Introduction to Aerodynamics

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Lecture No. # 26
High Reynolds Number Approximation

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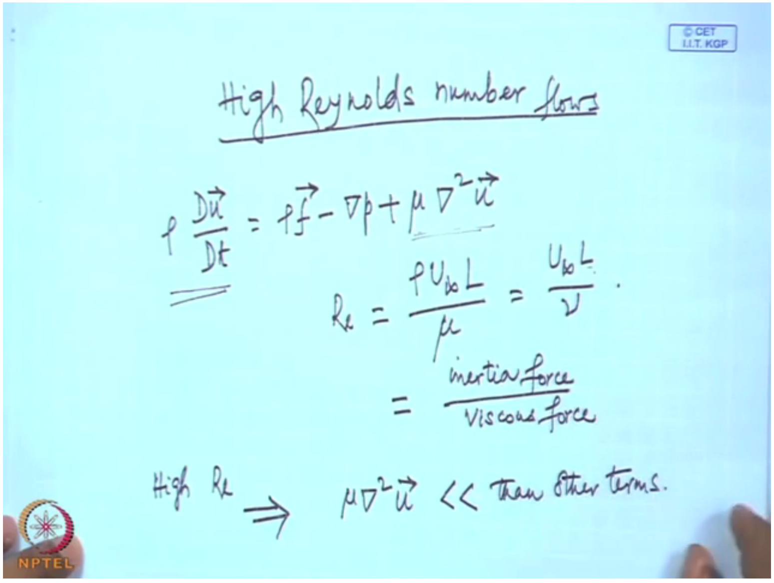

So, we have seen that for flow at very high Reynolds number the viscous stress term in the equation of motion can be neglected when we compared to other terms and the flows can be considered inviscid. This you have seen from our dimensional analysis of the equation of motion. Of course, we non dimensionalize the incompressible flow or the constant density flow equation but, this is even true if we consider the full compressible flow equation that when the reynolds number is very high the viscous force term present in the equation of motion can be neglected in comparison to the other terms. This term is much smaller than the other term present in the equation.

The Reynolds number which you have defined as a characteristic velocity, we took it u infinity I think, some characteristic length divided by the fluid viscosity or also quite often written like this. Also this is a ratio of the inertia force and viscous force. So, a very high

Reynolds number means that the inertia force are much larger than the viscous force. That the ratio of the inertia force and viscous force is given by this term that can be easily seen that the inertia force term in this equation is this term, this is the inertia force term and this is the viscous force term. And again if you take the order of magnitude the inertia force you can take any term in the inertia force there is either rho d u d t or say rho u d u d x.

The result is the it is rho u square by L, the order is rho u square by L whether you take rho d u d t because t is again of order of L by u. So, rho u d u rho d u d t or rho u d u d x any of these term that gives its order rho u square by L and the viscous force here you can see the order is mu u by L square. Say again the ratio is this Reynolds number. So, a very high reynolds number flow implies that the inertia forces are much larger than the viscous forces and under such condition as you say that this term can be dropped from the equation and.

So, high r e this implies

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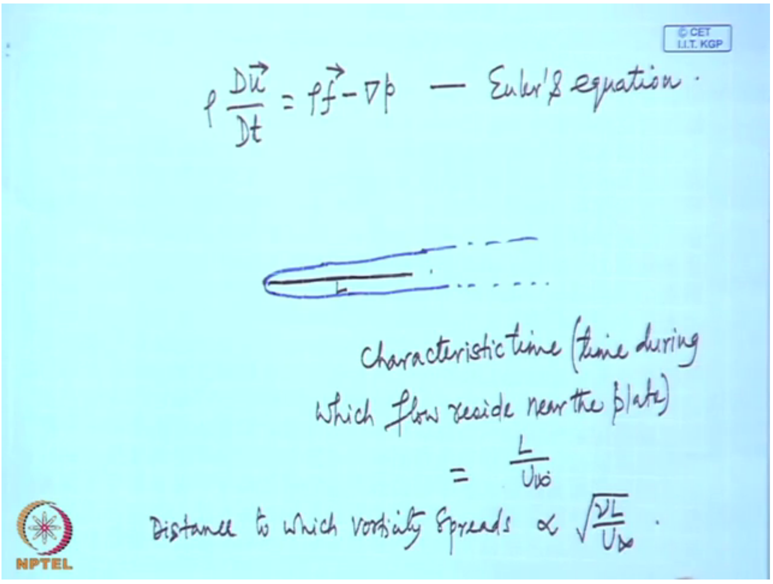

and the equation of motion then can be written as which we called that these equations are known as Euler's equations momentum conservation for an inviscid fluid or inviscid flow. As far as mathematical difficulty of this equation is concerned it remains almost the same because the difficulty in solving these equations comes beca[use]- form the nonlinearity of the equation and which is present again here also. However, we see a very important outcome of this simplification. When you neglect the viscous term from these equation, the equation reduces by one order.

