**Satellite Attitude Dynamics And Control**

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**Lecture - 24**
**Rigid Body Dynamics (Contd)**

Welcome to the 24th lecture. So, we have been discussing about the rigid body dynamics and we will continue with that. So, going to the previous lecture this we have discussed about the retrograde precession. Now consider the case where I 0 is less than I.

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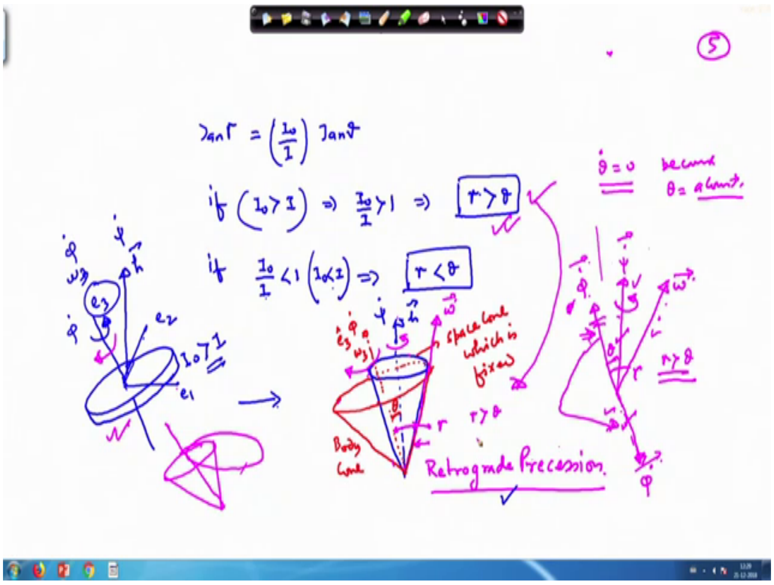


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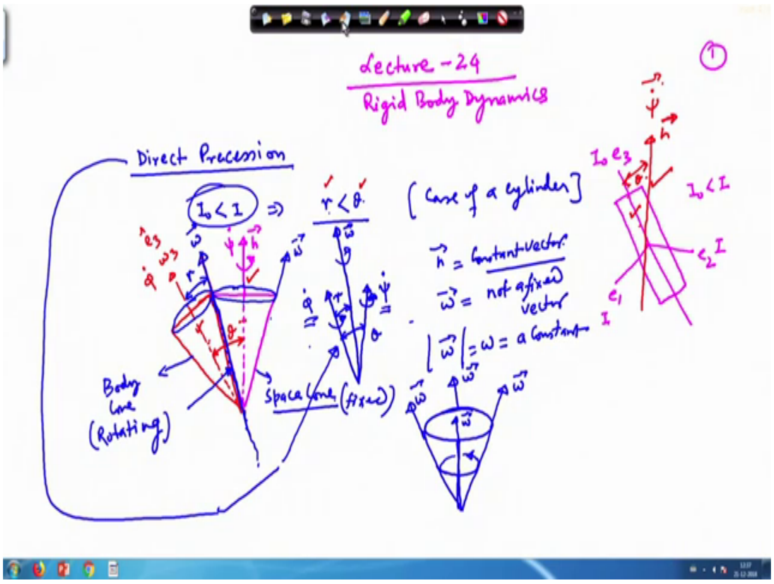


So, I 0 is less than I this implies that gamma will be less than theta. So, this implies gamma will be less than theta. So, this is the case of a cylinder such a torque free rotation of a cylinder how the motion will for the cylinder will appear.

So, here once we are describing in terms of disc. So, what it has shown that this red color cone, this red color cone signifying your disc, ok. And this e 3 along this direction itself your omega 3 is there then phi dot is also there. So, omega 3 phi dot all it is a shown like this. And as it rotates it is a rotating on it’s own axis phi dot and simultaneously it is also rotating about this h vector at the rate of psi dot. So, instead of this rotation being anti clockwise as shown here it is just clockwise, ok. So, the rotation sense just reversed, for this particular case as for this disc.

So, if we visualize in terms of this cone. So, basically this outer cone which is shown by the red, it is rolling over this inner cone which is shown by blue. So, this is called the space cone which is fixed and once it rolls so, you can see that this omega vector, it will as this is the touching point, ok. These two bodies surface line that meeting along this line as shown here this is meeting here in this place this is the meeting line.

