Satellite Attitude Dynamics and Control

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Lecture - 38
Spin Stabilization

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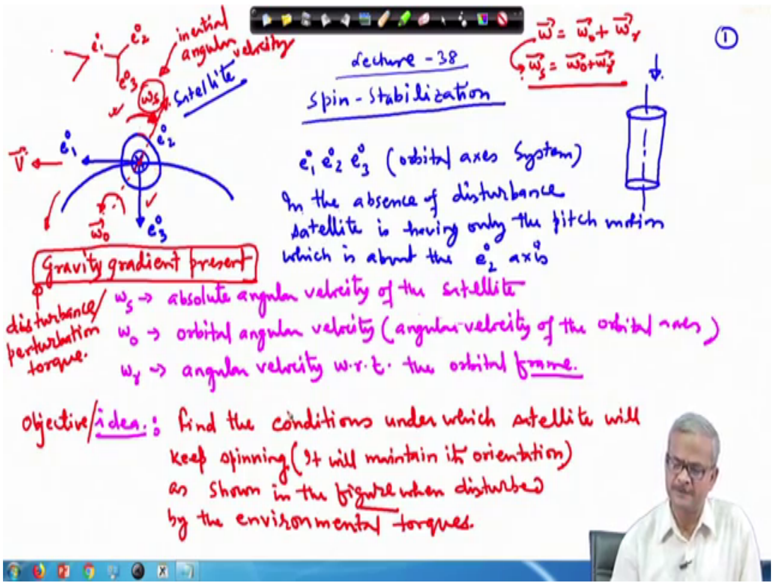

Welcome to the lecture number 38. So, today we are going to start with their new topic on Spin Stabilization. So, while we talk about the spin stabilization, a satellite is moving in orbit as usual we have this e 0 1, e 0 3 here in this direction and e 0 2 points inside the page. So, this is our orbital axis system; orbital axis system e 0 1, e 0 2 and here I am showing a satellite.

Now, the idea is to keep the satellite as it is shown in this figure means, if I have a say or cylinder; if I have a cylinder so, what I would like that, this cylinder if we so; for see it from the top. So, it will appear like this. So, it may be any other object and what we like that, this keeps its direction intact. All the time so, right in the beginning you are this is a satellite, which is rotating about this the e o 2 axis ok. So, when disturbance is not there in the absence of disturbance; in the absence of disturbance satellite having only the pitch motion, which is about e o 2 axis.

So, this satellite is pitching and now we have the, we define here omega S as the absolute angular velocity of the satellite and omega 0 we define as the orbital angular velocity. So, this the rotation or the angular velocity of; this is the angular velocity of the orbital axis system. So, this is the angular velocity of the orbital axis. And omega r this we define as the angular velocity with respect to the orbital frame.

So, idea is here or the objective, find the conditions under which satellite will keep spinning about; satellite will be keep the spinning or other way we can say this as, it will maintain its orientation as shown in the figure, when destroyed by the; when destroyed by the environmental torques.

So, here the velocity vector of the satellite is in this direction and we know that your e 0 2 its a going here in this direction. So, this is e 0 2 which is going inside the page and say something like this, this is e 0 1, e 0 2 and e 0 3. So, we will bring for the idea that omega the absolute angular velocity, this equal to omega 0 or the orbital angular velocity plus omega r and this omega we will write as omega s.

So, the satellite should always keep spinning and suppose that the satellite is spin direction is like this. So, this is your omega S and where what is the direction of the orbital angular velocity? Because the satellite is going here in this direction ok. So, the omega 0 is directed along this direction. So, the this your orbital reference frame is rotating and with respect to this the satellite will appear to rotate, because it is supposed to maintain its inertial angular velocity omega s inertial angular velocity.

So, already we have observed that if a satellite or if a rigid body it is free from the external torques and if it is rotating about some axis through is a possible to maintain that rotation or whatever the orientation in which its a rotating. So, its a possible to maintain that orientation under certain condition. So; obviously, whether its rotating about the minor axis or the major axis that is another issue that we have already discussed. But here this special issue is that, there is gravity gradient will be acting here in this direction.

So, gravity gradient this is present. So, gravity gradient is present and this will act as a disturbance torque; disturbance slash perturbation torque and we are supposed to maintain this direction. So, if it is disturbed from the disorientation what we are shown by some little bit amount and drop with respect to the orbital access by little bit amount. So, whether such kind of system will be stable in direction or not, this is what we are going to study in this.

