Introduction of Aerodynamics

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Lecture No. # 09

Kinematics of Fluid Motion (Contd.)

We have derived the mass conservation equation in the last class, and look for two special cases; one is when the flow is incompressible. You see that the mass conservation equation or sometime we will call it continuity equation has become simply divergence of the velocity is zero.

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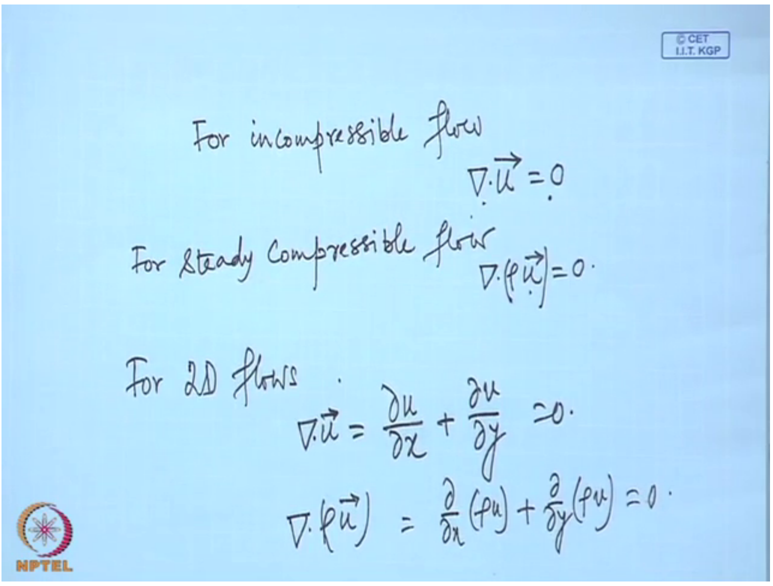

So, for incompressible flow for incompressible flow, we have divergence of the velocity vector is zero, and for steady compressible flow for steady compressible flow the continuity equation is simply divergence of rho u equal to 0. See for the incompressible flow this divergence of u equal to 0, this holds even when the flow is unsteady even when the flow is unsteady that is in this case this u can be function of time, velocity at a point is a function of time; however still the time derivative do not appear in the equation even when the flow is unsteady, in case of incompressible flow that is because of that constant density case.

In case of compressible flow, the equation becomes simply a divergence only when the flow is steady otherwise that d rho d t term will be present. So, for compressible flow this u or this rho is not function of time they are simply just function of special coordinates. Now the special it is in this case that is in both these cases the mass conservation equation is expressed simply as a divergence in one case it is divergence of the velocity field, and in one case it is divergence of the velocity into the density.

Now let us assume that further another simplification, that the flow is steady if the sorry if the flow is two dimensional. Let us further assume that the flow is two dimensional then this divergence is basically sum of two derivative. So for 2D flow this then equation becomes, for 2D flows in case of incompressible flow the equation becomes what do you write u v w x y or u 1, u 2, x 1, x 2 either way say let us write this divergence of u is simply d u d x plus d v d y equal to zero, and in case of compressible flow compressible steady flow in this case in both these cases where this divergence can be expressed as simply by sum of two derivatives.

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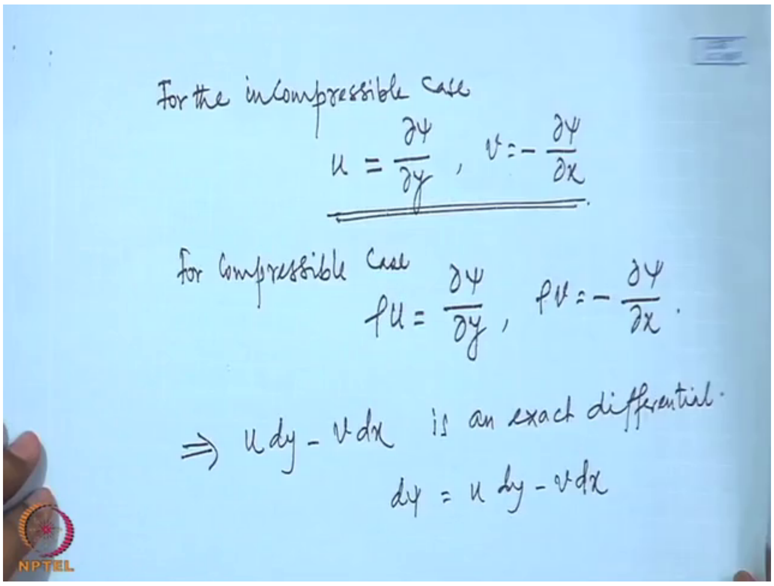

