Space Flight Mechanics

Prof. M. Sinha

Department of Aerospace Engineering

Indian Institute of Technology, Kharagpur

Lecture No. 32

Trajectory Transfer (Contd.) & Attitude Dynamics

In the last class, we have been discussing about the trajectory transfer. So, we were working out the non- coplanar trajectory transfer. So, we continue with this and further, we will try to do some part of the attitude dynamics, in this particular lecture. And thereafter, now, after, once finishing the trajectory transfer, we will continue with the attitude dynamics, which will take around 6 lectures to complete it.

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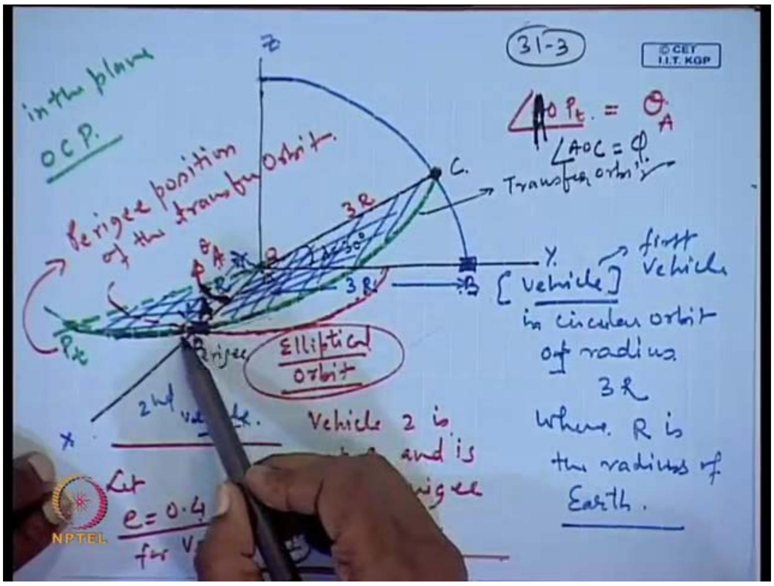

So, we had the problem of non-coplanar trajectory transfer. So, in that case, we saw that, we were working out this problem, where a satellite is at B, in a, and moving in a circular orbit of radius 3 R and another satellite, which is moving in an elliptical orbit and that is positioned at A and it is moving in the plane x y. So, unknown coplanar transfer was to be done. So, the impulse was required to be given at A, along certain trajectory, so that, the satellite from A, it reaches, catches the satellite B at the point C which is at 30 degree latitude.

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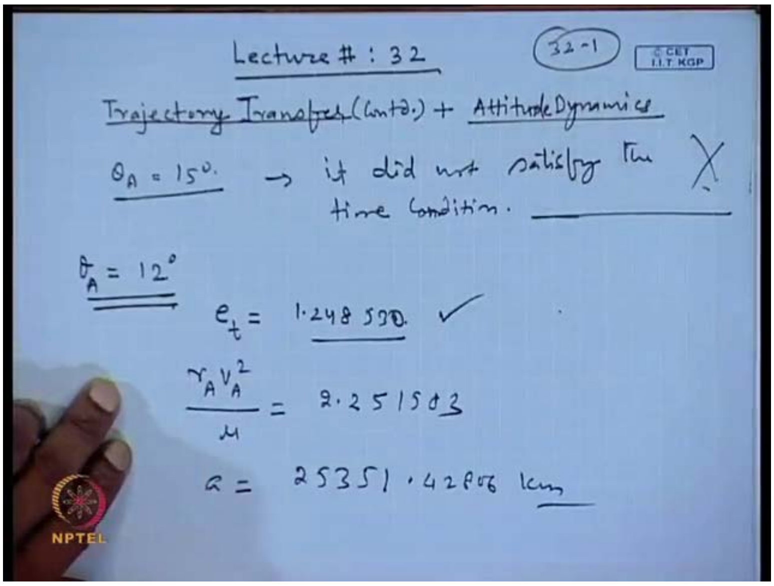

So, we solved the problem upto certain extent; few steps were remaining. So, what we saw that, once we choose theta A equal to 15 degree, so, it did not satisfy, satisfy the time condition. So, therefore, we need to revise it. So, what we saw that, last time, this was our last time lecture. So, theta equal to 15 degree, e t, the transfer orbit eccentricity turned out to be 1.14, and this was a semi major axis in kilometers, and t A C turned out to be 2292 and while t B C is 2195. So, t B C is the time which the satellite takes to go from this place to this place; satellite B is taking to move from this place to this place, and covering latitude of, angle of 30 degree in the, this circular orbit.

So, what we see here, in this place that, this quantity, this time is a small and this time is larger. So, this needs to be reduced. So, how we can reduce? This is the transfer orbit in which we have put the satellite and this was the perigee position of the satellite. So, and satellite location is, at the epoch time, at the beginning, it is located here, but the perigee is located here and this angle from here to here has been written as theta A. So, we reduce the value of the theta A. So, reducing the value of the theta A, what does it imply. So, if your perigee is far away from this point, means, by the time the satellite reaches this point, its velocity will, its velocity will decrease here, in this place. And therefore, or the, because the orbit is being taken, this is an elliptical orbit, and in this hyperbolic orbit, it has turned out to be, because e t has, is 1.1478, so, this is an hyperbolic orbit.

