Introduction to Aerodynamics
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Lecture No. # 27
Conditions for incompressibility

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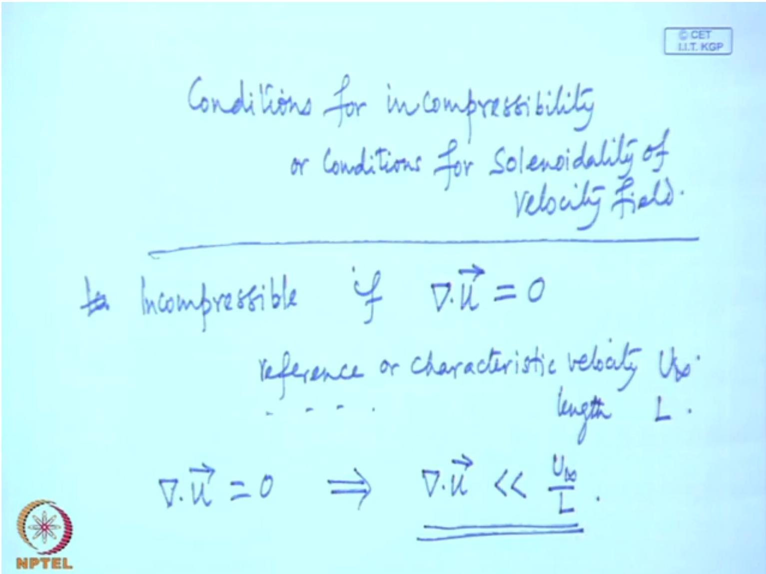

So, we will now try to find what are the conditions for incompressibility or conditions for Solenoidal velocity field, you can say Solenoidality of velocity field. They are anonymous, they are synonymous whether the flow is incompressible for only in the incompressible flow the complete velocity field is Solenoidal. If the flow is not incompressible compressible then also it has the contribution which is Solenoidal contribution, but the complete flow field is not Solenoidal only when the flow is incompressible the complete velocity field is Solenoidal.

Now, what is the condition for incompressibility? We have already seen the flow is incompressible, if divergence of u is 0. So, incompressible if now what is first of all see the, we will compare here again the order of magnitude. What, because these are basically approximation that we know that. What we are doing are basically approximating. It is not an exact condition; there is no fluid which is exactly incompressible. So, this is never exactly true, but approximately true, under what condition we can think that this is approximately true, that is what is always approximation means that, under what condition this is approximately correct.

I there were an exact incompressible fluid, then we would have got exactly divergence of u equal to 0, but since no fluid is exactly incompressible we are not expected to get divergence of u equal to 0 exactly. So, we are likely to get it approximately, So, approximation means under when we can treat it 0 or when we can neglect it. Now, what is the order of divergence of u, once again let us consider that we have a characteristic velocity scale, which is u infinity characteristic velocity is u infinity characteristic length is l. So, reference or you can say reference quantity, once again you call it u infinity and similarly, the length we will call it l and we will see that we will think divergence of u equal to 0 is satisfied if divergence of u is much smaller than u by l.

Divergence has the order of infinity by l each of the term has the order of magnitude as u infinity by l d u, d x plus, d v, d y plus, d w, d z each of them is having the order of magnitude of u infinity by l. So, we will call that divergence of u equal to 0 is satisfied if divergence of u is much smaller than u infinity by l. So, we will sort for this if we find some condition at which this divergence of u infinity is much smaller than u infinity by l that divergence of u equal to 0 is satisfied. Now, what is divergence of u you have seen that that is the rate of dilation dilatation or rate of expansion.

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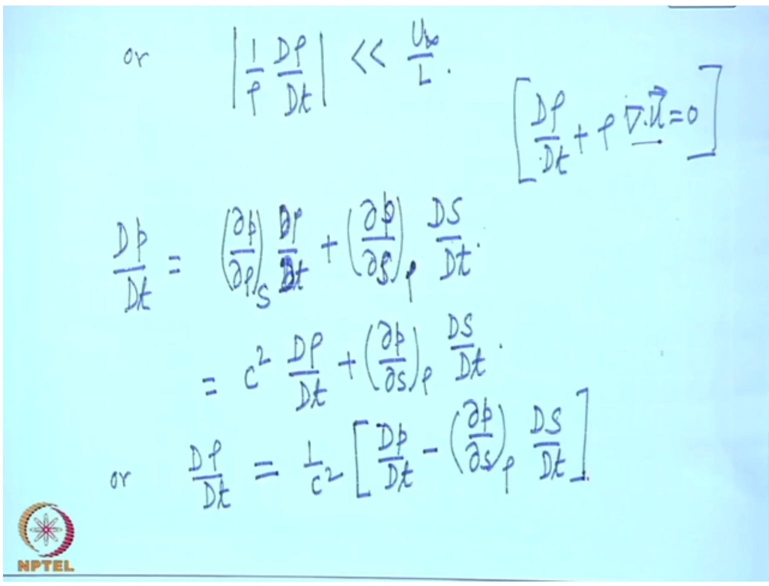