It is quite easy to introduce a scalar function a single scalar function which will satisfy the equation which will satisfy the equation say for the incompressible case just taking an as an example if the for the incompressible case define this way, define u as and you see the equation is satisfied, the divergence equation is satisfied see the simplification we had two unknown u and v, we have been replaced that by a single unknown psi. For compressible case similarly, you can define it is little we have to take rho also along with it see the difference between psi and psi in incompressible flow and psi in compressible flow. In case of compressible flow the density is also within the definition in case of incompressible flow it is not.

Now, hence forth we will consider the for our further discussion only these incompressible definition, but you can make everything whatever we are doing for incompressible you can just do the same thing for compressible flow wherever there is u you replace it by rho u, wherever there is v you replace it by rho v. So, everything will remain same only with that replacement. This definition suggests that u for incompressible case now we are whatever we are writing here is for incompressible u d y minus v d x is an exact differential, and let us that can be written as delta psi if you want to make it for compressible again simply replace u by rho u, v by rho v that we will do nothing additional. along any path along an arbitrary path let say this along an arbitrary path O to P the integration is carried out. Now looking to this integration u d y minus v d x can you say what does it imply, what is the meaning of that integration u d y minus v d x, imagine a surface of unit depth that is a surface or imagine that this line OP is translated by a unit distance in the third direction the z direction.

It will make us surface if this line OP is translated in the z direction by a unit distance it will trace a surface of unit width, think of that surface and then tell what is this, what is the meaning of this integration? u d y you can think of u z d y or z d y is then the area d z d y but d z is one fixed we have already fixed it so, 1 into d y become the area of that surface which is normal to x direction. So, what is this first term of the integration, what is the meaning of this? Normal component of the flow velocity into the surface area what is that? We have already used that term.

Flux, the volume flux so, this is the volume flux. The second term is the same, second term is again same this also is d x into 1 that it is a area normal to this y component, so this is the flux, volume flux that is entering through that surface, entering in the generalized sense it might be leaving, it might be entering. So, this integrant here or this complete integral is simply the volume flux that is entering through the surface traced by this line as it translates unit distance in the z direction. So, this is the volume flux across this we will call the volume flux across this line. So, this psi minus psi 0 gives the volume flux across this line.

There is a sign convention usually associated with this volume flux, if the flux is counter clockwise with respect to P with respect to P then it is taken as positive. If this flux is counter clockwise with respect to point P, then this flux is taken as positive. So, this is the volume flux, just to make it short usually not called in this way that the volume flux across the surface found by translating this line unit without telling, that much just called that volume flux across this line, where that across that line means that this line is traced in the third direction by unit distance, and the then the surface that is formed. So, this is volume flux across the line.

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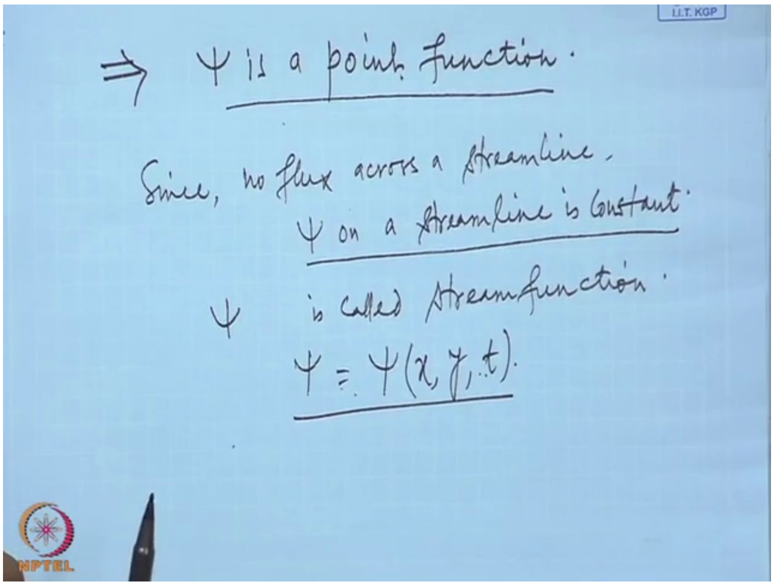

And hence, this give that (psi is a point function, and then the difference between the psi between two points is simply the value of psi between two points like psi P minus psi O that is the way we want we need only the value of psi at P and at O, it is independent of the path that it follows whatever path it may takes the difference will remain the same, of course, provided the path is one of two paths which completely encloses incompressible flow region otherwise not otherwise not.

