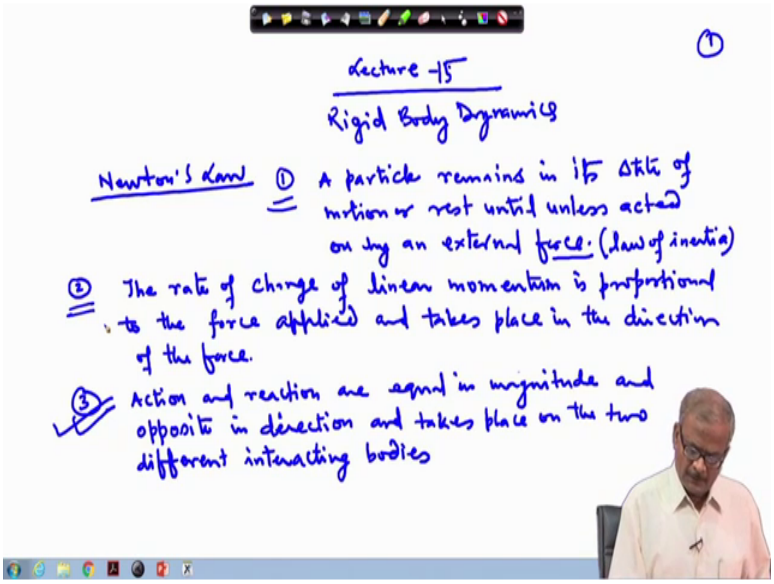
Satellite Attitude Dynamics and Control
Prof. Manorajan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture – 15
Rotational Dynamics – Rigid Body Dynamics

Welcome to the 15th lecture. Today we will start with the Rigid Body Dynamics already we have covered the rotation and rotational kinematics in details except few things which I have left for the discussion in tutorial or either I will upload as the supplementary material for your help. So, today we are going to discuss about the rigid body dynamics, but for that what we need? We need the basics of the Newton's law that must be very clear ok.

(Refer Slide Time: 00:47)



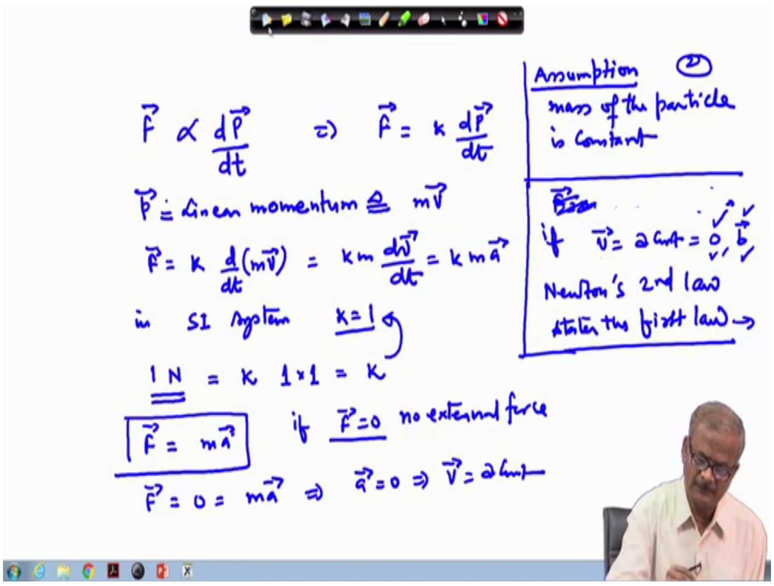
So, let us start with the Newton's law, popularly we have three laws and the books the credit Newton's for all the three laws. But, the reality is that only the third law is due to the Newton it is a continuation and other two laws the first and the second laws were already existing ok. And to that Newton added the third one.

So, we discuss the basic things of the Newton's law. The first law is states that a particle remains in its state of motion or rest until and unless acted on by an external agency or external force. And, quite often this is also called a law of inertia.