So, as it will roll over this inner cone the outer cone so, the point of contacts the line of contact will change. So, this omega vector will right now it is here after some time it will go here, it will go here, it will go here, it will come along this line then back again it will come to this place. So, it will keep revolving. So, you can see that it is a just motion appears like motion of a top, ok. So, this is the outer cone, and this outer cone it is revolving like this the center of this cone will go in a circle like this, ok. It is right now here sometimes afterwards it will come here in this place and so, on so ah.

And the rotation sequence as we have shown here to rotation direction it is like this, ok. This is in anti clockwise direction, this is in the clockwise direction. So, this part we then we have reversed and shown here it this way, this is not valid here for this particular one, this will not be in the upward direction, but here in the lower direction, ok. And submission of these two vectors, we are showing here in this place which is omega this is psi dot, this is phi dot these are the vectors. In the case of the cylinder, if we have a cylinder like this so; obviously, we see here in this case I 0 will be less than I, I is along this two axis e 1 this is e 2, this is your e 3.

So, here you have I, I and I 0 is along this direction. So, in this case gamma will be less than theta, ok. So, for this we can make another figure again we make a space cone and in this direction we show h h vector psi dot vector then we take another cone and show it like this. So, this is your omega 3 phi dot is along this direction e 3 cap we have shown along this direction.

So, consider this now, if we draw the line here so, this is your theta, ok. This is e 3 direction and this is your h direction and psi dot also it lying along this direction. So, the angle between this and this as you can see from this place this line. So, this line is shown here this particular line is shown here in this place and this line is shown here like this. So, angle between these two lines it is theta. So, the theta angle it appears like this, this is your theta, ok. And what is the gamma angle? Gamma angle is less than theta value so; that means, your omega here in this case, it lies along this direction, this is your omega and this is your gamma angle. So, gamma angle here in this case; obviously, you can see that gamma is less than theta.

So, again following the same way your phi dot is along this direction and here in this case psi dot is along this direction. This is shown here and omega as a combination of this the omega is shown. So, the result of this is omega is lying here along this direction, ok. And this angle from here to here, this is theta while the angle between omega and this is written as gamma. So, this is your space cone, now this is way of visualization, we are visualizing in terms of two cones.

So, how the motion of the cylinder will appear? So, the motion of the cylinder will appear as such two cones are given, one cone is fixed another cone is rolling over this fixed cone. So, here in this case, this red cone is rolling over the pink cone. So, this is the this one this is the line of touch or the these two surfaces are touching along this line, this particular line, ok it is touching along this line.

So, as this cone will roll over. So, we can make it like this. This cone is here and this cone is here and this is the line along which these two cones are touching each other. And then this cone which is the called the body cone this is fixed and this is rotating. So, this particular cone, it is a rolling over this body. And therefore, this omega will keep rotating, you can see that omega vector is rotating. So, omega vector keeps rotating h vector is fixed, h vector it is a constant vector. So, it cannot rotate constant vector, but omega is not a fixed vector, not a fixed vector or constant vector.

However, omega magnitude which we are written as omega this is a constant. So, magnitude does not change, but this vector direction it is changing. So, vector direction how it is changing, you can see that, right now it is here after some time this omega will come here in this place, ok. Sometimes it will be here it in the opposite direction and so on. So, all the cases, I cannot show here on this figure, but if you look if this is your space cone. So, omega lies along this direction sometimes it will be here, ok. Sometimes it will be here ok, sometimes it will lie on the surface like this. So, it is a rotating continuously it is going like this, ok. It is a rotating all the way from here to here.

So, this is direct precession and here in this case just by using the parallelogram rule you can see that. And why it is called direct precession, because you can see that this and this they are in the same direction, because gamma is a smaller than theta. So, therefore, it has been accommodated like this, if gamma is larger than this. So, omega will go either on this side or as on this side and no longer it will be between these two, ok. So, and if gamma is larger than theta, so, already we have seen that it will, it is bound to be in the opposite direction, not in this direction. So, here the sense is rotation is like this and also the sense of rotation of this is like this anti clockwise both are in the same sense, both have exactly the same sense. And therefore, you can derive some of the simple relationship, ah.

Now, before going into any other thing this direct precession I will a state that this direct precession directly comes from this figure, otherwise we will derive one equation and from that equation we will be able to see that through equation itself it will be visible that, ok. For this case, it will be direct precession and for this case it will be retrograde precession and that can be done by using the Euler's equation and Euler's angles Euler's radical equations, we have to use and Euler's Dynamical equation.