Now then since the difference of psi between two points is given by that integral what will be the difference of psi when the two points lie on a streamline? or what will be the value of psi on a streamline? see we know the streamline is a line to which the velocity is tangential. So, what will be the flux across a streamline? Zero, because the flow velocity is always tangential to the streamline so, there is no flux crossing any streamline. So that means, that integrant for the streamline is zero, if that point O to P lies on a streamline then the integrant is zero, and what does it means psi P is same as psi O, meaning that the function psi is constant on a streamline.

Psi on a streamline is constant, and following this function psi has a special name it is usually called the stream function, the psi function psi is called a stream function. (In this case of incompressible flow this is a function of position, and as well as time. So, psi basically for incompressible flow case psi (x, y, t), it is a function of position and time. Of course, if the flow is steady then it is not dependent on t. One thing we must remember that about the function psi the way we defined this psi and the volume flux we mention. It is quite clear that the psi is basically a many valued function it is not a single valued function. As long as the reason is completely occupied by incompressible flow psi behaves like a single valued function.

However if the region encloses some internal boundary in which there is somehow a creation of volume, creation of value maybe like say you have a flow region in which you are introducing some tube which is discharging some fluid there or also a very simple case remember that, your fluid region is water and thing there are some air bubble within it then as the air bubble expands or contracts your incompressible flow region or the water region changes, the volume of the water region changes.

So in that in that type of situation where there is some interior boundary in which the volume of the total incompressible fluid it can change. Then this function is no longer a single valued function if this rate of discharge or if this rate of increase is m, then every round you make the value of the stream function will increase by m, so if you make two round it will increase by 2m, if you make three round it will increase by 3m, and so on.

So however if that type of situation is not there, there is no internal boundary or then the function is single value but whenever there is an internal boundary then the function is not single valued. We will later on come back to it, and we will see that this needs some special treatment in some of our problems that we will handling. Now, the stream function and the velocity component in terms of it we have defined for our standard Cartesian system for other system also you can define it say one very useful either so called polar coordinate r theta coordinate system, for r theta coordinate system if you define again you can defined that volume flux exactly in the same manner and the velocity components.

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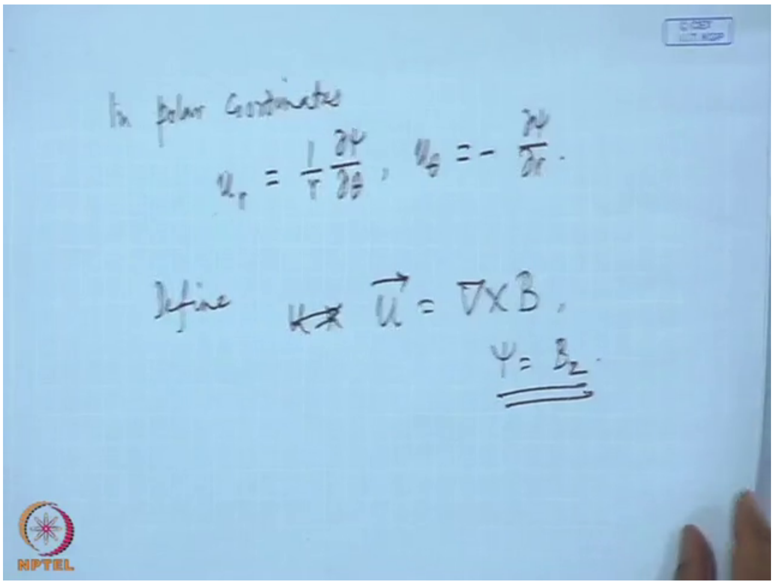

Let say if we call u r in polar coordinates, let us write u r u theta the standard notation, it is 1 by r, you can remember you see that in one case the sign is changing but it is easy to remember in this way that if you differentiate the stream function by in any direction you will get the velocity in the direction which is 90 degree clockwise, if you differentiate it any direction you will get the velocity component in the 90 degree clockwise direction that is all. You can check it with this that in both cases that is what it is, if you differentiate it with respect to y, you get the velocity with respect to x which is 90 degree to the clockwise. If you differentiate it with respect to x you get the velocity in the negative way direction which is again 90 degree clockwise with respect to x. So, differentiate by any direction you get the velocity in the 90 degree clockwise direction.