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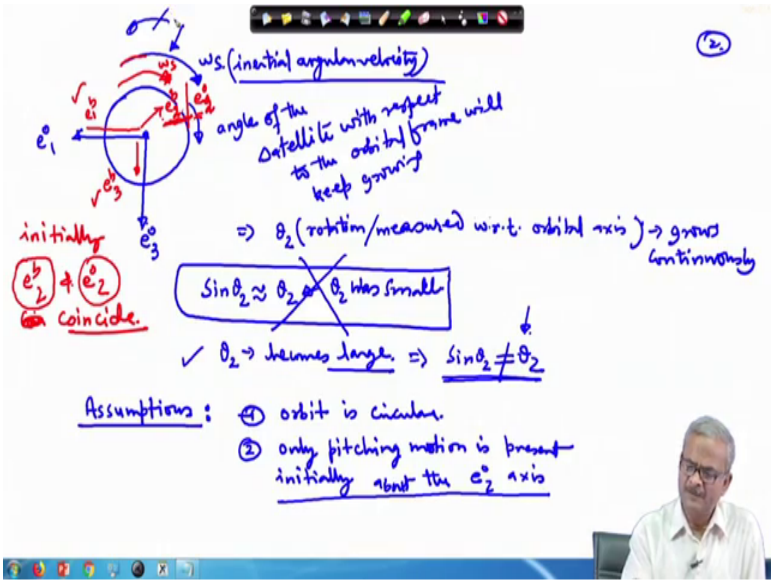

So, the particular point we need to point out here is that, this is my satellite and suppose this is rotating at omega S ok. And here you are showing your the orbital axis system. Now, if it is rotating so, you can see that because this is the inertial angular velocity inertial angular velocity. Because this is initial angular velocity so, in this direction the angle will; angle of the satellite with respect to the orbital frame will keep growing. This simply implies that the rotation theta 2 this rotation which you are measuring from, measure with respect to the orbital axis; orbital axis grows continuously.

So, so far what we have assume that, quite often you have seen in all our previous discussion while doing the stability analysis, we have approximated this terms like sine theta 2 equal to theta 2 because theta 2 was a small; this was a small, but no longer this quantity this condition is valid ok, so this is no longer valid. Now, theta 2 becomes large, because of the continuous rotation its a continuously spinning.

And therefore, this approximation is not valid and this implies sine theta 2 will not be equal to theta 2. So, because of this angle being large, we are not doing this approximation. So, this discussion it differs from all our previous discussion in that this angle involved here theta 2 will be large and therefore, any sort of approximation we are doing with respect to theta 2; for that theta 2 in the previous discussion they are not valid here. So, we have to take care of this particular issue in this place.

So, we start here now discussing; so, our assumptions are when the orbit is circular, this simplifies the working with the equations a lot otherwise the whole thing becomes very complex. So, to this satellite you are attaching the body axis, which is coinciding with this. So, write in the beginning suppose the body axis its a coincides like this. So, this is e 1 b, e 2 b and downward this is e b 3.

Ones the satellite is starts spinning here in this direction with omega s. So, this e 2 b this direction will not change, but e 1 b and e 3 b they will keep rotating and this e 2 b it will coincide with, we have written here b so let me do the correction here, this should b is on the top 2 is here. So, (Refer Time: 16:17). So, this two directions e v 2 and e o 2 they coincide initially. So, initially e b 2 and e o 2 coincide but as the system is perturbed ok. So, because of the perturbation, this e b 2 and e o 2 they will differ ok. So, we are going to write equations for all this things.

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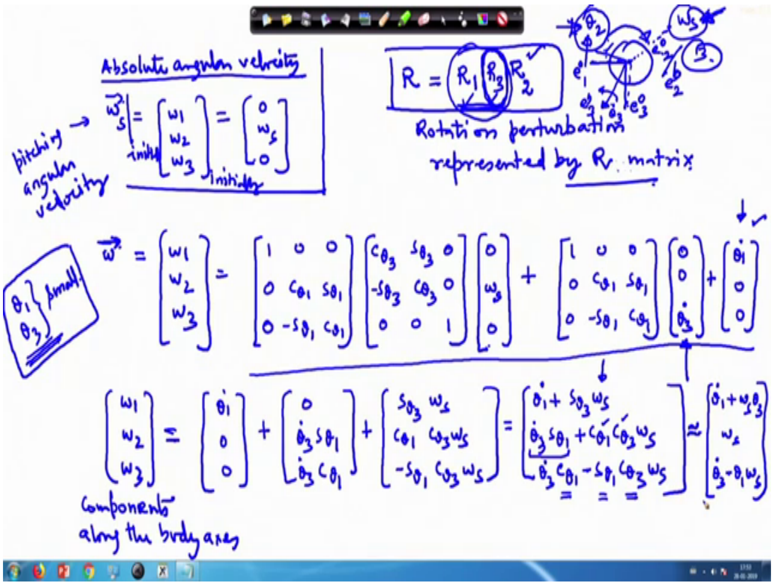

So, first of all let us write the angular velocity. So, this is the absolute angular velocity which enters into the Euler's equation and this will enter into the Euler's equation. So, omega this will be omega 1 omega 2 omega 3 and we put here s. So, this is the initial situation omega S this is your pitching angular velocity initially, and we can put here right here something like the initial. So, we understand from this point that only the second term will be present and rest others will be 0. What once the perturb system is perturbed so, the things will appear in a different way.