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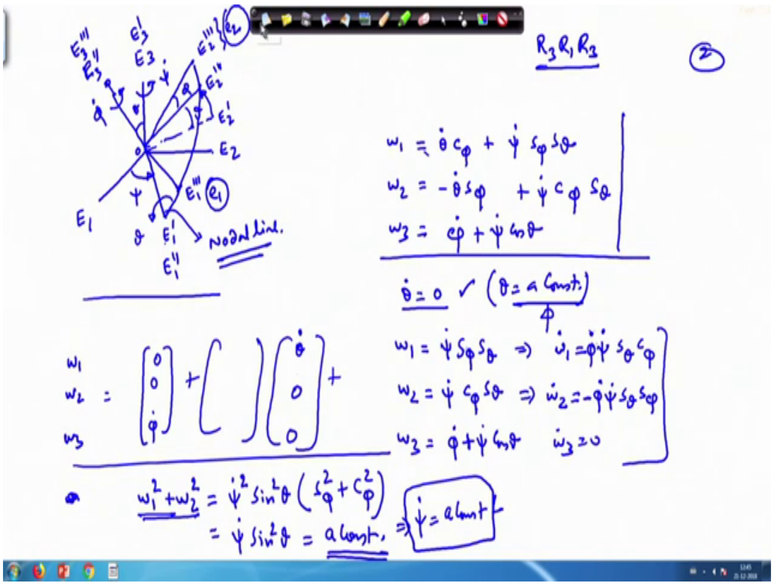


So, this I have given you the gist of how the angular motion will appear, ok. So, overall what we see that is we have finally, I am concluding this E 1 E 2 and E 3, ok. And this is the line you rotate it by psi about this so, this is psi angle here. So, the point which we have written as E 1 prime, this is called the nodal line. So, if the you show it by the psi dot rotation about this E 3 means this nodal line rotates at this red psi dot about this axis, ok. And thereafter; obviously, then you are giving rotation by theta along this. And then finally, so, this turned out to be ah, this E 2 will go from this place to this place, if you give rotation so, theta this will come out of this plane. So, it will appear in the here in this place. So, this is angle theta and then these two are combined together and it is a given rotation phi. So, this is your theta and this is phi.

So, if you have difficulty with this figure. So, we are going to take it up during tutorial again. So, this is E 2, E 2 prime, then E 2 double prime and this is E 2 triple prime, this part we have written as e two and again and again I am repeating this is E 1, the E 1 double prime is here, E 1 triple prime which we have written as e 1, e 2. So, e 3 is here E 3 triple prime is here and once it is a rotated, so E 3 double prime is here and also finally, given one rotation about this E 3 prime is here. So, phi is here in this place.

So, overall here this node is regressing, this called a nodal line if we write this as O. So, O and E 1 prime or E 1 double prime this rotates, in anti clockwise direction, because h we have taken along this direction and accordingly we have written. So, this figure is of great importance and we will do a little exercise to come to the conclusion, whatever we have done earlier this part through a little, but bit of theory. So, we have now, we will have to go back and recall that what is the relationship between the omega and the angular velocity described in terms of omega and Euler angles. So, we right here the expression for omega 1 omega 2 and omega 3, ok.

This expression from time to time if you can memorize it, it will be very good and, but it may happen that instead of rotation R 3, R 1, R 3 we are giving some other rotation. So, in that case whatever we have done in the previous class in the 22nd or 21st and 22nd lecture that will not be valid, ok. So, memorization will only work for only one case, ok, it will be difficult for you to memorize all the cases. So, I suggest that you approach the way I have described it like omega 1, omega 2, omega 3, first I described in terms of writing in terms of here phi dot 0 0 and plus this rotation matrix and then we used the theta dot, theta dot perhaps we have taken I do not remember along which direction I have written, but depending on theta dot here it was x direction so, maybe here theta dot written so, like this, ok.

So, if it is a long y direction. So, I will put here theta dot here in this place, then theta dot will appear in the middle. So, otherwise here this is 0 and then one more time was there. So, accordingly we have worked out and this then the omega 1, this expression we will complete it. And we will use it further for our purpose omega 1 is theta dot c phi plus psi dot s phi s theta and minus theta dot s phi plus psi dot c phi s theta. And omega 3, we have written as phi dot plus psi dot cos theta and we know that theta dot equal to 0, because theta is a constant theta a constant.

