Introduction to Aerodynamics

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Lecture No. # 12

Kinematics of Fluid Motion-Velocity with Specified Extension and Vorticity (Contd.)

The last time we consider what will be the general velocity field, if rate of expansion is specified everywhere or a vorticity is specified everywhere. And then as a specific example we considered a singularity in the distribution of rate of expansion where the rate of expansion is infinite at a point. But everywhere else it is zero and we approach this by considering a large rate of expansion which mathematically can be described by a step. A peak at a small region and then we approach this small region to a point by which we obtained a point source and we derived the potential function phi e and the velocity distribution that is obtained by this point source; which is a singularity in the rate of expansion distribution.

Because in this case there is a rate of expansion of infinity amount at one point and everywhere else it is zero or if not zero there might be some other values but which can be additive which can be added. Also if you look to this expression for phi e and the u, that we obtained the velocity that we obtained that both approaches to a very large value as r approaches zero.

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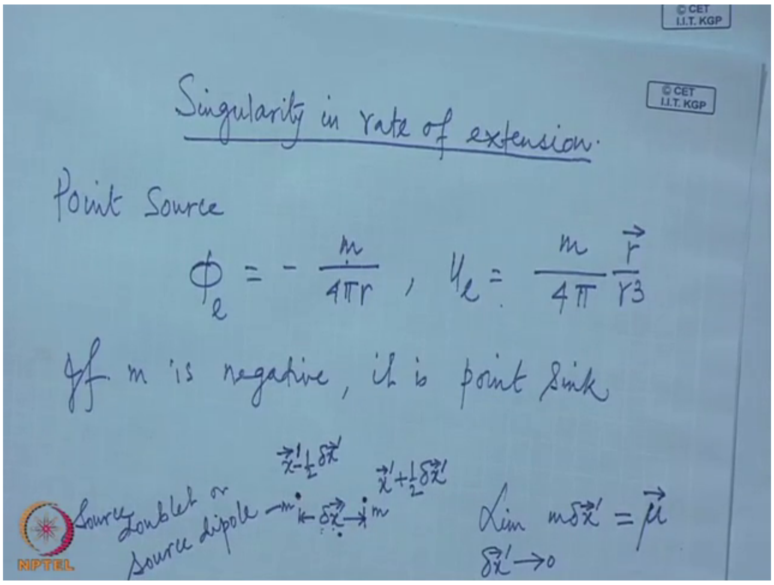

Also you will calculate this is point source, what we had phi e is minus m by 4 pi r and u e as m by 4pi to give the direction we wrote it r by r cube that is m by 4 pi r square in the direction of r. The velocity as you can see clearly is decreases with inverse square of r, and as r approaches zero the velocity become extremely large and at r equal to zero neither this potential nor the velocity is defined. This is why these are called singularity. If we consider any closed surface enclosing the point at which the source is located then the flux of volume across that close surface is m which we call the strength of the source. We will later on see that a single source a single point source is usually not created by any dynamical effect.

Due to certain forces usually wherever there is a source type of situation this is due to some external reason. That is either you inject fluid at certain point or you shock shocking certain amount of fluid at some point that gives rise to near about a source type of situation. Otherwise just by the application of the forces that usually is at exerted on a fluid this type of point source situation usually does not arise. If the strength is taken minus m then the singularities called point sink. If m is positive resource if m is negative it is called sink, m is negative it is point sink. Later on we will see that the utility of this point source or this is constructing more important more interesting flow field on its own point source really is not very important.

But its main importance lies in the fact that it can represent along with others a much more interesting and often some practical flow fields. But of course, we will come to those later.

The next singularity that we would like to consider is a combination of a point source and point sink placed a delta x distance away. Both the point sink and the point source have same strength. Let us consider that sink source as a strength of m the sink as a strength of minus m and they are separated by a distance delta x. Now think about that we are decreasing this separating distance delta x and simultaneously increasing this m, the strength is being increased while the separation is being decreased. In such a way that the product always remain constant, that is m into delta x that remain constant.

Let us consider this is the sink this is the source. This distance is we will call it delta x prime, this point is say x minus x prime minus half delta x prime and this point is x prime plus half delta x prime these are all vector, that means it is really the coordinate of the point. It is all right know and that will be

Since, the azimuthal component is zero. So, in the axial plain the velocity is like two dimensional the velocity field is two dimensional in an axial plain. And hence we can construct a stream function for it using the definition using that r theta phi coordinate system considering a r theta phi coordinate system origin at the location of doublet and theta equal to zero along doublet axis that is along the direction of mu. The radial component of velocity is given by 1 by r square sin theta d psi d theta where psi is the stream function which we have defined earlier.