So, divergence of u we can replace by this term that is the continuity equation isn't it d rho, d t plus rho divergence of u equal to 0. So, divergence of u equal to 0 means that d rho, d t or 1 by rho d, rho d t equal to 0, because our original continuity equation is this comes from the continuity equation d rho d t plus, rho divergence of u equal to 0, these equations you should remember particularly these continuity equation momentum and Navier-stokes equations or momentum equations Euler's equation. So, divergence of u equal to 0 means 1 by rho d, rho d t is 0 or divergence of u is much smaller than u infinity by l means 1 by rho d rho d t is much smaller than l.

This d rho, d t now would like to express in terms of pressure. To do that let see that what we have d p, d t what is d p? D t, D p, d rho, sorry but pressure is in general is a function of pressure is state variable and in general is a function of another two state variables assuming that the fluid is a pure substance then pressure is or any state variable is function of another two state variables. S, assuming that the pressure is the function of density and say entropy we know that these can be written in terms of these perhaps, we have written at del d you can make it capital d also then this is at constant entropy and again it will be oh sorry this d p, d rho at constant entropy is called the acoustic speed in that fluid speed of sound in that fluid.

The fluid which we are considering and usually denoted by c. If it is air then it is speed of sound in air if it is water then it is speed of water sound in water (and now then we can write it is two conditions. Now, we can write it as two conditions (sorry this has to be less than u infinity by l that implies that each of these individually must be less than this. So, what we stated so, simply that if the change in pressure is very small, then we can consider the change in density is negligible instead of that, we have come these two complex conditions 1 by rho c square D p, D t is much less than u infinity by l and even more complex 1 by rho c square D p, D x at constant entropy sorry constant density and then rate of change of entropy.

Now, let us see what each of this term implies or each of these mean for that let us try to find what is D p is? Let us get it from equation of motion. W have to find what is this first term D p, D t? D p, D t you can write in what way.

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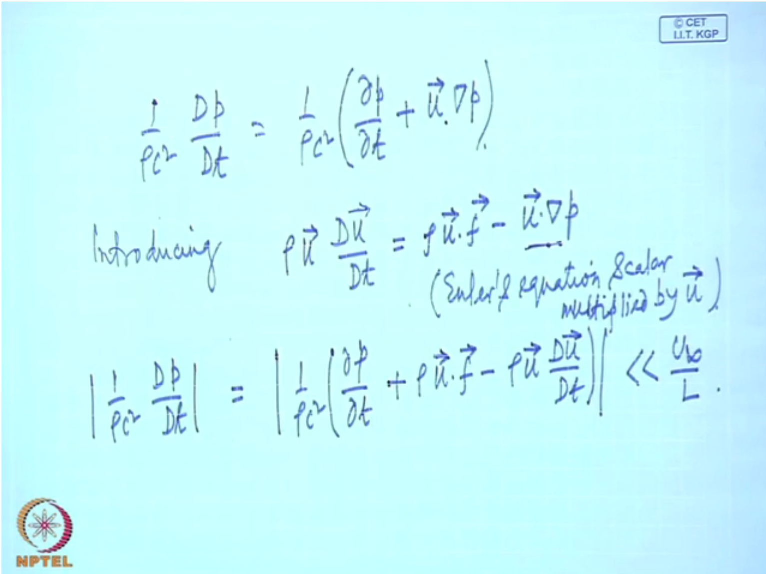

One by so, we put it we will it that later on. 1 by rho c square D p, D t, d p, d t is this and considering that this equation rho u, d u, d t a scalar product of the velocity vector with the Euler's equation. Euler equation scalar multiplied by u, this is Euler's equations scalar multiplied by u. From here you see this we can replace this term we can replace from here this is here. So, what we had this 1 by that q square is as before the scalar product of u. So, this will be satisfied if each of these term are individually less than u infinity l. Remember this is not the complete conditions, we have already split the condition into two and that one of those two is split into further three or meaning of that is these three. If all these conditions are satisfied then they are not y (( )) they are there will be more. So, these are the required conditions some of the conditions which must be satisfied. So, the flow can be treated as incompressible or the velocity field can be treated as completely Solenoidal.