So, we will discuss the fine points of all this laws. The second one it is states that the rate of change of, rate of change of linear momentum proportional to the force applied, and takes place in the direction of the force. And the third one it states that action and reaction are equal and equal in magnitude and opposite to each other, opposite in direction; opposite in direction and takes place on the two interacting bodies which are two different bodies on the.

So, this law it is a credited to Newton, the other two this were existing before him, but as of you know that all the three laws they are a credited to him. So, now we take this the second law of first. The second law is the state that the rate of change of momentum is directly proportional to where linear momentum is directly proportional to the force impressed and or otherwise we can write.

(Refer Slide Time: 04:53)



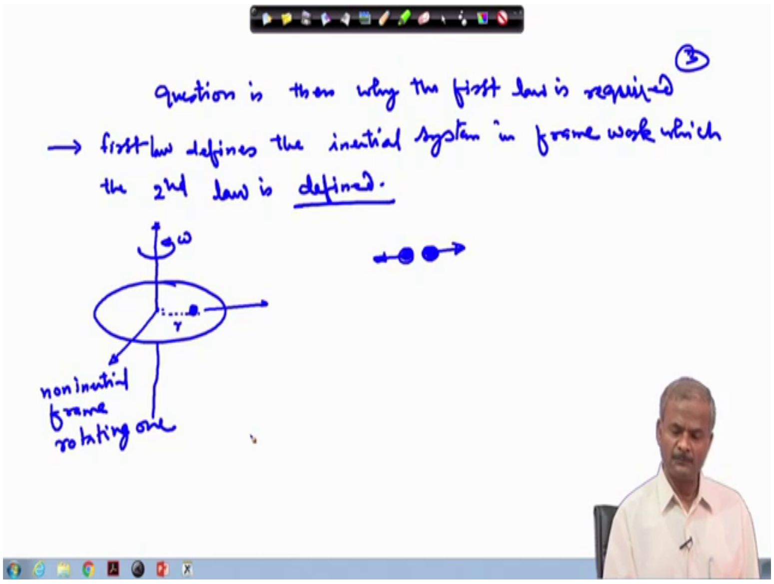
So, if it will be equal to k times dp by dt and by definition this our p the linear momentum this is a linear non-momentum by definition this is m times v where m is the mass of the body. So, here the assumption is that mass of the particle, mass of the particle is constant ok. So, Newton’s law it is applicable to a particle and it is so happens that then its extended to the different bodies.

Now if you look in to this equation so, this can be reduced to k times d by dt m v because m is constant; so this can be written as dv by dt ok. In SI system it is taken so that k turns out to be an SI system k turns out to be 1. So, how it is done that F on the left hand side it is a chosen as 1 Newton. So, on the right hand side so 1 Newton is defined as the force which produces an acceleration of 1 meter per second 1 meter per second is square in a body of mass 1 kg. So, we can see that if the body mass is 1 kg and dv by dt this is the acceleration. So, k m times a so this is 1 ok. So, that gives you here we have written magnitude wise the vector notation we are not putting so, this is k. So, k turns out to be 1.

So, their SI system the things have the units are defined such that k equal to 1 and therefore, your Newton's equation often you find in this form ok. Now, earlier also I have stated that if F equal to 0 means that there is no external force, no external force. If F equal to 0 so what will be the situation. So, this simulates F equal to 0 simulates the or the shape F equal to 0 it also states otherwise that in the first law, if you see that if until unless acted on by an external force. So, if F equal to 0 means there is no force acting on the system. So, either the system will be in the state of rest or in the motion which is visual from this place.

So, F equal to 0 this implies and this implies a equal to 0 and v equal to a constant. So, therefore if v is a constant so it can be either equal to 0 or it can be a either equal to say some parameter v ok. So, both are possible that either it is moving with constant velocity or either it is at rest. So, in other words Newton's 2nd law, it states the first law ok. This is what it implies, but then the question arises that what is the need of the first law then ok.