So, in this coordinate system this is what is the and how can we get u r? u r we can easily obtained as u dot r by r. This u already we have the expression that we have written that 1 by 4 pi minus mu by r q plus 3 mu dot r by r to the power five r so take a dot product with r and tell me how much is this or let us write 1 by 4 pi this r comes here the first term becomes minus mu dot r by r cube and the second term becomes 3 mu dot r by r to the power 5 and r dot r that is r square.

Minus we got.

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And the result is how much 1 by 2 pi mu dot r by mu dot r by r to the power 4. We can write this further that 1 by 2 pi mu cos theta by r cube mu dot r is mu r cos theta.

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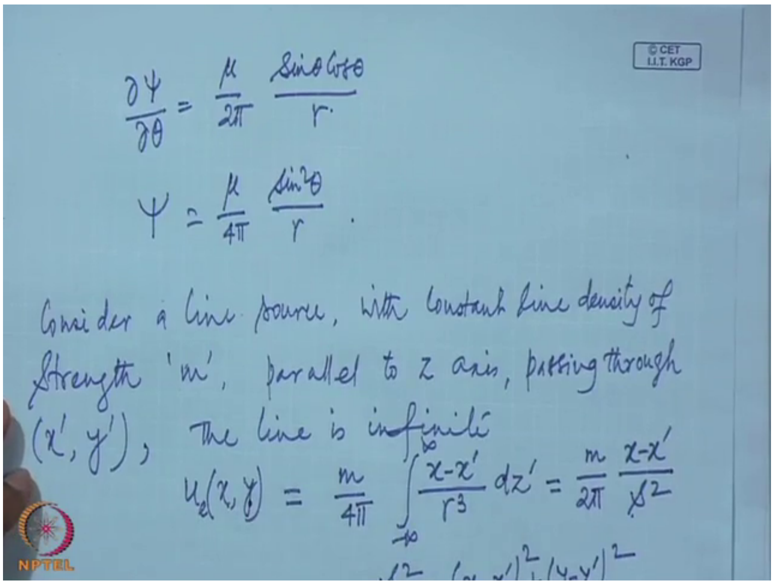

Now we can evaluate what is psi d is what d? psi d theta is mu by 2 pi sin theta by sin theta cos theta by r know. And so psi equal to mu by 4 pi sin square theta by r. The constant of integration can be a function of r and phi but that we are just taking it for granted. Of course, you can check it that to have the proper tangential velocity that constant of integration has to be zero. The velocity we have all ready found so we can find what is the radial velocity as well as what will be the tangential velocity or the transfers velocity. And the transfers velocity to get that proper transfers velocity you can see that this constant of integration will be zero, we are not doing it but if you want you can check it.

Now we can construct similar type of singularities even more complex or higher order singularities. As an example like we placed two source and then brought them closer we can do the same thing we can place doublet, one of strength mu the other of strength minus mu, and then you can imagine that they are being brought closer and closer such that the product of that strength and that separating distance. Both being vector it will be a vector product that should remain constant and we can get what is known as quadruple this is dipole. And then two dipole that will make a quadruple. We can obtain the quadruple in other manner think about a parallelogram of any set and then place two sources in the opposite corner and to sink on the remaining two opposite corners and then imagine that this parallelogram is sinking to a point.

Again you will get that same doublet quadruple not dipole same quadruple. However they are very rarely used this higher order singularity so we will not go into that. But we will talk about two more very important singularities in rate of expansion one is when there is singularity over a line. Initially consider a singularity at a point that the rate of expansion is infinite there and everywhere else it is either zero or may have some finite value. It is possible to consider a line at which again there is peak but peak is not at a point along a line then it is called a line source.

Similarly we can consider a surface on which there is peak in the singularity distribution at each and every point on that surface there is peak which we then call a surface distribution. And as the flux across a close surface surrounding the point for a point source gives the volume flux in case of a line also. If we consider the flux for unit length of that line then that will give the line density of the line source it is called line density of the strength line source. Similarly, for a surface if we consider a unit surface area and then flux from that unit surface area is also called the flux density or in this case surface density of the surface source or strength of the surface source. These will be very important and we will come back to them again and again.

But right now just consider one line source and to make it simple, consider this line source the strength is everywhere constant. It is not necessarily that the strength has to be constant or the line density has to be constant it is not necessary. But to make it simple we will consider that line density are constant everywhere and not only those we will also consider that this line is infinite. And again for simplicity let us consider that this line is parallel to the z axis and passing through points say x prime y prime, then so consider a line source with constant line density of strength we will call this again m. Consider it parallel to z axis passing through the point x prime y prime and this line is from the line is infinite.

Now to find the velocity due to this line source what we can do is? We can consider a small element of that line of length say delta z prime, and then the strength of that small line element is m into delta z prime. And this small line element can be thought of as a point and we know the velocity distribution for a point source of strength. In this case the strength is m delta z prime so knowing the velocity for that we can now integrate the velocity from minus infinity to plus infinity. That will give us the velocity. So, how much it will be? So, the u component let us write now in component form not in the vector form. How much it will be?

