Introduction to Aerodynamics
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Lecture No. # 29
Potential Flow Combination of Basic Solutions

So, we will try to now combine some of the basic solutions of the potential flow that is the solution of laplace equations which as you have already mentioned the basic solutions are the uniform flow, point source, point doublet they are all the basic solutions. We will now try to combine them and see what it happens. Of course, any super position is also a solution of the laplace equation because laplace equation is linear, so if we combine the 2, any of these 2 solutions or 3 solutions or even more solutions that also is a new solution..

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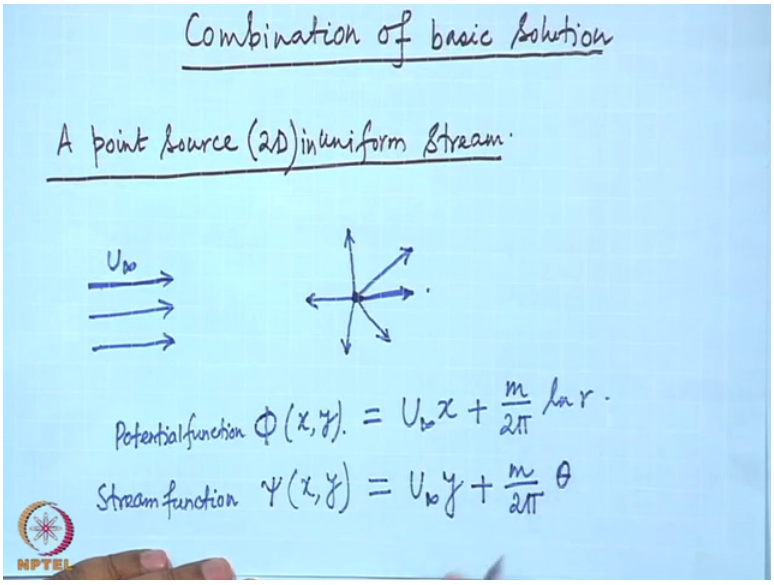

So, would now we will combine some of these basic solutions and see what do we get. So, the first 1 we will do it, a uniform flow plus a point source, a point source placed in a uniform flow. Let us call it two dimensional, a two dimensional point source again remember that two dimensional point source is actually an infinite line source. Let us take the uniform stream along the positive x direction that is from left to right, uniform stream from left to right. A uniform stream is always represented by just a few parallel lines with of course, arrow to give the direction of the flow and the flow velocity in the x direction we simply call it u infinity and we place a point source in it. This is the point source a two dimensional point source or an infinite line source and we take the location of the point source as our origin. This is just for convenience, as you know that the potential due to a point source is m by 2 pi log r where m is the source strength and r is the distance from the location of the point.

Now, the stream lines if we show some of the stream lines of this point source they are like this they are all radial, the flow which is from the source the flow is coming out in all direction radially. So, which is going to the left that will of course, be opposed by this uniform stream but, which is going to the right that will be supplemented or with the uniform stream increase. First of all let us write the potential. What is the potential of this? Consider this a steady flow, forget about any time in it so let us let phi at any point x y, call this plane to be x y plane.

So, what will be phi x y? Yes, potential due to uniform stream what it is?

Student: (( ))

U infinity x, and the point source.

M by 2 pi log r.

M by 2 pi log r. If we write you can write even a constant plus a constant also you can write that we will drop those constant all. What is psi? The stream function. What is stream function for this flow?

Student: U infinity (( )) .

