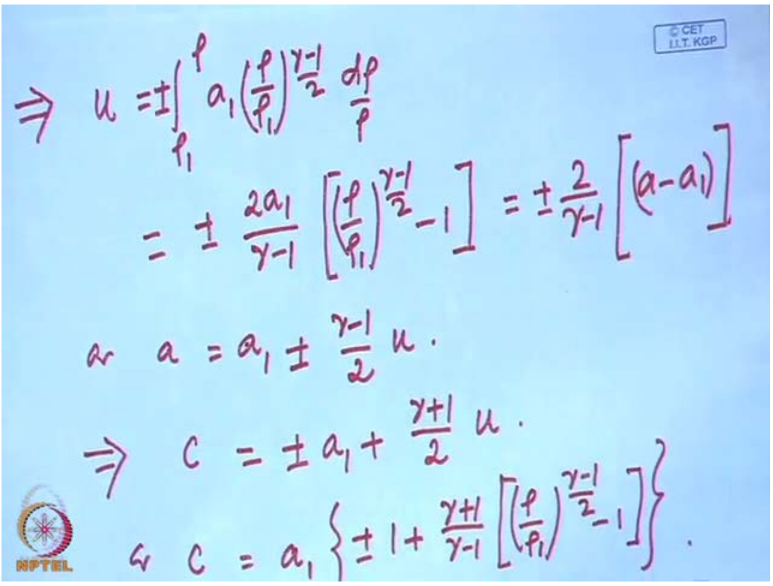


Now, this is integrable and we can see that u equal to ((no audio 28:13 to 29:00)) minus 1 by 2 minus 1 which of course, can be written as plus minus 2 by gamma minus 1 and taking a 1 inside and substituting a 1 equal to we have a minus a 1. This gives a equal to a 1 plus minus gamma minus 1 by 2. So, this relates the local acoustic speed with the acoustic speed in the undisturbed fluid and the particle velocity and substituting these in the relation C equal to plus minus a plus u we get C equal to or also in terms of  a 1 is the speed of shown in the undisturbed fluid and rho 1 is the density of the undisturbed fluid. Of course, the fluid in this is case is a perfect gas and this gives the local wave speed of a finite wave with finite amplitude or wave with disturbances which are not small.

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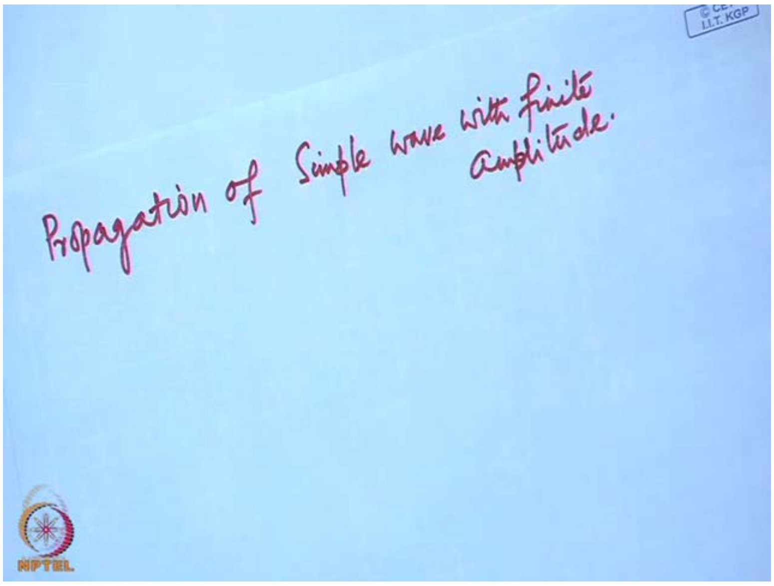


Now, once knowing this the variation wave, wave speed in terms of density, let us now consider the propagation of s simple wave of finite amplitude consider propagation of simple wave is finite amplitude that is the wave which has been produced by disturbance which are not infinitesimally small.