Now, let us see each of these, what do they mean what we can say about the first term what will be its order, what will be its order or? So, we start with this last term last term will, perhaps be simpler last term will perhaps be simpler see what will be the order of this term? what will be the order of this term D q square D t.

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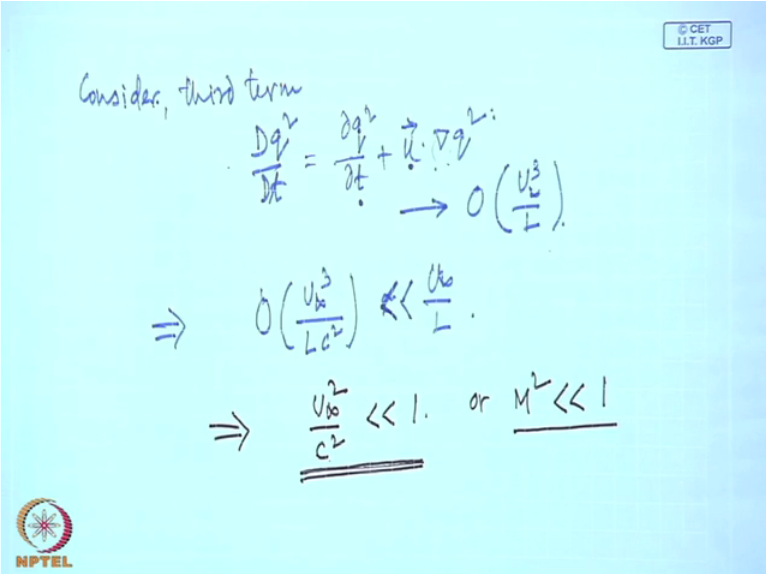

So, let us first consider the third term yes of course, we will consider all, but let us start with the third term third term contains D q square D t. We are we are interested in the order what will be the order of magnitude of this term, that is all this as you know by definition is this. So, its order can be obtained either from here or from here any of the term. So, what will be the order? The velocity is of the order of infinity the length is of the order of L. So, time is of the order of L by U infinity. So, its order is yes order is, let us say this divergence this is some gradient operator of q square.

U infinity q by l. So, that is u infinity square by L this term this U infinity. So, it is U infinity cube by L, if you look here it is again U infinity square by L by U infinity, again U infinity cube by L. So, this terms order is U infinity cube by L of course, this term has a multiplication factor 1 by 2 c square 2 of course, has no significance then what is this means. If we say U infinity let us keep c as c, c of course, you can express in terms of some order, but let us not worry about it this is some parameter. So, it is it. So, the first third term implies that the order of it is order of U infinity by L sorry oh much less than sorry not nearly this is much less than U infinity by L. So, what as it implies. So, what this condition says.

Student: (( ))

So, this is the condition, the first I mean one of the condition, but in particular think about a flow in which this body force has not much of influence and the flow itself is steady there is no time change. So, look out of these three, if the body force has no significance I mean not much of significance, this term is practically you need not consider body force is you not present or body force is insignificant meaning that we need not even worry about this term. If the problem is steady problem this term will also have no meaning this is present only if unsteady or. So, this is present that the first of the condition.

The first of the condition is only valid when there is unsteadiness time dependence, if there is no time dependence everything is remaining independent of time then of course, this is not of any concern and in particularly the other conditions which you have left here that the condition related to entropy. If this is totally negligible in that type of situation or the body force is insignificant or the problem is steady and where this change in entropy is again not considerable or negligible or the flow is isentropic. In that situation you see this is the most, this is the perhaps only condition or most important condition and what is the meaning of this term? That if this condition is satisfied, then because it has come from the first condition.

The first condition says about rate of change of time, this is significance of this condition this is an outcome of this term which is fractional rate of change of pressure. So, what it is means then that when body forces are insignificant, the flow problem is steady the entropy changes are also negligible, then all pressure changes are basically related to these term and if this term is very small, this U infinity square by c square. If this parameter is very small then the pressure changes are also very small. What is this U infinity by c square? This is a parameter commonly known as Mach number. So, a very important condition and at as you see one of the most important condition.

The flow field to be incompressible or Solenoidal, because that most often you will find the problem is really steady problem and particularly in aerodynamically problem. The gravitational forces are often negligible also in many practical problem. The gravitational forces are practically insignificant, it is not that the force is very small its changes is small, because most often we are interested in change not exactly the total amount of the quantity. How much it is changing? So, think about in much flow the change in gravitational energy is practically small, because the change in gravitational force will be considered considerable.