U infinity y is for the uniform stream eventually these are the stream lines plus m by 2 pi theta plus a constant

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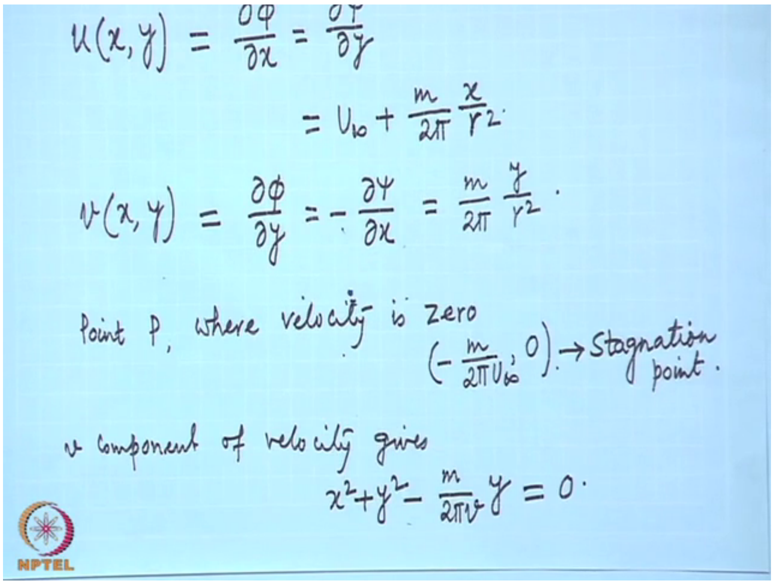

Find the velocity components? The velocity at any point u x y, the x component of the velocity u x y is you can write d phi d x or equal to d psi d y. How much is this? yes?

Student: (( ))

u infinity plus?

Student: U infinity plus m by?

Yeah

Student: M by 2 pi r?

D r by r.

Student: D r.

Wow why d r?

Student: M x square x by root under x square plus x 2.

Student: x by?

Under root x square plus 2.

Student: M by 2 pi x by.

Under root x square plus r square.

Student: Sir r square.

R

Student: R and r r square.

So it becomes m by?

Student: Two pi r square.

2 pi?

Student: R square.

X by r or r square?

Student: R square sir r square.

What about the other velocity component?

Student: M by 2 pi. M by 2 pi.

Only m by 2 pi?

Student: Into y by m square.

Y by r square.

So, this is the combined velocity field. This is the velocity field due to a combination of uniform stream and a point source. Is there any point where the flow velocity is 0?

Student: (( ))

If we look to this problem from here, if there is a point where the velocity is 0 that is likely be to on the negative x axis.

Student: (( ))

At theta equal to, that is theta equal to pi is there anything like that? What is that? Say, we call point p where velocity is 0 equal to minus m by 2 pi u infinity. Minus. M by 2 pi u infinity. M by 2 pi. U infinity. Comma 0 comma 0.

So, there is a point which is, which has no velocity. Velocity at that point is 0, we usually call this point a stagnation point. The point at which the velocity is 0, that is a stagnation point and all other properties associated to that point they are called stagnation properties. Like the pressure at the stagnation point is called stagnation pressure, density at that stagnation point is called stagnation density, temperature at this stagnation point called stagnation temperature.

However, remember in this case. This you should remember that this stagnation property is little more than what we are saying that velocity where the velocity is 0, that is what is stagnation. The stagnation properties can be associated to any point irrespective of the flow velocity. Here, we said the stagnation pressure is a pressure were the at the stagnation point, where the flow velocity is 0 but, in practice stagnation temperature, stagnation pressure, all these properties are associated to each and every point irrespective of the flow velocity. It is not necessary the flow velocity to be 0 to define a stagnation pressure at any point it is not necessary that the velocity at that point has to be 0. We can, the way we can define a pressure temperature at any point similarly, we can also define a stagnation pressure stagnation temperature at each and every point.

This is by assuming or imagining what would be the pressure or what would be the temperature if the velocity becomes 0 there. Say at point there is some velocity say 10 meter per second. Imagine and it has some pressure some temperature. Imagine what would be the pressure and temperature if that 10 meter per second becomes 0? It need not be really 0, just imagine then what would be the pressure and temperature that is actually called the stagnation pressure and stagnation temperature but, then comes something else, you are assuming or you are imagining that your speed is becoming 0 from say 10 meter per second, how it is becoming 0?

How we are making the flow velocity 0 in your imagination? You are imagining, all right but, how you are imagining that it is becoming 0, to make it 0 it has to follow certain process.