So, as we take it away from this point, it will take larger time to go from this point to this point, because by the time velocity decreases, if we measure the position from the perigee location. So, we need to push the perigee towards the point A; here, the point A has been shown by, this is now, it is almost dark. So, we can show here, this is the point A. So, let us reduce theta A, and we make theta A…So, this condition did not work and therefore, theta A, we need to make it, let us say 12 degree. So, with 12 degree, if you compute, the e t will turn out to be 1.248530. And this, we have computed earlier and the equations used, is already described in the last lecture. And, r A V A square divided by mu, this quantity will turn out to be 2.251583; semi major axis A will turn out to be 25351.42806 kilometers. So, you can see from the previous results, here, in this case, the eccentricity is 1.24. Earlier, the eccentricity was small, 1.14; eccentricity in this case has gone up. The second thing, the semi major axis in the previous case for theta equal to 15 degree was 42000, more than 42000; while it is turning out to be here, 25000 something. So, obviously, the semi major axis, the orbit size has reduced; semi major axis gives you the idea about the orbit size.

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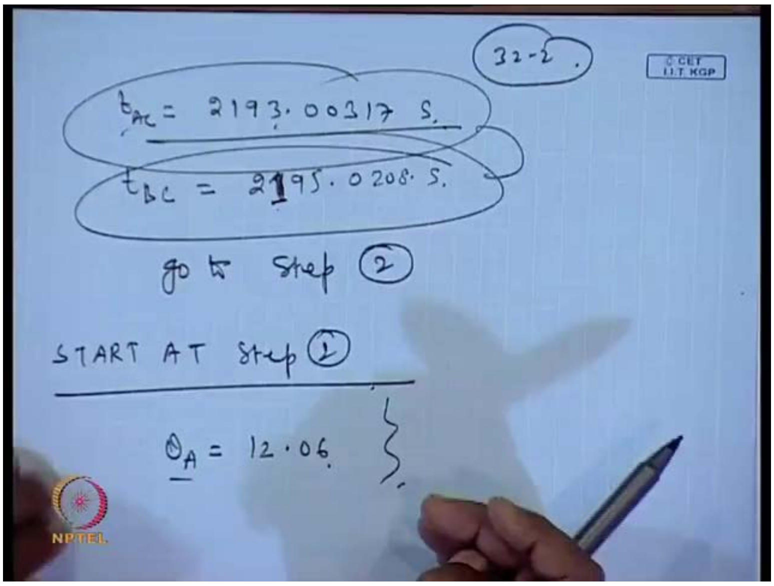

And, if we compute t A C, so, t A C turns out to be 2193.00317 seconds; while t B C is given to be, this we computed already in the last lecture; it is 2195.0208 seconds. So, now, you compare this two; the, it is a, this is 2195; still this t A C is short of t B C; but this difference is small, as compared to the, what we were getting last time. So, last time, the difference, it was coming larger 2292.46 something and here 2195. So, around 100 difference, 100 second difference was there; while in this case, the difference has reduced to around 2. So, now let us take…So, this is you, we start here, we write here, go to step 2. So, we have written here, again, go to the step 2, in the last lecture. So, start at step 2, start at step 2. So, by proceeding like this, theta, we assume, this is the assumed value, and we proceed, we get all this values and finally, we are getting t A C. So, again, it is not matching exactly. So, we try to change the value of theta A. So, let us again increase. So, here, we will write, go to step 2. So, start at a step 2. Now, assume theta A to be 12.06. I did this calculation on computer; I just programmed the whole thing and got the result. So, you can do it on a programmable computer, calculator, because it is a little bit iterative and then, the equations that you are using, it is lengthy.

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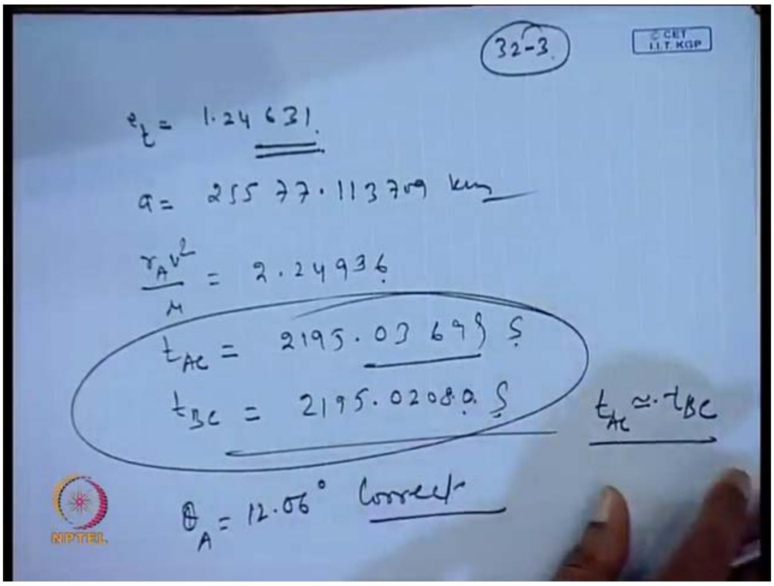

So, it takes a lot of time. So, instead of doing some manually, I just programmed and worked out the whole thing. So, with this assumed value of theta A, 12.06, e t will turn out to be 1.24631. Similarly, the semi major axis, this will turn out to be 25577.113709 kilometers and r A V A square divided by mu…936 and t A C turns out to be 2195.03699 seconds. And, our t B C, this quantity is 2195.0208 seconds. So, here…So, we can see from these two, that t A C, this is quite near to t B C. And therefore, the assumed value of theta A equal to 12.06 degree, this is correct; that is, perigee of the transfer orbit; this is the, we have shown it by the green line; this is, the green line is constituting one plane. So, the shaded portion, with the blue hatched portion, this is constituting one plane. So, the perigee position, from this point, it is lying around 12.06 degree away from this point.