So, if we use this information here in this place. So, your omega 1 gets reduced to psi dot s phi s theta, omega 2 get reduced to only this part psi dot c phi s theta and omega 3 remains as it is phi dot plus psi dot cos theta. So, this implies omega 1 dot differentiated ones. So, this will be psi double dot, before this we do one more step not to complete complicate this whole equation once we have got this now go back to this equation omega 1 square plus omega 2 square. So, from this place we get this as psi dot square will be common and sin theta is there. So, sin square theta will also come as common and you will have s phi s square and c phi square where this as the usual rotation of this is sin phi this is cos phi.

So, this is nothing, but your psi dot or sin square theta and this is a constant as per our earlier working. So, this quantity is a constant on the right hand side theta is a constant as we evolved earlier. So, psi dot must be a constant so, this implies psi dot, this is a constant. And this is why I told you just wait for a while we do something and then will differentiate it. So, once we differentiate here this. So, omega 1 dot this will be now no longer I have to differentiate this. So, I will take it as it is s theta will also remain as it is, because theta is a constant. So, psi dot s theta here this is s phi. So, this becomes c phi and then phi dot will appear. So, phi dot, we will write here in this place once we differentiate this, ok.

So, s phi sin phi differentiated this becomes cos phi and then phi dot appears. Similarly this omega 2 dot, we get from this place psi dot s theta remains as it is, ok. And c phi this becomes s phi and with minus sign this becomes phi dot an in omega 3 dot; obviously, this is 0 omega 3 dot 0, because this quantity is a constant. Now from this relationship it is a we can work out I will have to go on the next page put it in the Euler's equation and then work it out. So,, let us go on the next page.

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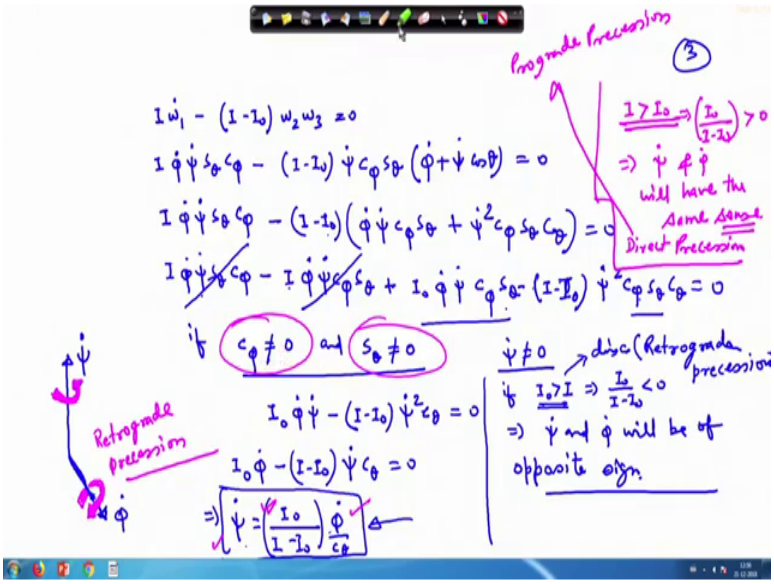


So, in the Euler equation I 1 is I so, I times omega 1 dot minus I 2 which is I minus I 0 times omega 2 omega 3 this equal to 0. So, the first equation in which we insert this so, this is I time omega 1 dot omega 1 dot just now we have worked out this is phi dot psi dot s theta c phi, phi dot psi dot phi dot psi dot s theta and c phi, phi dot psi dot s theta and s phi, minus I minus I 0 omega 2 omega 3; which we need to pick up from here omega 2 and this is omega 3 psi dot c phi s theta psi dot c phi s theta times phi dot plus psi dot cos theta, this is omega 2, this is omega 3 and this equal to 0. Expand it phi dot psi dot s theta c phi minus I minus I 0, if we multiply this. So, this becomes phi dot the first term this is psi dot c phi s theta and plus phi dot square c phi s theta and this cos theta, we will write as c theta this equal to 0 breaking it minus I times phi dot psi dot c phi s theta.

And similarly, so we will have total two terms here. So, if we break it we will have two terms ok, but what we can see that these two terms will cancel each other so, the case gets simplified. So, here the other term we can write as this plus I 0 times phi dot psi dot c phi s theta and minus I minus I 0 times s theta c theta this equal to 0.

So, these two terms they dropout and from here, what we see that c phi and s theta both are common. So, if c phi not equal to 0 and s theta not equal to 0, ok. This is the case c phi not equal to 0 see, because we are going to divide both sides by c phi and s theta to eliminate c phi and s theta so, if this quantities are non zero. So, then what we get I 0 times phi dot psi dot minus I minus I 0 times psi dot square c theta this equal psi dot will also psi dot is not 0. So, therefore, this gets reduced to we described in terms of psi, we will describe it in terms of psi dot.