So, it depends what that process is how we are making the velocity 0 there it will depend, the pressure at that point will be depend how we are making it 0. So, actual stagnation quantities need that this process must be isentropic, for temperature and enthalpy adiabatic is sufficient but, for all other properties the process should be isentropic.

That means you have to imagine that you are making the flow velocity 0 there isentropically, then the pressure at that point whatever pressure at that point will become that is what is stagnation pressure.

So, just if are asked that define stagnation pressure simply saying that where the pressure velocity is 0 the pressure at that point is stagnation pressure is not actually a correct definition. The correct definition is if the flow velocity at any point become 0 isentropically then the pressure at that point would be the stagnation pressure. However ,at present we are discussing incompressible inviscid irrotational flow and for such flows all processes are isentropic. So, we need not specifically mention every time that it is stopped isentropically, it is stopped isentropically but, you must remember that this process of stopping, of course, the imaginary process of stopping must be isentropic then only you will get the stagnation pressure, correct stagnation pressure otherwise not.

Look we have, if you look to this only this vertical component of the velocity v component of the velocity you can see that the v component of velocity are constant on a circle, the v components of velocities are constant on a circle. Take this v component, what it gives? (r square we have simply written x square plus y square. What is this? This is an equation of a circle with radius or center at 0 m by 2 pi v, circle with center at 0 m by 2 pi v sorry m by 4 pi v.

Student: (( ))

Circle with center at m by four pi v. So, for constant v, if v is fixed then all these points are on a circle so on each circle the vertical component of velocity is constant.

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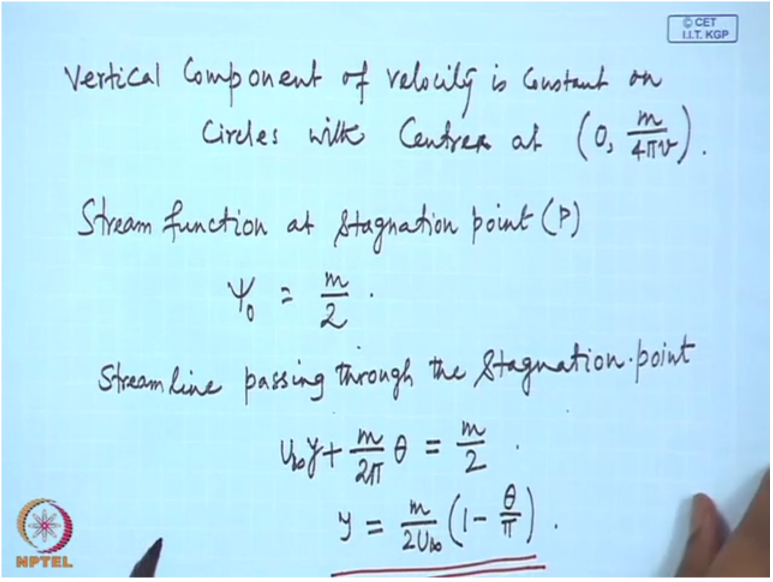

Find the stream function at the stagnation point or on a stream line which passes through the stagnation point? You know any streamline is simply psi equal to constant. psi equal to constant gives a streamline, so find the stream line which passes through that stagnation point. How much it is? Stream function is a function of x and y and you know x and y for the stagnation point.

Student: (( ))

So, how much? Let us call that to be psi 0. How much it is?

Student: (( )) m by 2 final.

M by 2. So, the streamline passing through the stagnation point what will be the streamline passing through the stagnation point? (Any streamline is given by psi x y equal to constant or u infinity y plus m by 2 pi theta equal to constant and since this is passing through the stagnation point the constant is already known m by 2. See, this equal to constant, this equal to constant is equation for all the streamlines, different streamlines for different value of the constant. The streamline which is passing through this for that we have already found a constant m by 2. So, in this case this equal to m by 2 the constant equal to m by 2 or we can write y equal to how much.

Student: (( ))

M by 2 u infinity into 1 minus theta by pi. Say, this is the equation of the streamline passing through the stagnation point. What type of line is this? Can you plot this line? Can you plot this line? A plotting a line in x y plane only.