So, this gives us idea how to do the trajectory transfer to, and these are the real things, that we apply in the real life. So, next, if you have to do satellite transfer from one orbit to another orbit, of course, in this, the two orbits were perpendicular to each other; even if they are making certain angle, you can follow the same principle and then, work out the whole thing. There is no difference between, if the, there the orbits are inclined at other angle, other than 90 degree. So, process is exactly the same. So, this completes the, the part, where we had to calculate the perigee position. Now, once these things are available, then, we can go to the next step.

So, once we have got this t A C equal to t B C, so, then, what is required that, how much impulse is required at point A to send it towards, in this orbit, towards C. So, for that, what you need, you need to calculate the velocity of the satellite at point A in the transfer orbit; and, at point A, in the original orbit, which is shown by, here, the red line, it is and it is lying in the x y plane; it is already known to you, because the eccentricity is given, and this is also given to be the perigee position. Therefore, you can calculate the velocity here, in this place, and the radius is also given; this radius, from here to here, this is 3, this is R; O A is actually R, but we need to find out in this orbit. So, once we have got this, that, this angle is theta A, now, we can compute all other quantities one by one.

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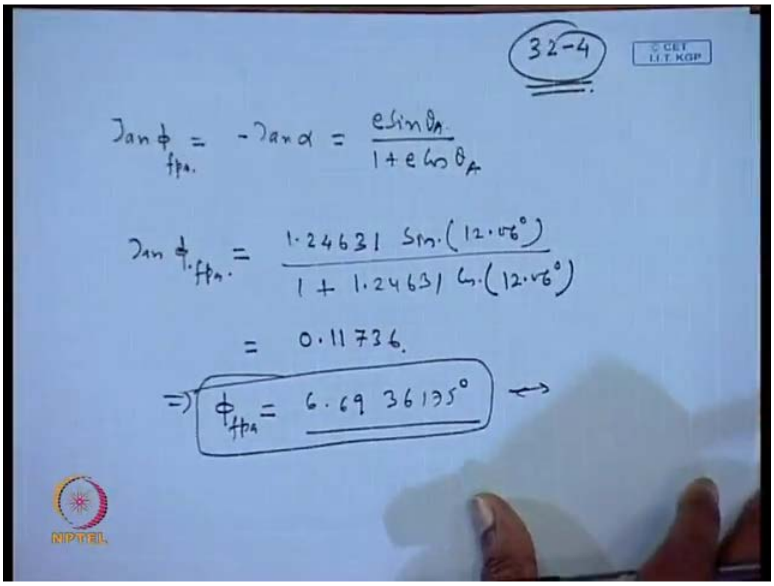

So, we need to furnish certain diagrams. First of all, let us write tan phi, phi we have used as the flight path angle and this is equal to minus tan alpha…This equation, we developed earlier, where theta is, A is indicating the position of the satellite, being measured from the perigee position; that is, this is the true anomaly. So, here, theta A is indicating that, the satellite is at this point and position is being measured from here. So, what will be the flight path angle, that we need to compute. So, we have tan phi, eccentricity the final, final thing we have calculated just now, this is 1.24631; theta A we found out to be 06 degree, 12.06 degree and 1 plus eccentricity again 1.24631 and cos theta 12.06 degree. And then, this turns out to be 0.11736 and therefore, this implies, phi f p a flight path angle equal 6.6936175 degrees. So, this is computer generated results. So, it is showing a lot of accuracy and if you do precise calculation, then, the result will be better; otherwise, a lot of truncation error will be generated.

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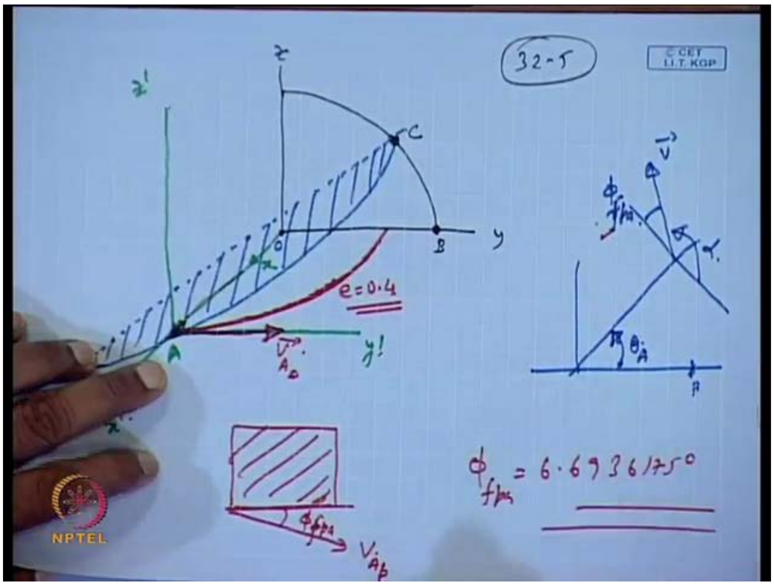