We will use the same notation u e since the m is constant it will come out of the integration m by 4 pi into what the velocity was m by 4 pi r square in the direction of r.

Now take the x component of that or in this way m by 4 pi vector r divided by r cube. Now take the x component the x component is simply x minus x prime, the x component of that r vector is simply x minus x prime. This x is no longer the vector x. Please make these differences because you are using in this case x as simply the one of the coordinate. In the earlier cases we are using x as the position vector or the point vector so that difference you should remember where you are using x as simply one of the Cartesian coordinate or where we are using x as a position vector of a point.

In this case with respect to me the source is right of me and the sink is left there will be similar set of lines on the bottom half also. I am showing only one half, this is symmetric about this line, only half is shown the top half, bottom half is not shown.

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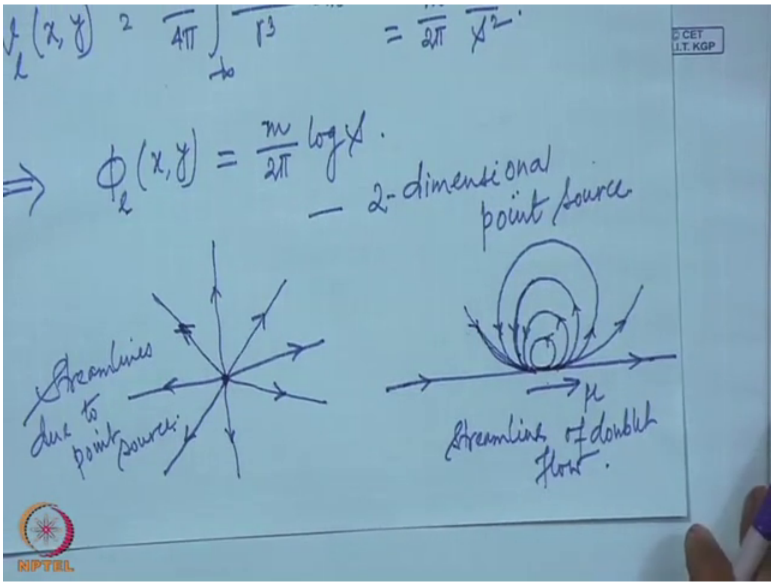

I have not written that but it is there. Next we will consider similar specific examples for vorticity distribution. That is certain singularities in vorticity distribution. However before we go to that vorticity distribution let us have some general properties of course, some simple general properties associated with vorticity.

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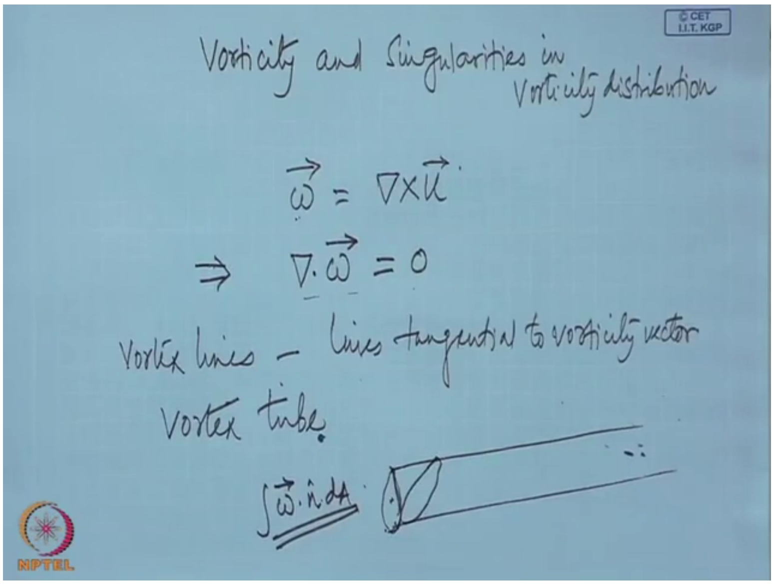

Then the integration of the vorticity over that surface area over that open surface area is called the strength of the vortex. Think about that this is a vortex tube. Now think about this surface think about this surface itself then this integration omega dot n d a is called the strength of the vortex tube.

And because of this property that divergence of omega equal to zero. We can very easily show that the strength of the vortex tube will not change. The vortex tube may change its shape it may have anything but the strength will remain same at anywhere and it is independent of this A that you choose. You may choose this type of A, you may choose a this type of A or any type of area, you choose this vortex strength that strength of this vortex tube will remain constant it will not change. And what is the consequence of that or significance of that?

That if we have a vortex tube then vortex tube will never end within the fluid with evidence means its strength is changing but it cannot. So, a vortex tube will never end within the fluid just because of this definition. We will continue it in next class the mathematical proof is very simple just one line mathematical proof but we will do it next class.