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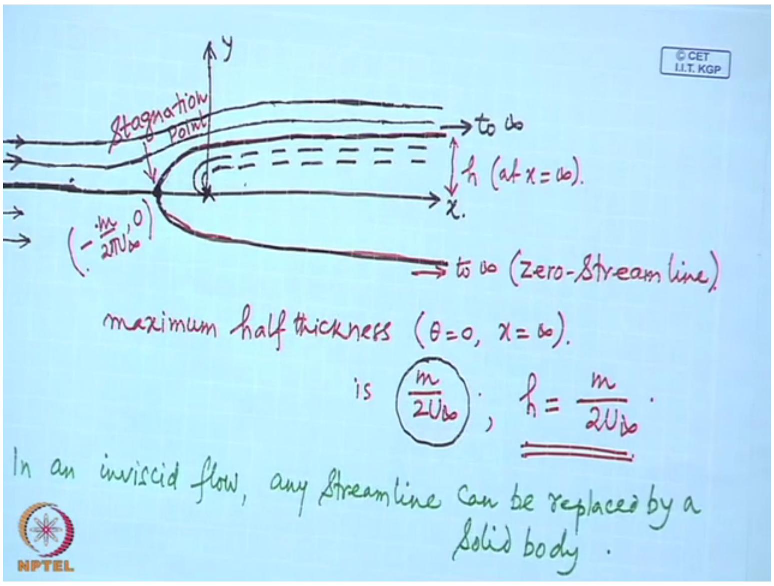

As you can see from x equal to minus infinity to x equal to, how much is that m by m by 2 pi u infinity, from x equal to minus infinity to x equal to minus m by 2 pi u infinity, this line will coincides with the x axis.

So, coming from minus infinity to plus infinity this is this streamline and let us say then this is that, this is that point which is minus m by 2 pi u infinity and what will happen after this? Looking to this equation as you can see as theta increases from 0 at theta equal to 0, this has a value of m by 2 to u infinity at theta equal to 0, as theta increases from m by sorry 0 to pi the value decreasing at theta equal to pi it becomes 0, that becomes this point only. Then as it again increase from pi to 2 pi this y become negative y become negative but, it is symmetric. The value at theta equal to pi by 2 is same as the value at theta equal to 3 pi by 2 but, with negative sign.

So, it is something like this. This geometry is symmetric and this is the location of source, this is the location of the source. So, this is the usually referred to as the zero streamline and the maximum what should we call? Thickness or half thickness? When theta equal to 0 the y is m by 2 u infinity or 2 phi u infinity, what it is? 2 u infinity m by 2 u infinity. You can call it half thickness because the on the lower half also there is a m by 2 u infinity.

So, the maximum half thickness. (Now, what is a streamline?

Excuse me sir sir that point on x axis whether it is m by 2 pi u infinity or m 2 u infinity.

What it is coming.

Student: (( ))

M by 2 u infinity.

Student: (( ))

When theta equal to 0.

Also this stagnation point was minus m by 2 pi u infinity.

Stagnation point is minus m by 2 pi u infinity.

This is not the stagnation point this is half thickness, this thickness that is the coordinate of that stagnation point this, coordinate of the stagnation point. (So, this is maximum half thickness this we will call it say h, let us call this to be h .

Student: (( ))

Which one?

Student: (( ))

That is a stagnation point minus m by 2 phi u infinity.

With the phi (( )) m by 2 infinity.

This is the height is m by 2 u infinity.

Now, what is a streamline? We have already defined or by definition streamlines are lines to which velocity is perpendicular parallel or tangential velocity vector is always tangential to streamline. So, what is the normal component of velocity to the streamline?

0.

Now, if we have a solid body in a flow, then an incompressible inviscid flow, not incompressible in any inviscid flow the boundary condition is normal component of the relative velocity is 0 or the normal component of velocity is 0 if the body is at rest that is the boundary condition.