Also look back to the definition of the streamline the mathematical expression for stream function streamline we did it last class d x by u equal to d y by v equal to d w by z sorry, d z by w forget that third component for the 2D case it is d x by u equal to d y by v, and you see that immediately get that volume flux is zero u d y minus v d x equal to 0.

So, from the definition of the stream function also we can come to it, u d y minus v d x is zero on a streamline. Any other component like say if we consider an anti symmetric case or symmetric case in spherical coordinate system again we can write what are the two components of velocity, and we will do that whenever we come across that type of situation.

And now let us look to something little different the relative velocity what exactly is the relative velocity leads to in case of fluid motion?

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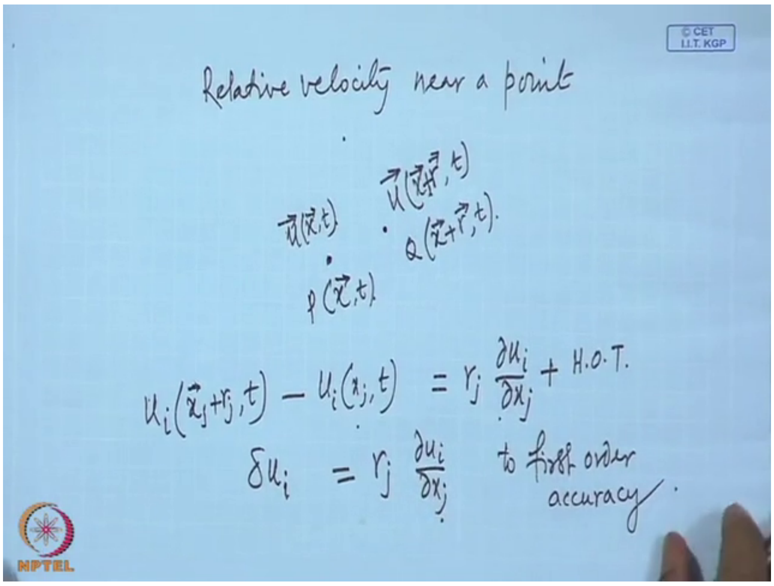

So, relative velocity near a point let say we have a point which we denote by the position vector x and of course, at time t, this is the point P, and let say at this point the velocity is u, let say a neighbouring point a neighbouring point which position vector is x plus r, instead of writing any delta x we are writing r the distance between two point is r which is of course, very small, and again at the same time that is the relative velocity at same instant simultaneous relative velocity we are not finding the difference in velocity at two different instants at one point at one point of time we have seen what is the difference between the velocity at two neighbouring point, and again the velocity will be function of sorry.

What will be the difference of these? see this can be obtained by a Taylor series expansion of this, this can be obtained as a Taylor series expansion of this so this equal to this plus something. So, what is that something? This is the first term of that Taylor series, this equal to this plus this then plus of course, r j square into second derivative of this, and so on, but assuming that are these small to the first order to the first order this is what is required plus higher order terms, and plus higher order terms which involves square of r j d 2 d 2 u i d x j 2, and so on. So to first order of accuracy you can have only these term, so this difference we will call it now delta u so the delta u between these two points delta u i is simply r j d u i d x j to first order to first order accuracy. Then look to this term d u i d x j what it is? see it has nine component d u 1 d x 1, d u 2 d x 2, d u 1 d x 3, and so on.

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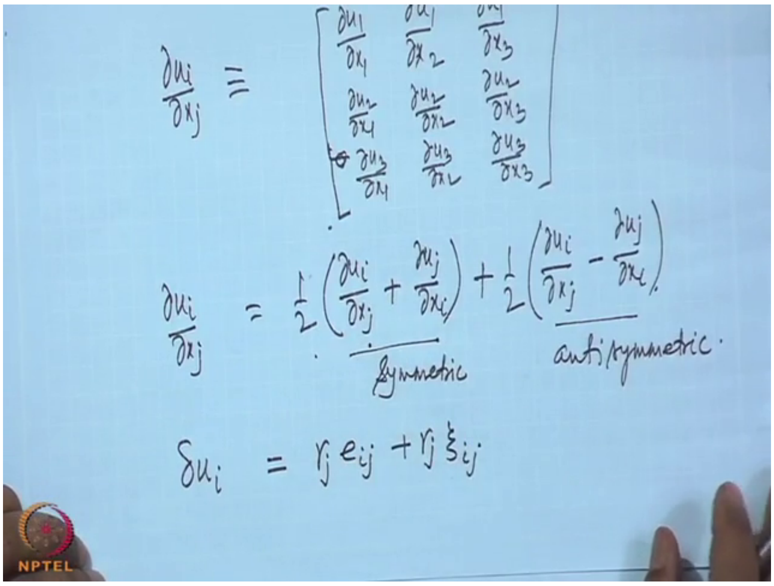