The original Navier-stokes equations a second-order equation while Euler's equations are first-order equation and the effect is that the Navier-stokes equation needs 2 boundary condition while this needs only 1 boundary condition. That means we cannot satisfy the that two boundary condition if you solve Euler's equations and the boundary conditions for most important problem of flow over any body as we said that on the surface there will be no slip, that there is no relative velocity which is expressed that normal component of the relative velocity as well as the tangential component of the relative velocity both are zero. If we make this approximation at high Reynolds number that the flow is inviscid then we can satisfy only one of these.

We can satisfy that the normal component of the relative velocity is 0 but, we cannot satisfy that normal component of tangential velocity is 0. Now, let see what actually happens in the physical problem considering a physical problem let see what happens. Just think about we have a uniform flow. In uniform flow all the streamlines are parallel and straight lines and let say at we call that time instant t equal to 0 at that at time t equal to 0 we place say as simplest possible body consider just a flat plate, we place a flat plate in that flow in the uniform flow. Now, this flat plate will occupy the position of one of the streamline when you place it in the flow. Now, before the flat plate was placed along that streamline the flow velocity was say u infinity.

Now, the instant we place the flat plate there because of that requirement of no slip condition on the surface of the flat plate which was earlier a position of the streamline, the flow velocity will there become 0. Now, on the adjacent streamline which is at a distance of 0 plus, the flow velocity still remains that u infinity. So, at t equal to 0 we have let say where we place the flat plate we call that thus some of the x axis so that is y equal to 0. At y equal to 0 we have flow velocity u infinity at t equal to 0 plus at y equal to 0 the flow velocity becomes 0 but, at y equal to 0 plus the flow velocity still remains u infinity. Now, what does it mean that there is a velocity gradient d u d y of the order of infinite because over a distance of 0 the flow velocity change from 0 to u.

Now, what is d u d y? d u d y you see the if we component of the vorticity the omega z is d u d y minus d v, d x or d v, d x minus d u d d y. So, d v d x is of course, 0 along x direction there is no change the flow velocity is 0 everywhere so at y equal to 0 we have created an infinite amount of vorticity. The instant we place the flat plate is simplification is that we have created an infinite amount of vorticity on y equal to 0, a vortex set if you consider it a two-dimensional case we have created a vortex set of infinite strength.

Now, we already talked about something on vorticity, we have also derived a vorticity transport equation. We have seen that a vorticity at any point changes because of its convection with the flow, because of its viscous diffusion and also because of some local redistribution due to its rotation and stretching. This stretching and rotation of course, is a local redistribution. So, it do not contribute much to the global transport. So, the global transport is by convection and diffusion. Now, what will happen then as t equal to 0 plus as you have created this infinite vorticity, we know whenever there is a very high concentration of any quantity whatever it is this viscous diffusion takes place, it tries to smooth out that concentration so that there will be not very large concentration at some reason.

So, the viscosity or the viscous effect will try to spread it in all direction even though it is created y equal to 0 this viscosity or the viscous effect or the viscous force whatever you call, it will try to spread these vorticity in all direction. So, that there is no high concentration at any reason. So, because of this viscous action the vorticity will try to spread in all direction that is to the front, to the back, to the sides, in all side. However, we now also have a convection and this convection as in this case is parallel to the body surface. The flow velocity is not parallel to the body surface. So, this convection will of course, try to take away the vorticity, away from the body surface to downstream direction only this convection is not trying to take it to other side.

The problem that we have considered in this case the convection will try to take it only along the flow and the flow is parallel to the body surface towards downstream. So, this convection will try to take the vorticity away from the body surface to the downstream while the viscous action will try to take the vorticity in all direction to the front, to the sides, to the back. So, out of this 4 direction you see to the back in both case is combined the convection is also trying to take it back, take it in the backward direction, viscous action is also trying to take the backward take it in the backward direction. So, there is no conflict.