So, we will write it like this, first let us eliminate this part I minus 0 times psi dot c theta this implies psi dot equal to I 0 divided by I minus I 0 times phi dot, ok.

So, look at this equation what does this say if I 0 is greater than I. So, the quantity which is present here, this will be negative I 0, this implies that I 0 divided by I minus I 0 will be less than 0 means psi dot and phi dot they will be of opposite sign. So, this implies this implies psi dot and phi dot will be of opposite sign ok. This is, what has been what the this is the case what we have been discussing that once I 0 is greater than I means it is a case of a disc. Disc and this gives rise to the retro grade precession here handwriting cannot be very good, because it is being written on the screen of a desktop.

So, exactly you can visualize from this equation earlier, we have done this to the geometry that psi dot and phi dot is of opposite sign. So, psi dot we assumed it to be in the positive direction. So, and means it is a anti clockwise and from here what it says that phi dot is bound to be if psi dot is positive. So, phi dot is bound to be negative and here one more thing is missing this psi dot equal to I 0 divided by I minus I 0 and this is c theta also. So, psi dot we are here we have to write here c theta or phi dot, you can describe in terms of psi dot whichever way you want ok.

So, if this is psi dot the sense of phi dot is not going to be the same, it must be clockwise means the psi dot to this phi dot must be here along this direction ok; that means, it say something like this is opposite way I will show by a better figure. This is the direction here the solid line this is anti clockwise ok. This is above this while for this case, this is rotating this way, but this is rotating this way. So, it is from the downside, these two are in different directions they have not the same sense ok so, this is for your retrograde.

On the other hand, if you have the case where I is greater than I 0, ok. So, this quantity in the bracket the quantity here in this bracket is going to be positive and therefore, both will have same sense psi dot and phi dot. So, in this implies that I 0 divided by I minus I 0, this will be greater than 0 and this implies psi dot and phi dot will have the same sense which we have written as the direct or prograde rotation precession which is the direct precession. And also it is a called the prograde this is retrograde so, this is prograde ok.

So, this completes the description for calculating the rate of precession basically what this gives you, this gives you the relation between the rate of precession. And rate of spin there are something that you can observe from this equation I will copy it on the next page.

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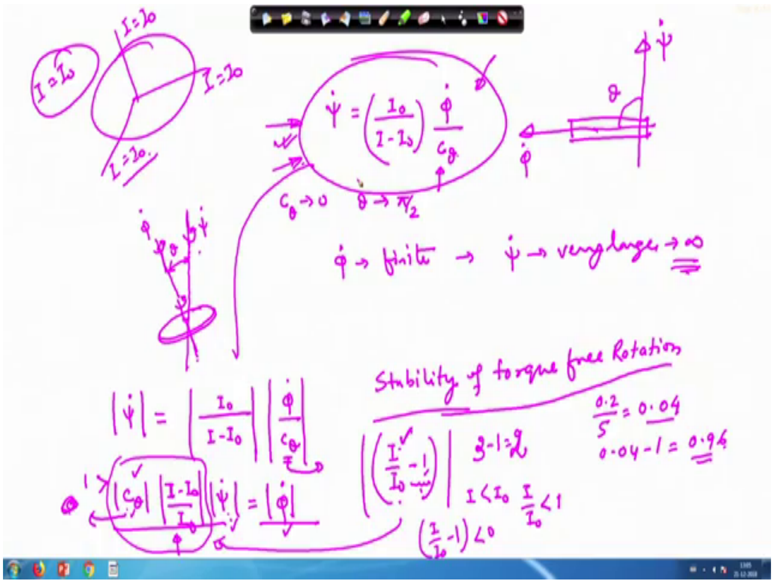


This is psi dot, this equal to I 0 divided by I minus I 0 and phi dot divided by c theta. So, what you can observe that if c theta tends to 0 means theta tends to phi by 2. So, this is the case where the, your cylinder or the disc, it has become something like this, this is your psi dot direction while the spin is along direction, ok. So, it is approaching phi by 2 so, in that case this blows up ok. Remember the case once we have derived here, we have written this should not be 0. So, we are not taking the case to be 0, but it is approaching that value limiting case. So, in limiting case as we do in the case of calculus. So, if we considered that case so, here this blows up ok. So, you can see that for phi dot, if it is finite psi dot will become very large means it tends to infinity tends to infinity.