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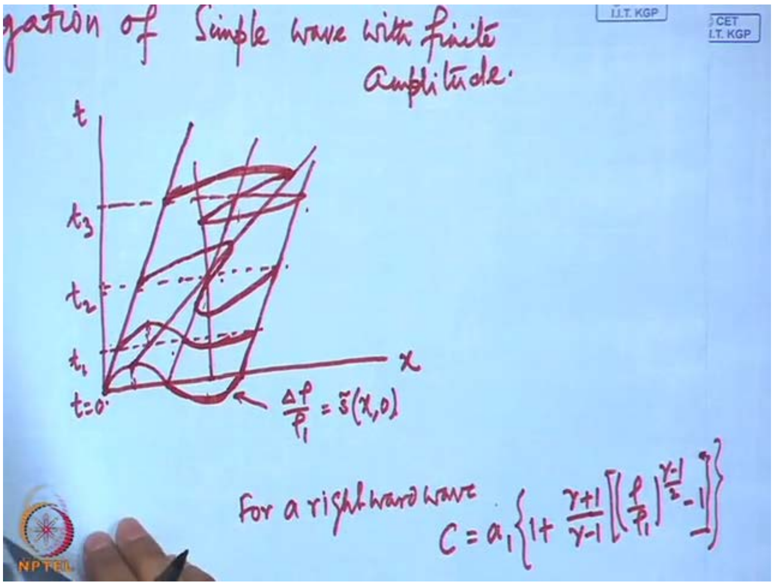

Now, let us consider a wave and let us say which is the density disturbance that is that condenses in parameter delta rho by rho 1 which we have earlier defined as s bar at time t equal to 0 this is t equal to 0 we consider for a rightward propagating wave for a rightward wave for a rightward wave the wave speed is the wave speed c equal to a 1 2 1 plus gamma plus 1 by gamma minus 1 ((no audio 34:41 to 35:27)).

Now, let us see what will happen to this wave, as time passes what you see that at the region where rho is greater than rho 1 c is higher than a 1 and where rho is less than rho 1 c smaller than a 1. Thus, the wave speed changes in the region of higher density the wave speed is higher in the region of lower density the wave speed is lower and consequently the wave as it propagates going to different speed at different point on the wave the wave shape will not remain as it was and it will distorted. And we see that region for the condenses in is higher that tends to overtake the region where the condensation is lower in the region of higher condensation. The characteristic lines are inclined more since the slope as we have seen earlier it is proportional to the wave speed. Consequently, let us say that at this point where the density is constant the speed will remain same as a 1 and the characteristics line will be like this power at this point which is the highest density the characteristics will inclined much more and will no longer remain parallel but, we will have much higher slope at this point of the wave. Again, the density ratio is 1 and consequently the wave speed will remain same as a 1 as it here and the characteristics line here will be parallel to the characteristic line here. Similarly, here also the characteristic line is parallel to the characteristic line here and here at this point where we have the lowest condensation this will have a slope which is which inclined much less and will move like this.

Now, let us consider a time t 1 ((no audio 38:48 to 39:57)) curve will take this shape. Now, what you see that we have already mentioned that the wave as it propagates is distorts the region of higher condensation. It tends to overtake the regions of lower condensation and the regions in terms of compression and expansion region; the net effect is that the compression region is steepened and the expansion region is flattened. In the compression region the characteristic lines converge; while in the expansion region the characteristics line diverge. You know in the compression region as we have seen that if we consider even a case like this ((no audio 41:13 to 41:44)) this will be the shape of the wave at…So, as the wave propagates according to this theory, this will be the shape of the wave leading to this situation at time t equal to 3. Of course, this is physically not possible because, it implies that there are 3 values of density at a particular given point in this region. This of course, cannot happen and consequently you can say that this situation will never reach. In fact, even the situation that is shown at t 2 will also not reach. The simple reason is that as considerably well before this situation occurs the velocity and temperature gradients, the velocity and temperature gradients reaches to a such a high value that the viscous effect and the heat transfer effects or not negligible and the diffusive actions are considerable and their effect will be such that to counter act this steepen intendancy.

So, before this situation reach the defining it is the diffusive actions are becoming considerable and they will counter act with these steepen intendancy that comes because of the non-linearity in the wave or the non-linearity in the equations and these two opposing effect will reach a balance and the compression part of the wave will become stationary in the sense that it will not change any further. However, this situation will eventually reach well before the time t 2 and in that case the compression part of the wave has become a shock wave.