(Refer Slide Time: 09:21)



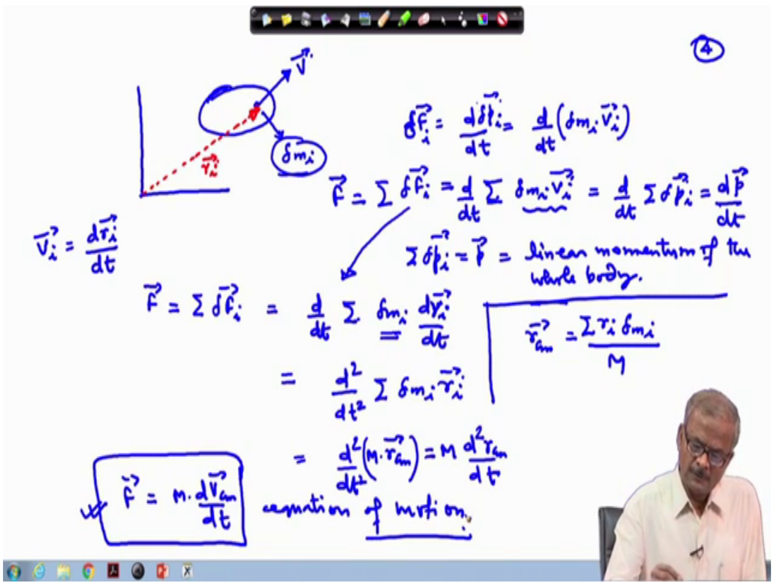
So, the question is then why the first law is required? See the thing is that the first law, first law defines the initial system in frame work of which the 2nd law is defined. Without any initial system your 2nd law is not valid just for take for example. This is a disk which is rotating with angular velocity omega and a particle is placed on at a distance of r from here ok. So, this will also be rotating, but as you know that if there is no friction available enough friction is not there so this will slide out ok. So, this will not stay on the board.

So, if you see that in this system there is no force acting this is a rotating frame this is a rotating frame which is a non inertial frame; is a rotating one. So, this is a non inertial reference frame. So, if your particle is here so it will slide out means apparently you are not applying any force there is no external force acting on the this particle, but still the motion is present ok. And if you consider the same thing in this system in this rotating system if you apply the second law then you will be in error ok.

Because, the second law is not valid in the for the non inertial system for that you need an inertial reference frame only then you can apply. So, the first law it defines the inertial reference frame and together with also it states something that there is something like the inertia existing in the system which does not allow the system to change its state or it may be particle it may be a body or whatever ok. So, a particle is there. So, if it is moving it will keep moving if a rigid body is there. So, it will keep moving if or either even a rigid body is moving so it will keep moving. Until unless act and on by certain force it may be friction force or any short of force. So, the first law it defines both the inertia and exists the frame work for the second law.

So, this setting the frame work for the second law this is not mentioned quite often in most of that text book, but it is the fact that it is used for setting the frame work for the second law. And also together with this also gives the notion of inertia ok. And the third law which is very obvious if we have two bodies just like the two masses colliding together. So, this will apply force on this here in this direction and this will apply force here in this direction and these forces will be equal and opposite to each other. So, within this framework then we are start working with the equation of motion.

(Refer Slide Time: 13:39)