So, for a finite spin, if you increase the angle of nutation here this is your theta angle, if you increase it up to 90 degree it is bringing it to this place. So, in that case the precession rate will be extremely high, ok. And quite often as sometimes in the beginning itself I, I might have mentioned that this kind of case which is the torque free case can be stimulated for the satellite or either say you have the point like this.

This is a point and if it is possible to rotate, it about this axis you rotate about this axis and then toss it making certain angles with the vertical theta, ok. And simultaneously it is a rotating about this axis by psi dot, this is by phi dot again here I am showing like this, but it will be in the opposite direction.

So, this case similar this is the coin rotating spinning about this axis, this at this particular axis and also processing about this axis and tossed up in the air ok. So, once it starts falling so, it's a free from gravity and because of the symmetry there is no torque acting on the coin and therefore, this case is perfectly simulated. Assuming that there is no aerodynamic (Refer Time:32:39) on the things or it may be a small for a short period, we an assume it to be risible,. So, this comes to a conclusion and we have to look into the stability of the system and what I will do that we can look at the same thing from another perspective and it will be very useful. However, it is going to take more time so, we have a few more lectures remaining on this. So, I will try to accommodate that part also, because if you get the another view of how to work out the same equation, it will be good.

But we have to also look into the stability of the rotation of the next topic for this will be the stability of torque free rotation which will take up maybe in the next to next lecture. There are few more things that we can conclude from here before winding up this particular lecture. I will go through this psi dot, if we look into the magnitude of this will be I by I minus I 0 magnitude times phi dot by c theta magnitude.

Now, this quantity what will do that I will bring this whole thing on this side. So, maybe c theta written here and I minus I 0 divided by I it is a written in here in this place and psi dot magnitude equal to phi dot magnitude. Now look at this equation, this quantity is always less than 1, ; this quantity is less than 1, because it is a c theta. So, either it is a positive or negative this bound to be less than 1 ok. This quantity whether it is a greater than 1 here this is I 0, this is I 0 here in this place here also this is I 0 on the previous page. Let us check I 0, we have written here.

So, I 0 is here and I minus I 0 thereafter, ok. What about this quantity? Now if I is greater than I 0, so, this we can write like this. If I is greater than I 0, this will be greater than 1 ok. Then this is greater than 1 and from there you can subtract this 1.Now how much it will be greater depends on this magnitude of this I say the I by I 0 is 2. So, this is 2 minus 1 equal to 1 ok. If it is 3 this quantity, so 3 minus 1 this will be equal to 2. So, this quantity this is always less than 1, this quantity is depending on the magnitude of this quantity either this will be greater than 1, it can also be other way if we look that if I is less than I 0 ok.

So, this quantity becomes less than 1 ok and therefore, this quantity will be small. So, this quantity then if I is less than 0, if I is less than I 0. So, I by I 0 this becomes less than 1. And therefore, I by I 0 minus 1, this quantity will be less than 0, this will become a negative quantity, because this is less then 1 and from there if we subtract it 1. So, this is going to be negative quantity.

Now, again how much these difference is how much difference can go, I can become small as compared to I 0, but how much it is going to be? The maximum it can be let us say this is 0.2. And here this turns out to be 5 I 0 the previous case we have taken to beok. We are discussing I less than I 0, fine. So, here in this case this we can write as 0.04 and from here if we subtract this mine this 1 so, this will be 0.96.

So, this quantity becomes 0.96 c theta is less than 1, and psi dot is here. So, this is less than 1; this is less than 1 magnitude by and these two are related by this relationship to what does it imply, here this quantity which you are multiplying. So, in this particular case, this becomes less than 1 magnitude wise ok. So, that implies so psi dot will be magnitude vise greater than phi dot, ok. This is what exactly it is applying. So, this is the way of looking into the, if you are given any problems. So, you can analyze starting from here if I keep doing this, it will be true strenuous and it will cover a lot of times I am avoiding this., I will wind up this lecture with this advice that always whenever you come across in the rigid body dynamics such equations or you are tackling finally, were come to conclusion with some equation.

So, always analyze this the limiting cases what happens do not fear as per here in this case theta dot theta tends to 0 or what happens with I and I 0, they become equal say I and I 0. Once, they are equal then what happens in that case I and I 0 becoming equal means it is a case of a sphere, ok. All the three are becoming equal I 0 this is a case of a sphere so, this becomes 0. So, what does this mean? So, this kind of situation will pick up take up in tutorial for the time being we stop here and go to the, we will continue here in the next lecture.

Thank you very much.