Now, the whole situation can be painted like this. We have one orbit here, y and z; this is the point B and where the satellite is initially located and while it reaches the point C, it has to be caught. So, the transfer orbit is given to us. So, now, what we will do, we will show the position of point A, by a different line; say, if we choose another coordinate system indicated by y prime, z prime and here, you can write, here, towards this, this is x and here, we will write, this is x prime plane and this is your point A. So, orbit transfer is taking place from here to here. So, somewhere, your perigee is located here, in this point and then, satellite is being sent from point A towards point C, along this trajectory. So, we can join the perigee point and this line and this constitutes one plane. So, now, the flight path angle is the angle…Earlier, we have described this as, if this is a perigee position, this is the true anomaly theta; this is the velocity vector V. So, this angle, we have described as the flight path angle f p a. So, in our case, this refers to theta A; that perigee from the perigee, the angle, true anomaly is being measured. And, alpha, we have referred to this angle. Now, once this is given…

So, here, already, the satellite is moving in a circular orbit of eccentricity 0.4. It is moving in a eccentric orbit, of eccentricity, not in a circular orbit; in a eccentric orbit of eccentricity equal to 0.4. So, the, initially, the satellite will have, because this is the perigee position of the original orbit, this is the original orbit, and this is the perigee position. So, your initial velocity lies along this direction of the satellite. So, this is V A; we can write as V A 0 or whatever you want. So, this is your, the velocity vector. In the new situation, where we are going along this trajectory, shown by the blue line, so, the flight path angle is shown to be, we have calculated this to be 6.6936. So, that implies that, this line is coming from here to here. So, the flight path angle at this point will be measured with the line we have shown here, y prime; but it will in the plane of, shown by this hatched lines. So, this, this is the shaded portion; in this plane, it is lying; its plane in this plane lying, but it is making certain angle with the y prime and z prime plane and that angle is shown to be here, so, to be flight path angle f p a.

So, in another words, if we consider this, the dashed, the hatched portion to be the plane in which we are looking, so, say, this is the hatched portion, in which the velocity vector is lying. So, your velocity vector is directed like this; after the impulse at point A, after impulse. So, we can write as V A post impulse and this is your f p, the flight path angle phi f p a. See, you can, this is not difficult to conceive; you just have to visualize it. So, this is the, hatched portion is the portion in which the velocity vector is lying, but it is a little lying off the y prime z prime plane by angle 6.936175 degree.

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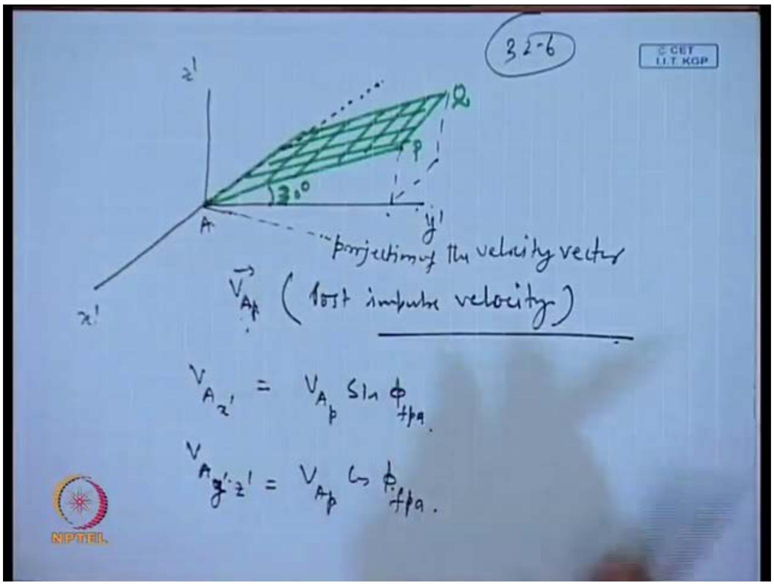

So, once this is known, now, the everything will be easy to work out. So, this is three dimensional view of the transfer plane. So, we name it as Q, this as P and this angle is 30 degree. This, we have seen already that, this angle is 30 degree, in the last lecture. So, now, your velocity vector, it is lying in this plane itself, but it is a little off the y prime z prime plane and that, I have shown you in the, this view, which is the hatched portion has been shown here. So, along with the hatched portion, the same thing has appeared here; velocity vector has appeared here, in this place. So, if the V A P is the velocity of, in the post impulse velocity, then, we can take, compute the components of this velocity along the different axes. So, we have, V A x prime will be equal to V A P, this magnitude of this vector V A P times sin f p a. So, your velocity vector is lying in this plane, plane which is inclined at a 30 degree angle. So, we can line it here, to make, this is a wedge shape; but it is lying out of the y z, y prime z prime plane. So, you can consider that, its projection will lie somewhere in this place. So, this is your projection of the velocity vector, which lies in the x prime y prime plane.

So, the velocity vector, once you are breaking it up, see, it will have two components; one component will go in the x prime direction; another component will go in the y prime z prime plane. So, we can write here, V A x prime, y prime and z prime is equal to V A P cos phi f p a. Now, this, this component which is lying in the y prime z prime plane, it can be broken into two components. So, how to break it into two components, because it is lying in this plane, this shaded plane; therefore, this angle 30 degree is the deciding angle here. So, let us say, this angle we write as alpha.