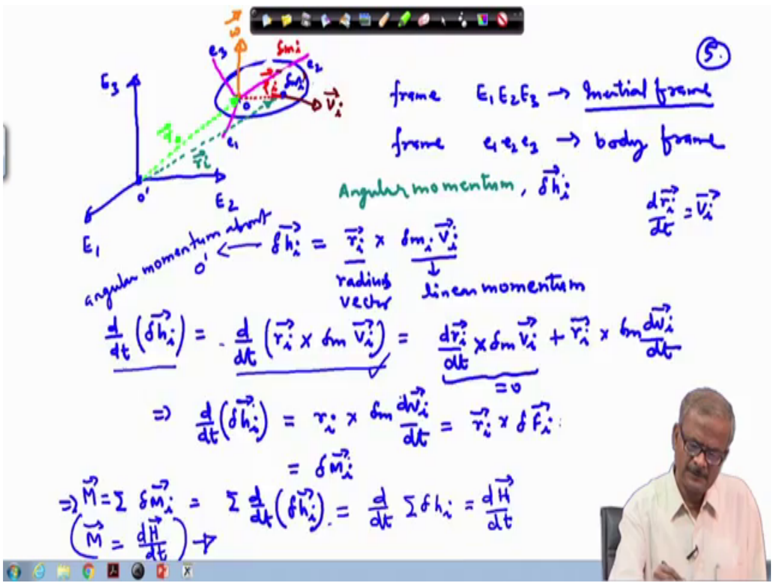


Three in a frame if we have a rigid what we say here and this is a mass of the rigid body which we can say this is delta mi and it is a velocity is here in this direction v. And let us say, this is the position vector of this point which will indicate by r i ok. So, if now if we apply the Newton's law to this particular point; so, we can write after the whole reduction we have written F equal to k is equal to 1 so this is dp by dt.

So, we will write here d F or delta F we will write this as the delta F and this p instead of writing here p we can write for this particle as delta pi so this is a linear momentum of the particle delta mi and which is nothing, but delta mi times vi. Now, if we sum over all the particles so this is delta F i and this differential operator and the summation operator is a linear operator we can exchange it and we can write this as delta mi and this is nothing, but your delta p i. Means this we can write as dp by dt where summation delta pi it extends over the whole body which we are writing as the p this is the linear momentum of the ok.

So, if this particular equation we can play with this little bit more. So, we have delta F i we can put here F. And this we can write as d r i by dt we know that vi the velocity of the particle i this is the it s particle here which we have shown ok. So, if this particular equation we can also write it in this way and because the mass is constant mass is not a variable and therefore, we can take out this d by dt outside and this can be written this way. And we know that centre of mass is defined as where M is the total mass of the body. So, this can be written as d square M times r c m by dt square and which can be reduced to be M times which you often write as M times ok. So, if this is the equation of motion.

(Refer Slide Time: 18:19)



Now, on the other hand if we look for the rotational motion of the body then what happens. So, here in this case the issue is little bit complicated. Now, let us consider this is a mass this will not put here delta mi. This frame will write as E 1 E 2 E 3 this is a point o prime this is point o and this will write as E 1 E 2 E 3. Frame E 1 E 2 E 3 this we call as the inertial frame and which we can write the equation of motion we can apply Newton’s law ok.

And frame e 1 e 2 e 3 we call this as the body frame and this is a rotating frame. So, it is a rotation direction we can show as let us say this is a rotation direction omega. And also we assume that some way of say v i is the velocity of this point and also the radius vector of the origin of the body frame this we write as r o ok. And radius vector of this point so little bit change I will make here. Let us say this point is lying somewhere else. Let us say we have a point here which is delta mi this is delta mi ok. And velocity of this point is v i. And this we indicate as rho i and if we join this points here this vector will show as r i ok.

So, we know that the angular momentum will indicated by for this particular particle as delta hi ok. And we are writing here in terms of delta hi because we have to integrate it over the whole body finally, to summit over the whole body and calculate the angular momentum of the system ok. So, if this delta hi we can write as r i cross the linear momentum of this particle which is delta mi times vi ok.

So, if this is a radius vector linear momentum. Now, if we differentiate this ok. So, differentiating this it gives me dr i by dt cross delta m vi and we know that this quantity dr i by dt is nothing, but vi this is the velocity of this particle ok. I am assuming here that you are well aware of the vectors because it is a matter of the twelfth class and already we have discussed a lot about the vectors and its various operations. Now here in this part so this is vi cross vi this will be 0.