Is it not? This d u i d x j if we expand it, it is d u 2 d x 2. So, d u i d x j is the velocity gradient tensor, it is a tensor it has nine component. So, that expression for the difference in velocity delta u i can be written as(we will call this symmetric part as e i j r j e i j r j xi i j, where this symmetric part we are calling as e i j and the anti symmetric part we are calling as xi i j.

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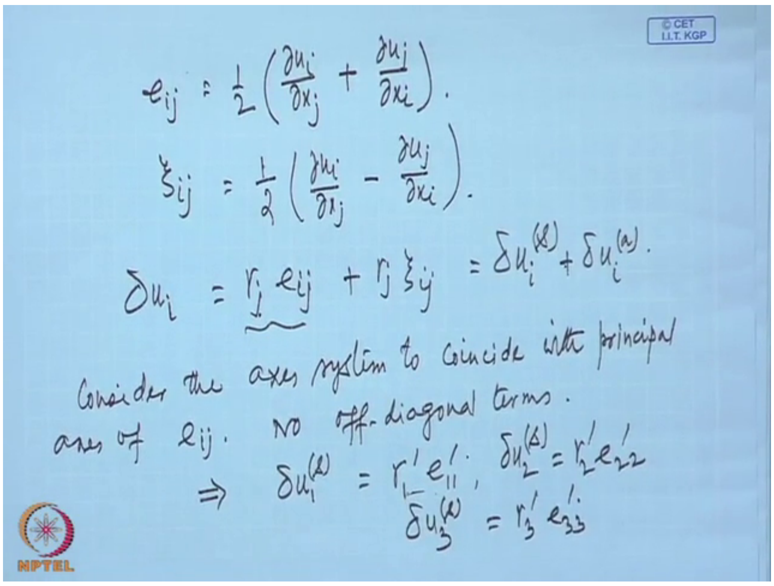

And this two part of the velocity difference or contribute two contributions one by the symmetric tensor, one by the anti symmetric tensor. We will just call it say symmetric contribution and the anti symmetric contribution, because you would like to see the nature of each contribution separately. Now consider first this r j e i j what is the contribution to this velocity? Or what is the nature of this contribution? to understand this contribution let say that our axes system is such that just take it that our axes system is such that it coincides with the principal axes of e i j even if it is not we can orient our axes system so, that it coincides with the principal axes of e i j. We can do it then of course, there will be no diagonal term we are doing it so that have less number of terms, and it is easier to interpret it is easier to interpret the meaning that is the only purpose why we want to consider it that consider the principal axes of e i j or let say our axes system is represented by that then we have and let say that components that coordinate system just to have a difference are denoted by prime, that is instead of e i j call it e i j prime.

When it is the principal system, and we will have then only three component only three component no off-diagonal terms, and these delta u i the three component I have in a very simple form, and delta u 1 will be delta u 1 (s) will be simply r 1 prime e 11 prime, this prime is that we are now having the principal system. Similarly, delta u 2 will be r 2 prime e 22 prime, and delta u 3 (s) will also be r 3 prime e 33 prime thats all. In a general case of course, these are not only one term there will be other terms also. In this case delta u 1 will be r 1 e 11 plus r 2 e 12 plus r 3 e 13, delta u 1 will be r 1 e 12 e 11 plus r 2 e 12 plus r 3 e 13 but now we have only one term r 1 prime e 11 prime. Also since this tensor is also have the same invariant that is e i i is same as e i i prime.