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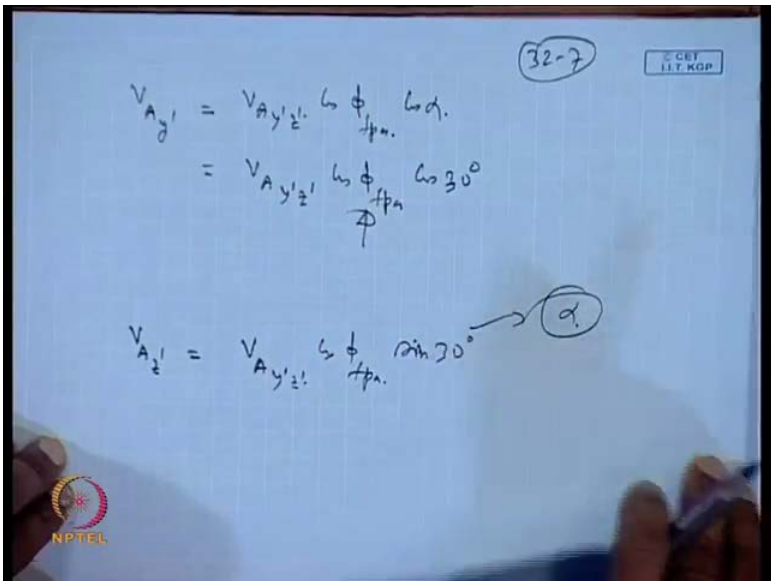

So, therefore, V A y prime you can write as, V A y prime z prime cos phi f p a. Now, y prime, obviously, you have to take cos alpha here, to get the component. So, this is cos alpha; put the value of alpha; cos phi f p a and then, cos 30 degree; and also, insert the value of phi f p a. V A z prime will be similarly written as, V A y prime z prime cos phi f p a times sin 30 degree; that is the component here, we get in this direction; where 30 degree, instead of writing alpha, we have directly written here, in terms of the actual value; we will come to this and compute the values again.

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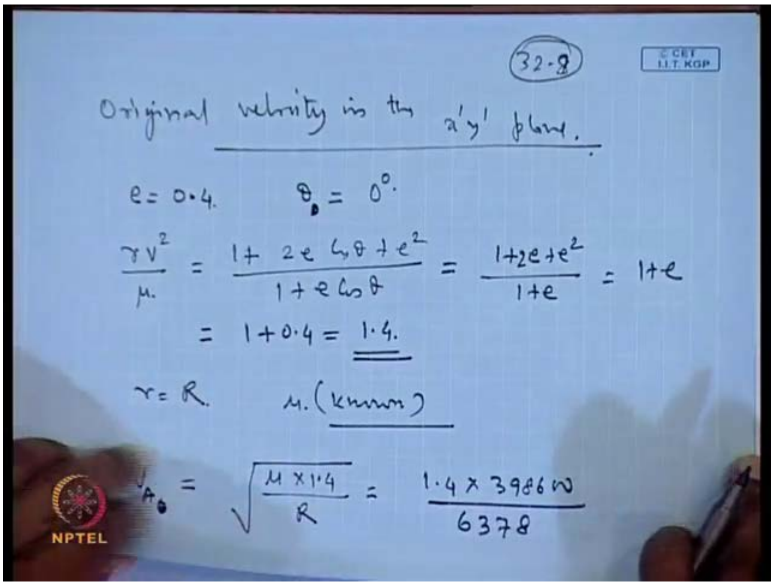

Now, let us see the original velocity, velocity in the x prime y prime plane, e is given to be 0.4, and obviously, the satellite is at perigee and therefore, theta, theta becomes equal to 0 degree. So, r V square divided by mu, this equation we developed earlier; we insert the value for e and theta here, in this equation. So, we will get this as 1 plus 2 e plus e square divided by 1 plus e and this gives us 1 plus e. And therefore, 1 plus 0.4, that makes 1.4. r is known to us. r is equal to nothing, but R; mu is known to us, this quantity is known.

Therefore, V can be computed. So, V A 0, we can write as, or we have written this as, this is post impulse and prior to impulse, we have just written as V A. So, V A is, now can be written as mu times 1.4 divided by R under root. So, insert this values, 1.4 times 398600 and R is, which is the radius of the earth in this case, we have taken.

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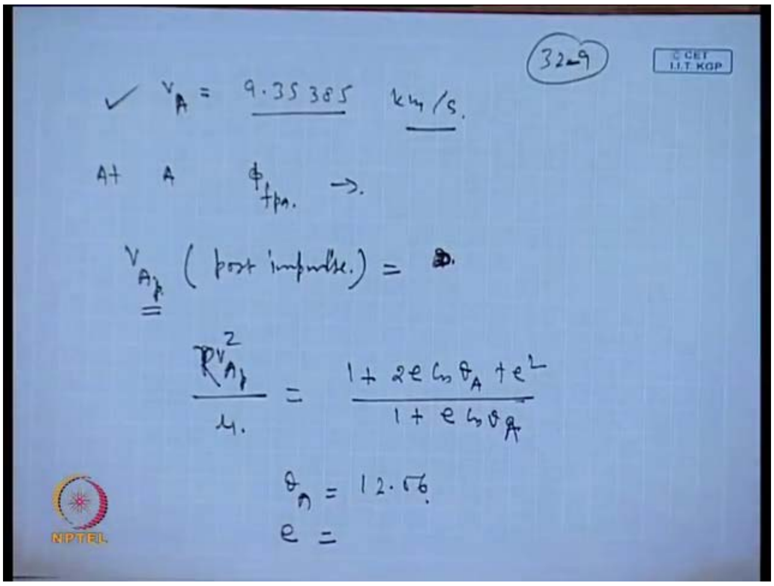

So, it is a, the consideration that we have taken; it is a little hypothetical condition here. As we can go into the our previous lectures, see, in the previous lecture, we assumed that, r B to be 3 R and r A equal to R, where R, we have taken as the radius of the earth; but suppose, in that case, A, in real situation, it will touch the surface of the earth, if we are taking this. But, it is just for our convenience, we have chosen like this, R and 3 R; you can choose some other value and try it out. So, once we have got this V A, now, at A, phi f p a, this is known. So, therefore, we can calculate the V A, V A the velocity in magnitude, in the post impulse scenario.