The effect will combine. However, as far as the viscous action towards front and towards side you see that will be opposed by the convection, that will be opposed by convection and this convection if it is strong enough will not allow the viscous action to spread the vorticity to larger distance in the front and as well as in the sides. So, in case where the convective forces or the convective action is much stronger than the viscous action what will happen? This vorticity will remain confined very close to the body in the front as well as in the side. However, in the back it has no objection it will go on.

So, what will happen that if let say this is the body and the viscous action will or the balance between the convective action and the viscous action will keep the reason to which vorticity can be spread to this part, this is to the infinity in this side there is no (( )). So, to these 2 sides as well to the front the convective action will not allow the vorticity to spread further by viscous action. We will now come back to that problem that we considered that a suddenly started plate, a plate suddenly started moving. This is equivalent problem whether we move the plate suddenly or we place the place a plate suddenly in a uniform flow the problem is equivalent.

So, in that problem when a body or a flat plate started moving implosively, we saw that the vorticity spreads or diffuses by viscous action to a distance which is proportional to root nu t, if you remember the distance to which vorticity diffuses is proportional to root nu t. Now, in this problem what will be a characteristic time looking to this problem where we have placed a flat plate in a flow, what is the characteristics time? We can take this is the characteristic time during which flow remains in the vicinity of the body. Now, since the characteristic speed of the body is u characteristic speed of the flow is u infinity and the characteristic length of this flat plat is L that is the length of this flat plate is L.

So, what will be the time during which the flow will reside near the body its L by u infinity, this is the order of the time during which the flow will remain near the vicinity of the body. So, this time we can replace by L by u infinity. So this is a characteristic time, that is time during which so the distance vorticity spreads root nu t, we have replace that t by L y u infinity. Now, how much is this how much is this root nu Ll by u infinity, how much is this? Is it of the same order as L, let say what is the condition at which it will become of the order of L. If you say this is of the order of L and it will happen then if u infinity L by nu is of the order of 1. So, if this (If the order of reynolds number is of 1 then the spread will be of the order of L. If the order of the reynolds number is much larger then the spread will be much less than L. Very very small compared to L and you see then vorticity is confined, the vorticity is confined to a very narrow reason near the body. Outside that reason there is no vorticity (that is coming back to this figure that all the vorticity will be confined within in this part, outside this there is no vorticity.

This narrow reason later on we will call it boundary layer, so name of this narrow reason is called boundary layer and the thickness of this reason become smaller and smaller as Reynolds number increases. This concept was postulated or brought in by Prandtl one of the greatest fluid dynamicist little over hundred years ago, to be precise in 1903, he postulated this and before that what is known as the subject of classical hydrodynamics had no answer to many of the problems, some of those problems we will encounter shortly, not now but, within in a few days.

So, what essentially it means that when the flow reynolds number is very high where the viscous forces are much smaller compared to the other forces present then we can treat the flow to be irrotational that is without vorticity except in a very thin reason near the body, except in a very thin reason near the body where the vorticity cannot to be neglected and so the viscous action that near that within that thin reason, within that thin reason the effect of viscosity and vorticity they cannot be neglected however high the Reynolds number is.

However as we will see that in many practical cases where you are concerned about meters of distance or meters of length, a millimetre can be neglected, this will see of the order of the some millimetres or few millimetres in a practical problem thinking about say the flow passed an aircraft this reason is of the thickness of few millimetres while we are concerned with say 10 meters, 20 meters, 50 meters or even higher larger distance. So, in that 1 millimetre is practically negligible. So, we see that in all practical cases we can neglect the effect of viscosity as well as vorticity. The effect of viscosity and vorticity both can be neglected at very high Reynolds number except in a very thin reason of the flow. However, will see later on what will be the reparation of this theory.

Student: Sir (( )) effect of this boundary layer is much i mean (( )) or not

Sorry I could not get you

Does the effect of this vorticity and this we will have any significance (( )) body motion. We will see it, definitely it has. As I said that without this concept, see this is the concept, it is called that this concept bridge the gap between classical hydrodynamics and the practical aerodynamics that without this many many questions cannot be answered, many questions cannot be answered. So, even though we are saying that it is very small reason 1 millimetre and 1 millimetre is a very small reason so it hardly matters but, we will see that it matters greatly later on though it is quite natural to think that this 1 millimetre is hardly of any worth you can safely ignore it.