Now, you see that all streamlines satisfy that. Streamlines are lines on which normal component of velocity is 0, if you have solid body in an inviscid flow then the boundary condition on the inviscid on the solid body is that the normal component of normal component of the velocity is 0 on the solid body. So, the solid body are same similar to streamlines. As far as inviscid flow is concerned solid bodies and streamlines are same or any streamline can be thought of as a solid body. You can write that in any inviscid flow.

Student: (( ))

Yes?

Now, in this case this is a streamline. We can have other streamlines let us say some other streamlines we can show approximately (these are also a streamlines these interior dotted lines.

Now, let us say that this body what we are calling the 0 streamline if we replace it by a solid body then what it is? This problem now then become uniform flow over a body solid body of this set. Of course, a semi infinite body.

Student: (( ))

So, if we replace this. this particular streamline what we are calling the 0 streamline, what we have called the 0 streamline this part we can forget only this, only this much, only this much.

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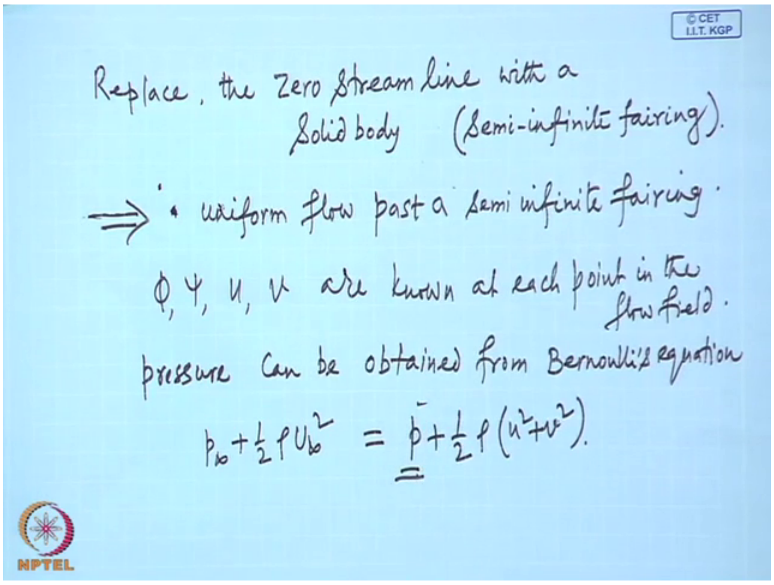

And this body is usually called a semi-infinite pairing of maximum thickness of m by 2 infinite into 2 then this problem now becomes what we have started with a combination of uniform stream dot and a source.

Now, become uniform stream passed a semi-infinite pairing. The problem now becomes and these particular streamlines these dotted streamlines that I have shown here they now become streamlines within the body have no significance, this source that is also now inside the body nothing it is not in the flow.

So, it is not creating any rate of expansion anywhere within the flow. It now does not exist only what you have a uniform flow and a semi infiniting semi-infinite fairing emerged in it or in other way we have solved the flow over a semi-infinite fairing. Only thing that when you started of course, you did not know that what body we are going to get or whether we are going to get a body really that was also not known to us but, you see the idea is this that we started with this and now we see that this is what is the practical case this is a practical problem of semi-infinite uniform flow over a semi-infinite fairing and we have solved it by indirect approach. Of course, it gives an idea that perhaps some other combination will give flow over something else .

We now know the potential function stream function associated with this flow with this problem the velocity at any point within the flow field, the important flow field is now only this, only this much which is outside this. This becomes a solid body. At any point here we know the flow velocity already you have found out what are u v.

So, you can find the flow velocity at any point the unknown and you can apply the Bernoulli's equation to find the pressure. Looking from the symmetry of the problem since the velocity at any point on the upper half and the lower half will be same the pressure will also be same. So, pressure on the upper half of this body on this upper surface of the body and pressure on the lower surface of the body will be same. This we can say without even calculating the pressure but, calculating the pressure is quite simple you just satisfy the Bernoulli's equation.

Take a point at the infinity or far away from here where we have flow velocity only u infinity far away we have flow velocity only u infinity pressure p infinity so at that point from Bernoulli's equation p infinity plus half rho u infinity square and take any point here where the pressure is p the velocity is square of the velocity is u square plus v square that u and v we have found.