This is the post impulse velocity. So, the eccentricity, we calculated to be, sorry, we had the equation, this equation we are using again. So, r V square by, r V A post impulse V A square by mu and this r we are replacing by capital R, and this is nothing, but 1 plus 2 e cos theta A plus e square divided by 1 plus e cos theta 0; so, theta A. So, theta A is known to us. This we have written as 12.06 and e also we have computed in the last part; e was 1.24631. So, e is known here; theta A is known. So, right hand side is known to us; mu is known; r A is known; therefore, V A p can be computed.

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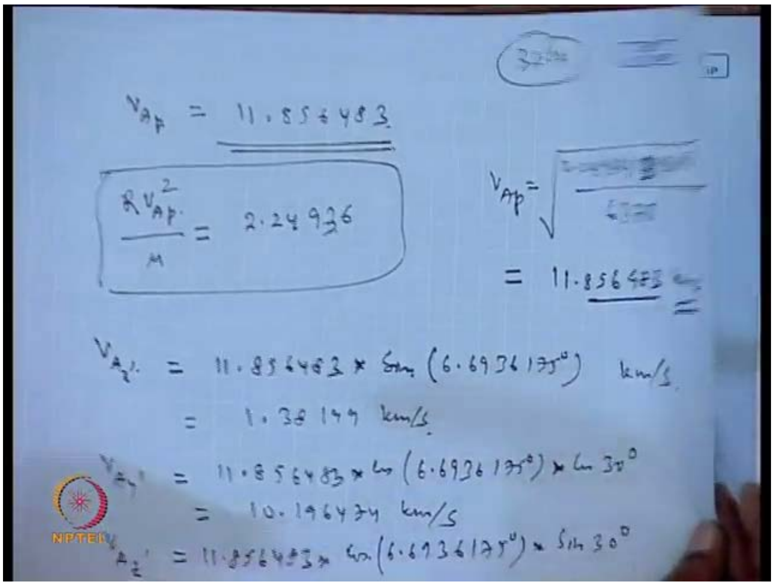

So, V A p, if you compute it, this will turn out to be the quantity 11.856483. Actually, the value of R V A p square divided by mu, this will turn out to 2.24936. So, from here, we are computing this V A p to be the, this quantity. So, you can write, V A p into 2.4936 into mu is 398600 and divided by R, which is 6378 under root. So, once we have got this, then, V A x prime, this will turn out to be 11.856483 times sin theta y a, we have, y a is 6.6936175 degree. So, this comes in kilometer per second, this velocity. So, this will be equal to 1.38199 kilometer per second. Similarly, V A prime, this will be 11.856483 into cos 6.6936175 degree times cos 30 degree. This will turn out to be 10.196474 kilometer per second; and V A z prime will be 11.856483 into they are all into cos 6.6936175 degree times sin 30 degree. So, V A z prime, this will be equal to 5.5886937 kilometer per second.

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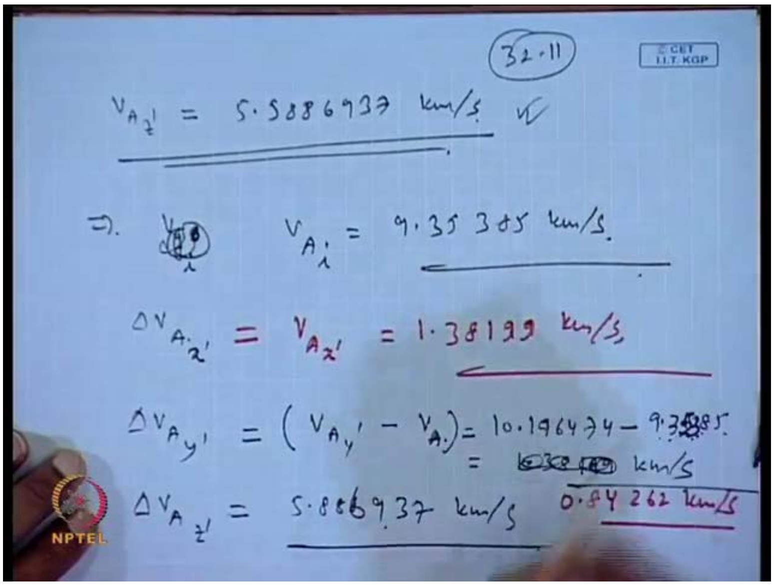

So, this implies…Now, we can compute the impulses required. So, we computed the actual velocity for the, initial velocity, that is, V A i to be 9.3585, and this is in the direction y prime, that is because it is at perigee. So, V y prime, or we write V y, or we have used as V A I, so, V A i 9.35385 kilometer per second. Now, calculate delta V A in the x direction, delta V A in the y, y prime direction and delta V A in z prime direction. So, if the satellite is moving, originally it was moving in this plane. So, there was no component in the z direction. So, simply, whatever the z we have got, here, in this place, that will turn out to be delta V A z prime. So, this is 5.886937 kilometer per second.