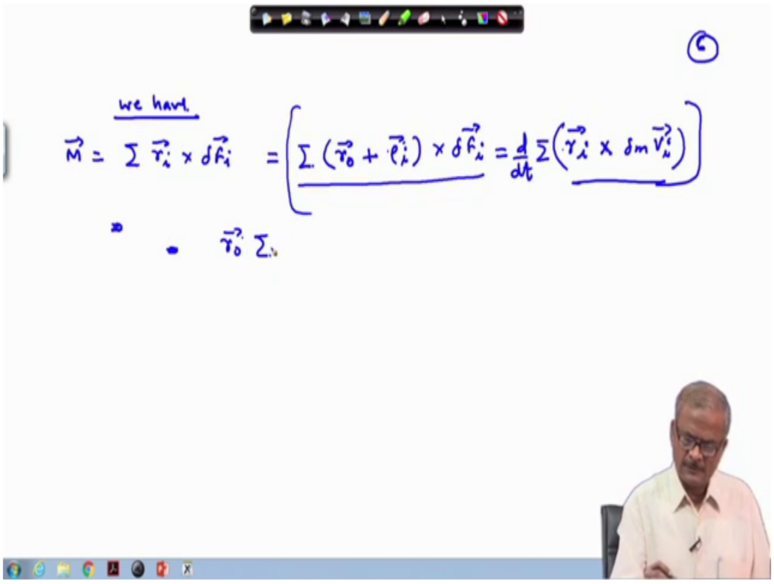
So, this part gets reduce to 0 so this quantity this equal to 0. And therefore, hi this gets reduce to r i cross and as we know this quantity already we have written this is r i cross delta F i. So, what we get from this place and this we write as delta mi which is the mi stands for the torque you can use also some other notation, but this is ok. Sorry, this is here we will write this is here in this place, this quantity then write as delta mi ok. Therefore, this if we take the summation over this all the particles.

So, as we know this summation operator and the differential operator can be exchanged. So, we can write this as this we can write as d H by dt. And on the left hand side here we can write this as M. So, what this gives me M equal to d H by dt. But, see here in this case what we have got this is the angular momentum we have calculated about this point o prime. So, this is the angular momentum about o prime this is not the angular momentum this v v I have written and this is o ok. So, the story does not end here we have to extend it because, this is the say if you are an aircraft or the satellite and you are sitting on the ground.

So, the angular momentum as you are aware of it most of the time you are calculating about this point o which happens to be the centre of mass. Here, in this case we have taken that a general point o and right now we have not chosen this as the centre of mass ok. So, if and this is the point we are we are calculating the angular momentum. So, you can say that this is the absolute angular momentum of this particle because, this we are calculating in the initial frame about this point o prime ok.

So, we need to extend this further to go to the point that we want to describe quite often it is a very easy to describe the system angular momentum about any point in the body itself. So, we reduced it to this that form and we see that what benefit we get out of that ok.

(Refer Slide Time: 28:21)



So, if this is the m dot we have written finally, we have got it in this format and this can be written as r o which is the radius vector of this point and plus rho i. So, this r i is summation of this vector and this vector so the green and the red one. So, r o plus rho i cross delta F i ok. On the right hand side, so this quantity will be equal to this quantity this dh by dt which we are calculating about this point.

So, we write about this point d by dt and the angular momentum of the whole system which we have written as r i cross delta p which is nothing, but the linear momentum of the particle. So, delta m times v i. Initially, we differentiated this quantity and then we have changed the operator so looking at in this place somewhere we have done that ok. So, if we sum over this one summation here we have shown if you can take this equation. So, here this is a part we are discussing about. So, if we summit over this point the summation will go inside.

So, you have d by dt will you can take it outside so this particular equation we are choosing and using here in this place. Now the left hand side this particular part will use it this whole thing in our expansion. So, this we can write as because r 0 it is not dependent on this summation r 0 is a particular point in the body. So, what we have done that in this frame we have assists the point here itself and we have not considered this to be the centre of mass. And, the centre of mass will evolve out of this during course of our discussion ok. So, we utilize this equation to formulate our whole problem.

So, we stop here this is our basic equation and we will continue here in the next lecture ok.

Thank you very much.