Yes, we can ignore it for certain respect but, there are some questions which cannot be answered even without considering this very very very thin reason. And the reason is as I mentioned earlier that what it makes it is simplifying the equation a little bit that is not matter that we have dropped this term the equation, the most important effect comes because we are dropping one boundary condition and we know that what is the importance of this boundary condition in any practical problem because at boundary condition alone decides what the problem is. See as far as the flow of air is concerned whether it is a cyclonic flow over the building which breaks down the building, brings down the building or the same airflow over an aircraft which gives it the lift force to fly, the same the governing equation is the same, same Navier-stokes equation no change the same Navier-stokes equation applies equally to all these problem or even to the oceanic flow or the oceanic drifts all are governed by the same equation.

Not only that even the say the the galactic flow or flow within a star they are also governed by the same equation, some forces perhaps we have not written explicitly in some particular problem. Some forces may be considered, some forces need not be considered what we have mentioned simply a body force that body force can take different term but, as such the equation is concerned it is the same equation but, obviously we will not say that what is that river flowing gently and a cyclone flowing over a building and then breaking it down they are the same.

The difference is the boundary condition that boundary condition alone gives the character of the flow and what happens to this simplification? Dropping that term is not important. If we could take the boundary condition even with dropping that term perhaps you will get much better result that we do not get many thing because we are unable to satisfy that boundary condition when you drop this term from the equation.

Now, with this say high Reynolds number approximation or inviscid flow approximation let us say what happens to the incompressible flow vorticity transport equation so vorticity or incompressible vorticity transport equation at high Reynolds number.

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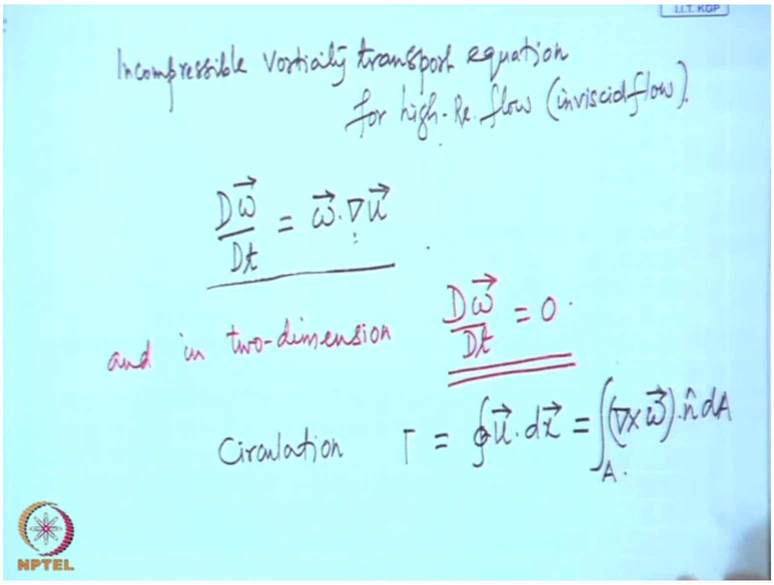

The equation was D omega D t equal to whereas, plus nu laplacian omega at high reynolds number the viscous terms are negligible. So, that nu laplacian omega becomes practically 0, we drop it. So, this is what happens to the vorticity transport equation at very high Reynolds number or inviscid flow. Out of these, these you have already said that this is this represents the rotation of the vortex filaments as well as stretching or deformation of the vortex filaments and this contributes to a local redistribution of the vorticity. It is not a global transport phenomena, it is a local redistribution phenomena. How the vorticity in a particular region is distributed its simply its simply alters that and this term is exactly equal to 0 in two-dimensional flow because the vorticity is then perpendicular to the plane of flow. In two-dimensional flow the vorticity is perpendicular to the plane of flow.

So, in two dimension this is exactly 0. So, can say in two dimension. So, this of course, we saw earlier without considering the dynamics from the kinematics itself we saw something like this that the strength of the [virti/vortex] vortex filament remain constant which we termed as Halmos theorem. So, again from dynamics we get the similar observation that the strength of the vortex filament remain constant. Vortex filament cannot end in within the fluid either it continues to infinity or it ends in itself that means it forms a closed loop so all these are obtained from here also.