Now, in the compression part the isentropic relations are varied until the diffusive actions are considerable or until the discuss friction and heat transfers are important. When are a when the stationary balance between diffusive actions and the non-linear steepening effect is reached the condition across this wave front are given by the shock wave relations which we have derived earlier and the intermediate part where the diffusive actions are considerable. However, they are not exactly in balance with the non-linear steepening effect. The states are non isentropic and they can neither be considered within the isentropic relations or with the shock relations and the complete unsteady equations including the viscous and heat transfer terms are required to analyze that part that will be somewhere between t 1 and before t 2. Looking to this case, however, we see that expansion wave always remains isentropic and as it flattens, as it propagates and the gradients are continuously decreasing, it never achieves any stationary condition and the isentropic relations always hold over these isentropic expansion part and since no stationary condition is achieved, there can there are no expansion shocks steep.

So, what we see that if we have a finite amplitude wave then, the wave different part of the wave will propagate at different speeds and consequently the wave will distort and what you see that the compression part of the wave steepens while the expansion part of the wave flattens. Due to this steepening effect the velocity and temperature gradients increases considerably and subsequently, after certain time a balance is reached between the steepening effects due to non-linearity and the diffusive action due to viscosity and heat transfer and the compression part of the wave will reach its stationary state and it becomes the shock wave. However, the isentropic the expansion part it flattens as it propagates consequently the gradients becomes smaller and smaller and the stationarity is never reached and no expansion shock occur; that is the expansions part always remain isentropic.

Next, we will consider this centered expansion wave. Let us consider that wave of fluid; certain amount of fluid is allowed to expand suddenly. This can be thought of let us say that certain amount of fluid is enclosed in a piston within a duct enclosed by a piston. Another piston is withdrawn impulsively. In that case, the distribution of the particle velocity in the first instant is a step and expansion wave develops and since the expansion waved flattens as soon as the wave starts propagating. So, at some time after t 1 the particle velocity will have a linear distribution and similarly the pressure will also have a corresponding distribution.

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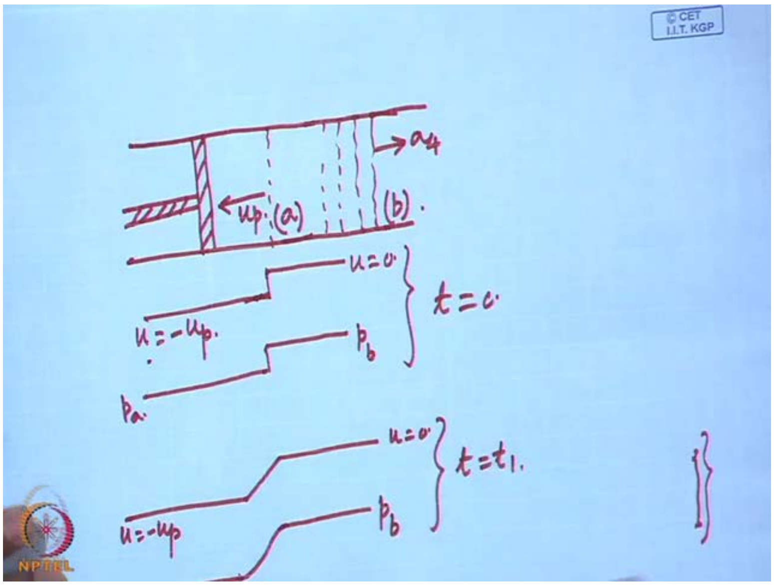


Let us say that we have we have a duct in which there is a piston and this is ((no audio 50:11 to 50:54)) the velocity distribution we can let us say this is the original ((no audio 51:07 to 51:46)) at t equal to 0 and expansion wave develops instantaneously as the expansion wave begins to flatten as soon as the wave starts propagating. So, after certain time t 1 and similarly the pressure distribution will be time t 1.