And we can find the pressure there. So, just we can write that phi psi u v (from here you can find p at any point, in particular if you find the pressure on the body surface on the body surface you will see the pressure on the upper part and on the lower part are exactly identical meaning that there will be no net force in the y direction the body is symmetric about x.

So, the pressure distribution is also symmetric about x. So, there will be net no net force in the y direction that means there will be no lift force in this case. Even you can say that the x component of the force that will also be 0. So there will be no net force acting on the body. This can also be thought of as flow over a clip if you consider just only half of this body say upper half looks like a clip, it looks like a clip.

So, it is a flow over a clip also and as you see that in that case at the stagnation point the flow velocity is 0. So, that is the most sheltered place. So, if you are near a clip and then suddenly a very strong storm is coming up and you want to be sheltered you can know that the stagnation point is the most sheltered position near about stagnation point and it is near about is the sheltered position .

So, that is the place where you can go and take your refuge. You can extend this problem to three dimension also. You can extend this problem to three dimension also of course, in that case then you will find a body of revolution like this, you revolve it, that is the body you will find and in three dimension of course, you have the real point source potential given by m by 4 pi r. Anyhow, we will not do that. We will rather take up another case which is all most similar but, now instead of a point source we will take a point doublet again two dimensional, two dimensional point doublet or an infinite line doublet.

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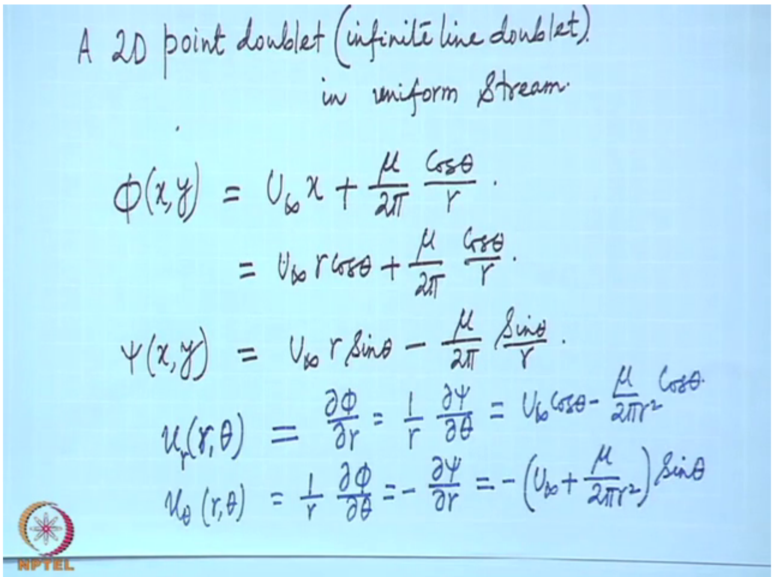

Next, case we consider. What is the potential due to a point doublet?

Student: (( ))

Mu by.

Student: 4 pi

4 pi actually becomes for three dimensional case that is the point or either perhaps infinite line doublet, you call it infinite line doublet. potential function mu by 2 pi yes? mu by 2 pi cos theta by r. This x r theta we can replace this x also, we can make it r cos theta so that we have only (the velocity components u and v, what are u and v? r r theta. The radial component of velocity it is d phi d r or 1 by r d psi d theta, the radial component it is should I write that u r u theta, this u is now the radial component not the x component. I think perhaps better to write that subscript. Is d phi d r equal to 1 by r d psi d theta. How much is this?

Student: Infinity cos theta

Minus.

Student: Mu by 2 pi cos theta by r square.

Mu by 2 pi r square cos theta, we should have written the cos theta once. What about that theta component? It is 1 by r d phi d theta or minus d psi d r. This is what both the term on negative minus u infinity plus. Inside the bracket there should minus (( )). Now look to the expression for u r where is u r 0 other than that cos theta equal to 0 and so theta equal to 0 or pi which is just the x axis other than that.