We look for another very special result from here which is known as Kelvin's theorem on circulation. Circulation, we as you remember is an associated quantity to vorticity. If you remember that circulation we defined as line integral of velocity over a closed circuit or over closed path which by stokes theorem can be written as the flux of vorticity across an open surface bounded by this closed path. So, this circulation is flux of vorticity across an open surface bounded by the closed path on which you are finding the circulation and how this circulation changes in a particularly in an inviscid flow is known as Kelvin's theorem. Kelvin's theorem on circulation.

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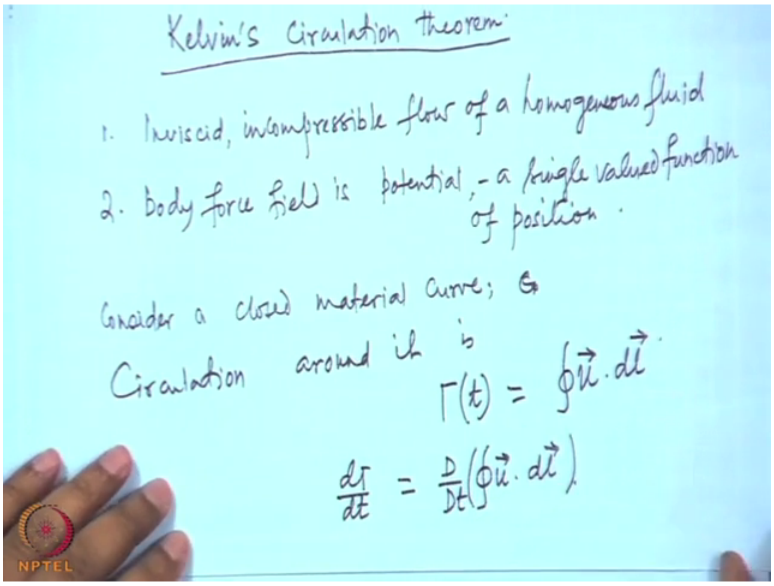

We will consider the inviscid incompressible flow of a homogenous fluid and also we will consider the body force is derivable from a scalar potential which is single valued function of position, a single valued scalar potential f position. So, inviscid incompressible inviscid incompressible flow of a homogenous fluid if you want you can even it write it as a function of time because now what will be rate of change of this circulation around this closed curve would like to find that.

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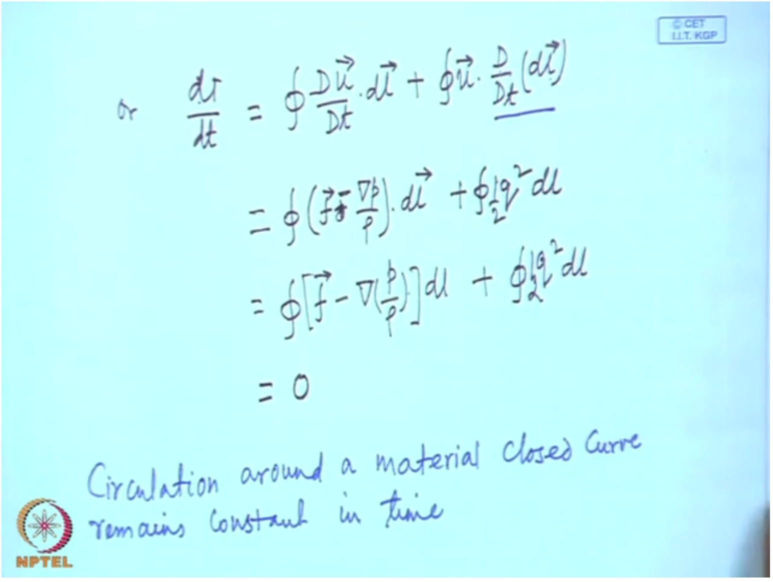

Sees this circulation is around a material closed curve and we are always keeping that material closed curve in this whether you write this conventional small d or that capital d for material derivative is immaterial but, on the right hand we have to write that substantial derivative because that u, the velocity u is defined for at a point, is not defined for a material point, it is defined for a special point and this becomes here also you can make it small d, no problem because this length element is of a material element. So, here this small d itself means a material derivative or substantial derivative. So, the first term on this can be expressed in terms of the governing equation what is that? what is this? what is this? So, this is not plus ,no this is minus sorry what is this? This term, what is this?