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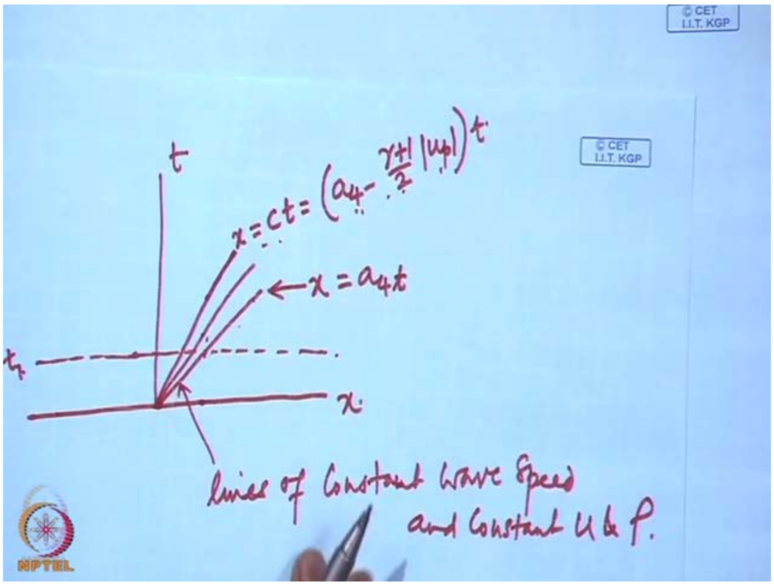


Now, the front of this isentropic wave hope we consider this in an x t diagram ((no audio 53:02 to 54:23)) the front of the expansions isentropic wave propagates at the speed of sound of the undisturbed fluid. Hence, the front of the wave propagates with speed let us say a 4 into the undisturbed fluid that is in the opposite direction of piston or in the fluid motion the wave speed in the portion of the wave which is behind the front is given by.. according to that theory that we discussed earlier as C equal to a 4 plus gamma plus 1 by 2.

Now, see that the wave speed decreases continuously since this u p is or u absolute of u p is positive of course, and minus u p. So, the wave speed continuously decreases as we can see from here the wave speed continuously decreases this plans of straight lines that we are showing here a lines of constant c and sub constant u and rho these are lines of constant wave speed lines of constant wave speed constant wave speed and of and of constant u and rho. Hence, these are basically characteristic lines. As time increases, this becomes wider and the wave becomes flatter and flatter and the gradients of velocity density pressure temperature. They become smaller and smaller thus the waves always remain isentropic the terminating characteristics is given by a 4 minus gamma plus 1 by 2 u p into t. And this will slope to the right or left terminating characteristics terminating characteristics t.

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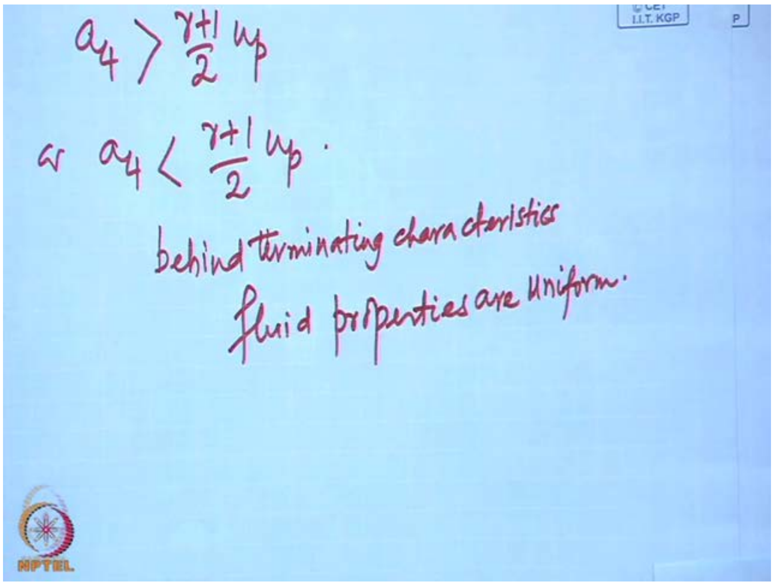


And these slopes to right or left depending on depending on whether a 4 is greater than gamma plus 1 by 2 up or a 4 less than gamma plus 1 by 2 up. Between the terminating characteristics or behind the terminating characteristics, the fluid properties are of the uniform values. Behind terminating characteristics terminating characteristics fluid properties are uniform. We will continue this in our next lecture because few more things have to be…