If the change in altitude is quite high otherwise not just about a few centimetre, even a meter or couple of meters difference in altitude hardly affects the gravitational force. Gravitational force will be of the order of other forces or change in gravitational forces will be of the order of change in other forces only if, we are considering an altitude which is quiet large say except perhaps, in atmospheric flow in most cases the gravitational effects have not much of contribution.

Anyway so, this is one of the condition Mach number and we will see that quite often people answer that what is incompressible flow. Incompressible flows are those flows for which Mach numbers are less than point 3 that, we will see common answer though of course, answer is completely wrong, but the source is here, because the Mach number is one of the most important parameter which decides whether we should treat the flow as incompressible or not. So, what we see that one of the important or the most important condition. When the flow can be treated as incompressible or the velocity complete velocity field is Solenoidal is whether the Mach number is small or not if the Mach number is very small we can.

Let us now consider the second condition or come back to the first term due to importance (Now, with we have also characteristic length L, L and characteristic velocity U. So, what will be this term then d p, d t the order what pressure is.

Student: (( ))

What is pressure see it is again (( )) change in momentum pressure is simply change in momentum. So, to find what is this pressure term what essentially you have to find out, what will be the order of momentum change or over this characteristic length L perhaps, you have studied a kinetic theory of gases. Where you first found out how to find the pressure by simply considering? The momentum as the molecules collide with the containing box that gives the pressure eventually that is what is pressure is in thermodynamical sense its simply rate of change of momentum due to those collisions.

So, now what we will are interested in what will be the change in momentum over this characteristic length l yes we have u l n. N (( )) by c square

Student: N (( )) by c square

You are talking about the entire term

Student: Yes

No, I am not going to entire term first of all let us see what will be that order of the pressure order of the pressure, then order of this time derivatives of pressure will be of the order of.

Student: (( ))

Since, the order of the frequency is n so, n times this change in momentum will occur place occur no n time that is why this rho u l and then this d p, d t is. What is n?

Student: N n is frequency as you say that characteristic frequency

Frequency of

Student: Frequency of oscillation is changing with time no unsteady flow (( )) that is coming (( ))

How many times it is occurring this assuming that this l there is a box and the particle. So, n times will that particle some particle will hit over that so, we have now u infinity writing. Remember this number this non-dimensional parameter n L by U infinity we found this when in non dimensionalized, this equation and call this Strouhal number denoted by s d Strouhal number in the last class. So, this can also be written as (and of course, as you see that this condition is required only if the flow is unsteady. If the flow is not unsteady flow, if the flow is a steady flow of course, we do not need to check whether these condition is satisfied or not yeah, automatically satisfied there is no change in time.

So, for an unsteady flow then both these condition must be satisfied. The earlier condition as well as this condition and you see in that situation this condition is even more stringent it may be possible, that the Mach number condition is satisfied that Mach number is very small, but even then this may not be satisfied just look them to the situation. If the frequency is of the order of or larger than U infinity by L, it can quite often happen the frequency is larger than U infinity by L, then U infinity by c square is satisfied much less than 1, but here it will not be satisfied. So, if the frequency of the dominant frequency or the characteristic frequency is larger than U infinity by L, this condition is more restricted than the other one.

So, even when the speed is very small the flow Mach number is very small, but if something is oscillating at a very high frequency very large frequency, then the flow resulting flow need to be considered as compressible and divergence of u equal to 0 will not be satisfied. The third condition, the other condition that remains I think that was that we wrote in between the second in our writing yeah it was at second u dot.

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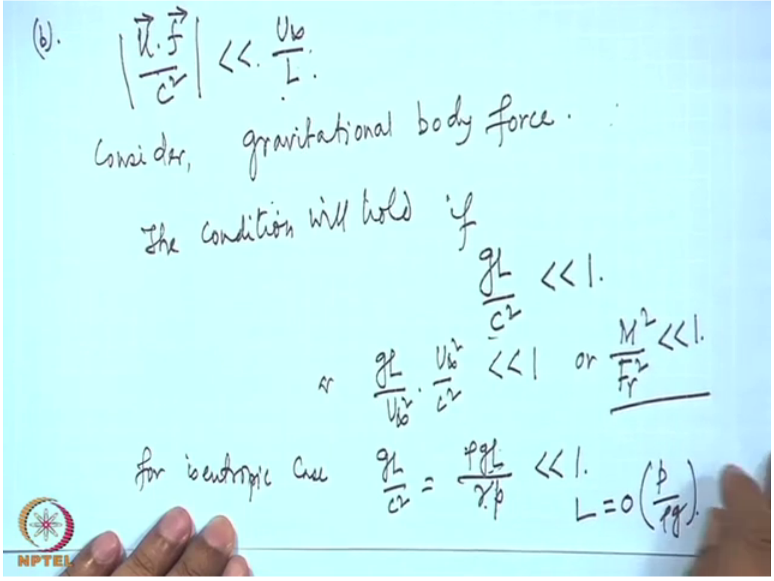

f of course, further we can say anything if we know what is the nature of this body force f so, consider this f to be only gravitational.