Now, delta V A y prime, this we can compute from, subtracting from V A y prime minus V A. So, this is 10.196474 minus 9.35385. So, this will give you 1.38199 kilometer per second; sorry, this turns to be not one point, this is 0.84262 kilometer per second. And, V A x prime, now, look into the V A x prime, in the original figure, because this is at perigee, so, the velocity is, original velocity is directed along the y prime direction. Therefore, we do not have any, any component along the x prime direction. This is the figure, here. So, original velocity is directed along this direction. So, we do not have any component along this direction. We do not have any component along the z prime direction; and therefore, whatever we got as V x prime…So, delta V x prime will be nothing, but V A x prime, and this is equal to 1.38199 kilometer per second. So, this solves our problem completely, and this, our trajectory transfer problem is complete now.

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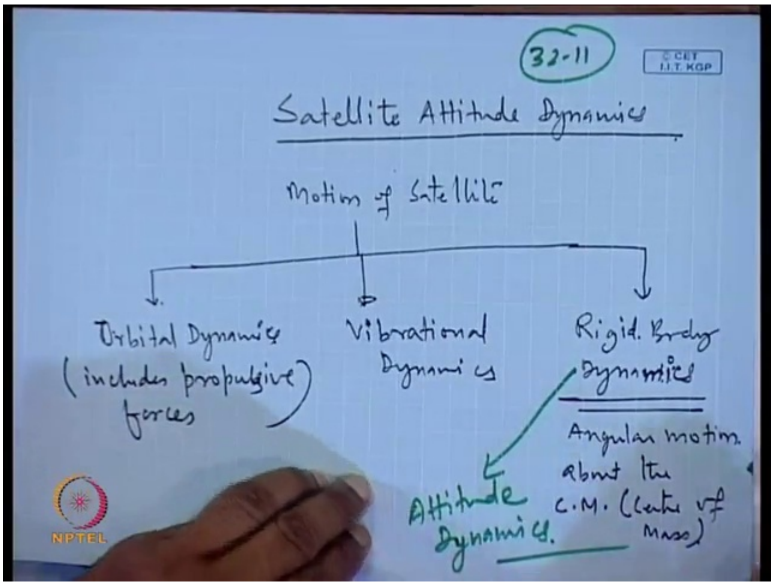

So, now, we go into the satellite attitude dynamics. So, motion of the satellite, it can be broadly categorized as orbital dynamics, or what we call as the trajectory dynamics, or vital dynamics, so, in which category, we have already seen, the trajectory transfer also falls under this category. And then, so, this includes, includes propulsive forces, propulsive forces. Then, we have the vibrational and the third one, we have the rigid body dynamics. For more complex, we can have a flexible body dynamics, where the rigid body, there are other vibrational modes. So, it is a rigid body, not the…Here, the, we, we will be considering the purely rigid body dynamics. In the vibrational dynamics, you can consider only the vibration; but once you are considering the system, which is rotating and together with its a vibrating, so, both of them are combined together. So, that does not come under the rigid body dynamics; rather it will be the flexible body dynamics; flexible body attitude dynamics. So, here, we will be dealing with angular motion about the center of mass and this is referred as attitude motion. This is written as attitude dynamics or attitude motion.

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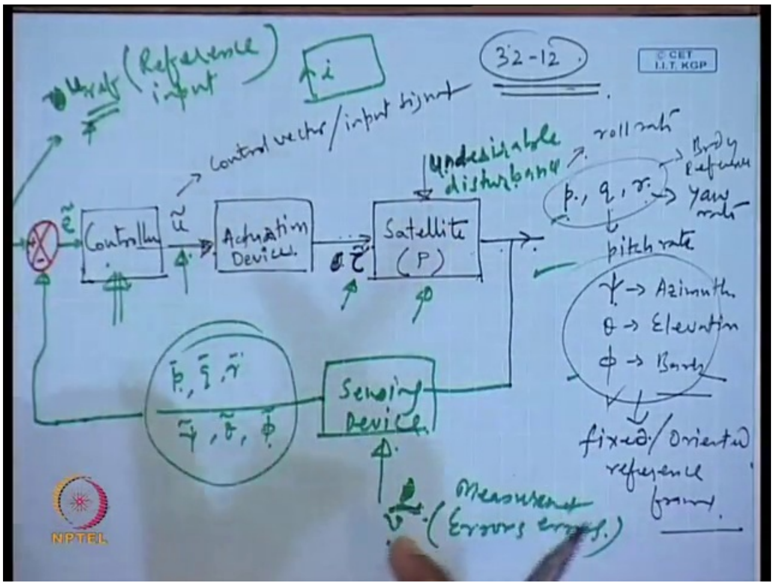

Now, let us say, we have a satellite, which is shown by this block diagram. Often, we indicate it by P, to indicate this is a plant. So, to maneuver the satellite in the orbit, you need certain amount of movement along the different axis. Without that it cannot be maneuvered. So, those movements can be passive movement, or it can be the active movement. So, what is the passive movement, active movement, we will come next to, in this discussion.

So, for actuating this satellite, what you need is some actuation devices, actuation devices. And, these actuation devices are built inside the satellite itself. So, this maybe the rockets; it maybe the reaction (( )); it may be the magnetic actuators. So, this actuation devices, now, nowadays, more, in the rockets will have the iron thruster rockets, and many complex things have come up. So, this actuation device, how much actuation has to be done, that has to be decided. So, that decision will be taken based on certain logic, or certain rules, which we called as controller. As a controller gives the signal, say, u tilde, u tilde we are writing as a vector, this is the control vector; control vector, or we call this as the input signal. And here, this is the actuation signal, which is going into the satellite. So, the actuation signal, we can show by, say, rotation of certain reaction, will further (( )). So, we can show this as tau, or tau tilde.