Student: (( ))

Velocity, that is the velocity again. So, this becomes u dot u and which as before we will write like this. Now, we have consider a homogenous fluid and for that this incompressible constant density. So, this becomes actually one half will come. Now, how much is this integral? First of all look term by term f d L around a closed path at which the body force per unit mass, how much it will be the integration over a closed path?

Student: (( ))

How much it is magnitude wise

Student: 0

0 it is a that is why we consider the body force as a single valued potential if it is not single valued it need not be 0 but, see that that is most practical is most often the body force will be the gravitational force and it is a single valued potential. So, this is 0. Again this is a perfect differential, this is also 0 p by rho gradient of p by rho integrated over a closed path is again 0. What about this last is 0 integration of say the kinetic energy around a closed path is 0. You come back to the same position, so it its 0. So, the entire right hand side has become 0. So, you see that the it says that in an incompressible inviscid flow of a homogenous fluid provided the body force is a potential single valued body force is derivable from a single valued potential then a rate of change of circulation of material closed curve is 0 or the circulation around a material closed curve will remain constant.

So, circulation in a material closed curve will remain constant. (The flow may be unsteady we are not saying the flow is steady but, the circulation will remain constant. Of course, provided the flow is inviscid, incompressible, the fluid is of homogenous fluid and the body force is derivable from a single valued potential but, you see that all these are not very strict restriction, are not very strong restriction body force derivable from a single scalar potential single valued scalar potential that is the most natural case. That is the most natural case so when we put that assumption we are not restricting much. The fluid is quite often homogenous other inviscid, the flow needs to be high reynolds number then it is practical inviscid and incompressible.

We have not yet see under what condition the flow is incompressible but, we will see shortly. And in particularly you see then that if the circulation is originally 0 then it will remain 0.

So, if there is no circulation when the flow starts then it will always remain 0. For this type of flow inviscid incompressible. So, you see that as we have already mentioned the flow over a body is simply some of a uniform flow plus due to extension, flow due to of vortex plus a flow field in which there is no vorticity, no extension or solenoidal irrotational velocity field. So, uniform flow is always there so that uniform flow from which we starts the flow there is no circulation in an uniform flow there is no circulation so it seems that subsequently also there will be no circulation. According to Kelvin's theorem since the flow originally had no circulation, it will never have any circulation or circulation will become 0.

Now, we have seen that there are certain situation particularly when the flow is at very high reynolds number, we may consider the flow to be rotational except within a thin region and that thin region we can practically neglect or we can say whatever solution we have obtained that solution is valid outside the boundary layer. The solution is valid outside the boundary layer, the boundary layer is neglected. However, we are continuing for some time that incompressible flow. We have already defined that incompressible flow are flows in which the change in density is negligible due to change in pressure that is density does not change because of change in pressure but, does its really happen that a density is not changing because of changing pressure?

We have seen what will happen if the flow is incompressible, if density does not change with pressure so many things will happen. We have the continuity equation the simplest possible form divergence of u equal to 0 if the flow is incompressible and the incompressible flow is those flow or incompressible fluids are those fluids whose density does not change due to the change in pressure. The most common fluid the most important fluid to us the air we know it is not so. If pressure changes the density of air changes, what are still perhaps little acceptable you change pressure density of water does not change much negligible but, air is too difficult too hard to believe that air is a fluid whose density does not change due to change in pressure. You see the very small pressure itself changes its density.

Then can we use all these anything for air now we will see that this is what we say the definition of incompressible fluid that density does not change because of change in pressure but, is there any situation where this can happen that density is practically not changing or change is negligible if the pressure changes. One perhaps without doing anything we can say that if the pressure changes themselves are very small, if the pressure changes themselves are very small so the pressure changes by say 2 percent, 3 percent of the pressure, whatever pressure we have it changes by 2 or 3 percent or 5 percent perhaps with that small change in pressure the density will not change. So, when the pressure changes are very small we can say that the density changes are practically 0 because of that density so only small pressure change perhaps we can consider but, that is of course, a qualitative answer. We look for now some quantitative answer that under what condition the flow can be considered incompressible and the result of that incompressible flow that the velocity field is solenoidal. So, we will see under what condition of flow or fluid can be considered incompressible and its velocity field is solenoidal. This is what we will try to do now and of course, then we will look for some solution of that type of flow.

Student: (( ))