Student: (( ))

c square then what is the meaning of this condition what it says hm

Student: (( ))

What I could not hear

Student: (( )) significant (( ))

No, but see this is gives that under what condition this will be this term is small or this term is negligible. If this is the body gravitational body force only then this is g, u, c square is much less than U infinity by L. What does it mean then? Remember see this is not what we have, what we want to have. We want to have this condition, we want to have that this must be small than smaller than this it is not that it is by default this is like this. We want or we would like to see when this will be smaller than this is what is required this is what is not automatic. We would like to see whether this is going to happen or not and if it is going to happen then when.

So, this will be satisfied or the condition will hold. If g u, g u infinity by c square less than u infinity will be less than u infinity by L, if g L by c square is much less than 1, we wrote something g l by c square g l by that was g l by u infinity square this also can be written like this g L by U infinity square again U infinity square by c square is much less than 1 or u infinity square by c square is m square and if you remember that g L by U infinity square we defined as Froude number.

Anyway let us try to see little more of this term g L by C Square for C Square. Let us consider an isentropic situation considering an isentropic case c square is gamma p by rho thus speed of square of speed of sound is gamma p by rho. So, for isentropic flow g L by c square is rho, g L by gamma p and this has to be less than 1. Now, rho g l we know is the difference between the static pressure at between two points, where the distance of l altitude of l this distance is of course, in the direction of g vertical direction. So,

if the difference between the two altitudes or altitude of two points is l then this term actually represents the ratio of this hydrostatic pressure and this pressure. So, you see this gamma we can forget gamma is of the order of one gamma is of the order of one. So, gamma has no contribution, if rho g L by p, now p lets say consider here where p is the atmospheric pressure which is again of the order of 10 to the power 5 Pascal. So, this will be of the order of 1, if rho g L is also of the order of 10 to the power 5 only then, if it is not if it is much smaller than that then of course, this term is satisfied this condition is satisfied. So, as long as the altitude or the difference in altitude between the two points is not considerable.

Which is equal to this about p that means, when the altitude difference in altitude of the order of p by rho g, only then this term will become important. So, if l is of the order of p by rho g, only then it is this term or this condition is important otherwise it is. So, for air it needs about something eight kilometres. So, if we are considering distance of eight kilometres between two points then we have to think about this term and we will see whether the condition of incompressibility will not be valid, but see most often we are not interested an altitude of the order of eight kilometre for any particular problem.

We do not like do not want to see what is happening over eight kilometre of the atmosphere. So, usually this condition is by default automatically satisfied at least for all practical aerodynamically problem or even any industrial fluid dynamical problem. This condition is however, if you are interested in meteorological problem we would like to think about the climates weather then of course, we need to consider about 8 kilometre and we cannot neglect the effect of this term and since this term is considerable. We cannot consider the equation also to be incompressible then you have to consider compressible equation, but if you are not this condition are automatically satisfied not to hold the condition.

So, we see that out of these three this condition we can say more or less always satisfied for all practical problem we need not bother. As far as the effect of Froude number is concerned or effect of the body force is concerned towards compressibility. Compressibility effect of body force towards compressibility is not considerable unless we are interested in a very large height of the order of few kilometres. So, that condition we can say always satisfied. So, in any problem we have to just check the other two condition or the Mach number is considerably less than 1 and if the problem is unsteady look to that of course, we have one more condition still unexplored that is related to entropy, but we just mention it later perhaps no time today, but you will see that that condition is also almost always satisfied for all practical problem unless you are considering very small size.

The second condition relating to that entropy and all that part will come only when you are considering very small size or something like that otherwise that is automatically satisfied. So, we see that what will happen that flow field will be treated as incompressible or velocity field Solenoidal. If the Mach number satisfy the condition that the square of Mach number is much smaller than one or Mach number is very small and in case the problem is unsteady, then you are more important that n square l square by c square is much less than 1. However, if the problem is not unsteady that condition is also not important and only the condition of Mach number remains you see that ultimately,

we have come down to that only the condition of Mach number is important to determine whether the flow is incompressible or not, but for steady flow for unsteady flow. The other condition is even more important that n square l square by c square, but for steady flow this is the condition Mach number and that is the reason people often just take it as a thumb rule if the mach number is less point two point three then flow is incompressible.