And, the resultant will be that, satellite attitude is changing; its angular rates are changing. So, angular rates of the satellite along the three axes can be indicated as p, q, r.

This is the roll rate; this is called the pitch rate, and this is yaw rate. And, simultaneously, we can have the satellite attitude in terms of angles, which we called as the psi; this we will call as azimuth; theta, this is called as elevation, and phi, this is called the bank angle. So, and these angles are measured with respect to certain inertial axis, (( )) say, there is a fixed direction with respect to which you are measuring this angles.

While this p, q, r, these are measured in body reference frame; while these are measured with fixed, or either oriented in a particular direction, fixed oriented reference frame. And, the same set of angles are also used to describe the attitude of an aircraft and the, the same rates, symbol can be used to describe both the aircraft and the satellite. So, we have the output signals available from here; but in the satellite, there may be certain undesired, undesirable signals, which are disturbance signals; undesirable disturbance, it is going into the satellite.

So, this can arise from various things like, the solar pressure radiation; if you have the magnetic actuators, then, in that case, it may be due to some residual magnetic moments arising from the currents in the various circuits; there are various wires inside. So, those wires, when interact with the current bearing wire, they interact with the magnetic field of the earth, and they create, they produce unwanted magnetic moment. So, for producing the magnetic moments, we have separate magnetic pulse; but if you have the wires here and there, so, they also create the magnetic moments. And, so many problems are created. So, various sources of disturbances are there. So, satellite output, certain attitudes and rates, so, they may be sensed here.

So, we will have sensing device. So, the sense signal, you may have again, error built into this. So, we show it by some nu bar, or nu tilde, to indicate this is the measurement errors. So, we can show here as p bar, q bar, r bar and similarly, psi, theta, phi, putting here tilde over this. These are not vectors; these are all scalars, but putting tilde over this, we are indicating that, this is now contaminated with noise. You need to follow, your requirement might be that, the satellite should point in a particular direction, in the space. So, that is inertial pointing; it may be required that, your satellite, it is always pointing towards the particular point on the earth; it is a called the earth pointing satellite.

So, accordingly, you may have a tracking signal here, which we call as the u reference. So, this is the, called the reference signal, or the reference input. This is also called reference input in controls, reference input. Now, this will be compared with the, we put a plus sign here and minus sign here, and this signal is fed back from this point to this point; and whatever the error is occurring, so, that error signal is fed to this controller; and then, controller decides, how much actuation signal has to be given to the actuating device; like in aircraft, it may be a servo motor, or it may be a hydraulic motor, hydraulically driven device, servo motor and hydraulic, hydraulically driven device, so, they may be used to deflect the flaps, in that case.

So, here, in this case, actuation device, as I told you earlier, this may be the reaction (( )), or in our magnetic actuators, the actuation devices will be our magnetic actuator, which are basically coils. These are the coils in which the currents will be passing. So, this coils, once will, it interacts with the magnetic field. So, it generates the magnetic moments. So, this indicates then, the magnetic moment and this is basically, the whole thing is situated inside the satellite. Then, satellite then, under the action this torque, it starts rotating. So, you give proper signal, you, your idea will be to produce this proper signal u tilde, so that, the satellite assumes the desired attitude, or maybe, even the desired angular rates. You want to kill angular rates about certain axis, or you want to induce angular rates about certain axis. So, all these things can be performed using the controller.

But this controller need to be designed, and there, the whole the classical control theory, the modern control theory, then, various new streams have come up, like the intelligent control, in which the neural control, quasi control, all these things fall. So, they have been developed, and any one of them can be applied to achieve the objective. But it depends, which one we are utilizing, it depends on the situation at hand. If we have the more complex objective to be done, so, accordingly, we need to choose. So, the classical control, till now, it is being utilized in most of the satellite control problem, at least in India, as far as I am aware of. So, the classical control is the old control method, which is worked out in the frequency domain. So, using that, you can linearise the system and then, you can device the control and you work out all sorts of things. So, this constitutes our primary, basic configuration in which the satellite is there. Then, we have the actuation device and the controller. So, whole objective of the attitude dynamics, and the control, is to achieve this objective of tracking the reference signal.

But before that, what we need to do, we need to work out the satellite dynamics, which is the dynamics part of the satellite, that if we apply certain amount of moment on the satellite, then, how the satellite is behaving. So, for that, we need to develop the complete equation; we need to study the stability of the satellite, whether, in what configuration, satellite is stable; in what condition, it is not stable; say, a satellite is rotating…This is, suppose, this is my satellite here, like this and this is rotating like this. So, this is the angular momentum vector. It is directed in this direction and the, it is a continuously rotating and pointing in a, some inertial direction; like the Hubble telescope, it is pointing in the sky towards a particular star.

Now, if you want to change the orientation from here to here, then, you must apply the torque. Without that, you cannot accomplish it. So, for that, you need to study the dynamics of this. And moreover, even in this configuration, in the initial configuration, whether the satellite will remain in this position for longer time or not, that you need to study. So, this study, it, it is part of the stability analysis of the system. And, once we do the control design…So, thereafter the whole system is to be analyzed, whether as a whole system will be stable or not, under this reference signal; once we give the input signal here, so, whether it will stabilize or not. So, various issues are there. So, we are not going to study the controls part in our attitude dynamics; rather we will concentrate with this part only. And, we will look into the dynamics of the satellite and its stability condition and other things. So, we stop here in this lecture. We continue with the next lecture, the same attitude dynamics. Thank you very much.