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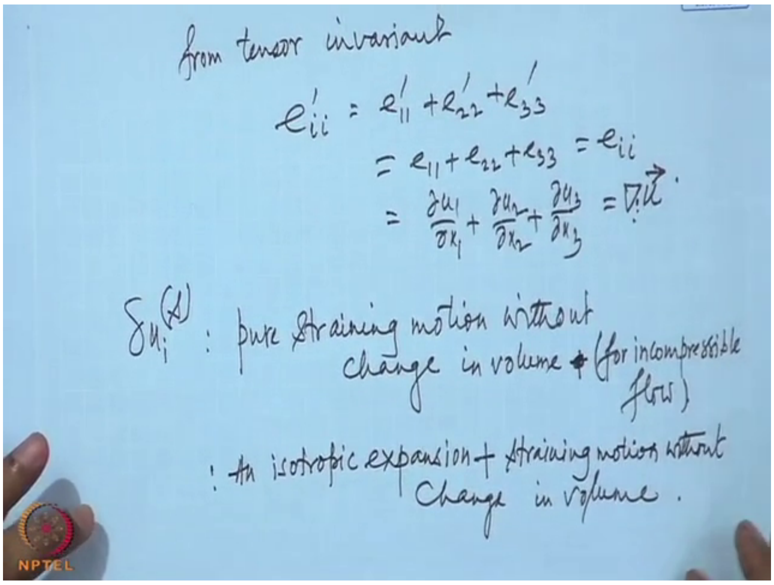

From tensor invariant, e i i prime that is e 11 prime plus e 22 prime plus e 33 prime is same as e 11 plus e 22 plus e 33 that is equal to e i i. We have already seen what it is? you can write this explicitly what is e 11 or e 11 prime? Look to that expression what is e 11 prime or e 11. e 11 is simply d u 1 d x 1 look to the definition of e i j e 11 is d u 1 d x 1. So this means d u 1 d x 1 plus d u 2 d x 2 plus d u 3 d x 3, this u 1 u 2 u 3 you can replace them u v w u x 1, x 2, x 3 you can write them x y z what is this? Divergence of u this is divergence of u divergence of u and which also you have seen is the same as the rate of dilatation or rate of expansion.

So, this is rate of expansion or rate of dilatation which you have already seen. So, now look to only one component or imagine the situation just an a imagination I am not saying this can happen that only u 11 prime is non zero e 22 prime, e 33 prime they are zero, then you have only delta u 1 which is given as r 1 a 11 prime. So, this velocity is simply proportional to the rate of expansion in the same direction. So, each term here represent a motion whose which is given by the rate of expansion in the same direction then think about now say sphere, think about now a sphere, and let say that this motion is given to or this sphere has this motion thus centre of the sphere is at x and it circumferences at the all those points x r at, x plus r.

Now what will happen to this sphere? That in each principal direction or in each orthogonal direction it will move by a distance r 1 e 1, the velocity will be there as r 1 e 1 in each direction. So, in every three measured diameter direction you if you take it each direction is being extended by e 11 prime, e 22 prime or e 33 prime or let us call it a, b, c is being extended at the rate of a, b, c. Obviously, it will no longer remain a sphere it will become a ellipsoid. So, this contribution to the motion is such that it will try to make a sphere with which will then ellipsoid. The element which are on the principal axes they will not experience any rotation they will not experience any rotation but simply experience a stretching in the same direction. The other line elements which are not along this direction they will experience both a stretching as well as rotation, but remember this rotation is such that it is consistent with the stretching along the principal direction.

This contribution to the relative motion or obviously the motion is called as training motion. Now like for the special case if the flow is incompressible we know that this is zero this divergence of u is zero so, in case of incompressible flow this motion represent a pure straining motion such that the volume do not change without change in volume, a pure straining motion without change in volume for incompressible flow that is the meaning of the term that delta u (s), a pure straining motion without change in volume of course, if the flow is not incompressible then there will be change in volume, but in that case again this motion can be decomposed into two part one is, an isotropic expansion that is a equal expansion in all equal direction given by one third of e i i in equal in all direction an isotropic expansion, and another straining motion without change in volume. So, this delta u (s) represent a pure straining motion without change in volume, if the flow is incompressible flow, if the flow is not incompressible then delta u (s) represent an isotropic expansion plus a pure straining motion without change in volume.

So, to summarize let us write that this delta u i (s) represents a pure straining motion without change in volume plus sorry, for this is only for incompressible flow only this much, and for compressible flow an isotropic expansion that is equal expansion in all direction we will stop here today, but we would not be able to complete it today we will continue it we will continue it next class.