Now so, therefore, omega the angular velocity afterwards it can be represented as omega 1 omega 2 so this is initially, omega 3. So, what we are going to do that the first rotation is about the 2 axis; the second body axis. So, its a continuously rotating and therefore, this angle will keep growing.

Now, the second rotation we will give about the third axis and the last rotation we will give about the first axis and therefore, this is our the rotation transformation to be followed. So, your body axis it is a moving away from the orbital axis system. So, rotation, perturbation represented by R matrix this is the rotation matrix ok. So, how do we represent it the first rotation. So, your satellite is continuously rotating. So, this is the first rotation you are representing and let us represent this as we are showing the rotation about the 2 axis. So, this is theta 2.

So, it is a continuously rotating. So, it is already this angle will be very large. So, as you know that if this is rotating and then we are perturbing about the third axis under the first access. So, therefore, the rest of the this transformation we have to indicate in terms of only R 1 and R 3 this we have discuss in details. So, I will not discuss it here again ok. So, this will be 0 0 0 0 and here 1 0 0 0 0 and now this angle about the first axis we are rotating by theta 1 so, this is C theta 1 C theta 1 S theta 1 minus S theta 1. And about the third axis we are rotating about theta 3 so this is C theta 3, S theta 3 and minus S theta 3 and C theta 3 and upon what it will operate, because the angular velocity omega S we are trying to convert.

So, what will be the; this is the absolute angular velocity along the two direction. So, that this theta 2, its results theta 2 we are measuring from the; remember that we are measuring it from the orbital axis system ok. But the omega S is the absolute angular velocity which is taking place about the o 2 axis ok. So, e o 2 axis and to which the e o; e b 2 also coincides. So, it is a continuously rotating about that so, we will right here omega S. So, this is the absolute angular velocity along the 2 direction. So, this is the first perturbation so we are thinking this like a perturbation. So, ah, but here it is a big perturbation, it will continuously grow and the others will be 0.

The next rotation of course will be 1 0 0 0 C theta 1, S theta 1 and 0 minus S theta 1 C theta 1 and this will operate on; now, the rotation about the this already rotating. So,now rotation about the third body axis ok. So, third body axis here it is a continuously already because of this rotation, third body axis is rotating like this, but once we are looking into this one, so here we will write this as theta 3 dot 0 0. And the last one we will write as theta 1 dot 0 0. So, carefully look into this part, this is just being operated by this is the angular velocity theta 3 dot means, your body axis wherever it is so for here with respect to this orbital axis system this is e o 3 and e o 1. So, with respect to this the body axis now suppose it is here in this place and so this is like this then you are this is your third body axis e b 3.

Then you will be perturbing about this, because the rotation the, we are giving this rotation thereafter. So, you are perturbing about this; so, once we perturb about this theta dot 3 will appear here in this direction. And this we have to convert to the body axis system. So, further this is being the rotated about this one axis, there after the resulting first axis; body axis it will be rotated and therefore that we do not need to do any conversion because this will be along the first body axis itself. So, this is the equation. So, this we have discus in a details long back and it takes a lot of time discussing about this things. So, this would be very much clear and you should keep in mind how to go about this.

So, omega 1 omega 2 omega 3 this is the absolute angular velocity of the satellite and which components along the; these are the components along the body axis, components along the body axis. So, if we multiply it so this term will right here, theta 1 dot 0 0 multiplying this particular part so, the first part is 0, the second part will be theta 3 dot times S theta 1 and the third part will be theta 3 dot times C theta and this is C theta 1 so this is C theta 1.

Now, the other part will be doing from this place. So, we need to multiply it and work out. So, same way we can multiply and write here in this place. So, this part I will skip and write directly the result you can check yourself verify it, as a C theta 1 C theta 3 omega S and minus S theta 1 C theta 3 omega S. Adding all of them so, this is theta 1 dot plus S theta 3 times omega S, this will be theta 3 dot times S theta 1 plus C theta 1 C theta 3 time omega S and the other one. See this value is quite large. So, we are not going to do any approximation with where ever the theta 2 is involved.

So, here we see that there is no theta 2 involved, but theta 1 and theta 3 they are a small, theta 1 theta 3 these are a small and therefore, approximation can be done for the theta 1 and the theta 3 term. So, this will get reduced to theta 1 dot and here S theta will get replaced by. So, this is approximately equal to theta 1 times omega S theta 3, similarly here in this place S theta 3 dot and then S theta 1. So, this term is of second order this particular term. So, we can neglect it and this part this is 1 or approximation to 1 approximation to 1. So, therefore, we get here omega S and this part again, this is theta 3 dot because cos theta C theta 1 this will be equal to 1, and here this is a cosine term and this is a sine term so, this is minus theta 1 times omega S.

So, this is the angular velocity we have got. So, what we have done? We have only approximated in theta 1 and theta 3, not in terms of theta 2 anywhere.

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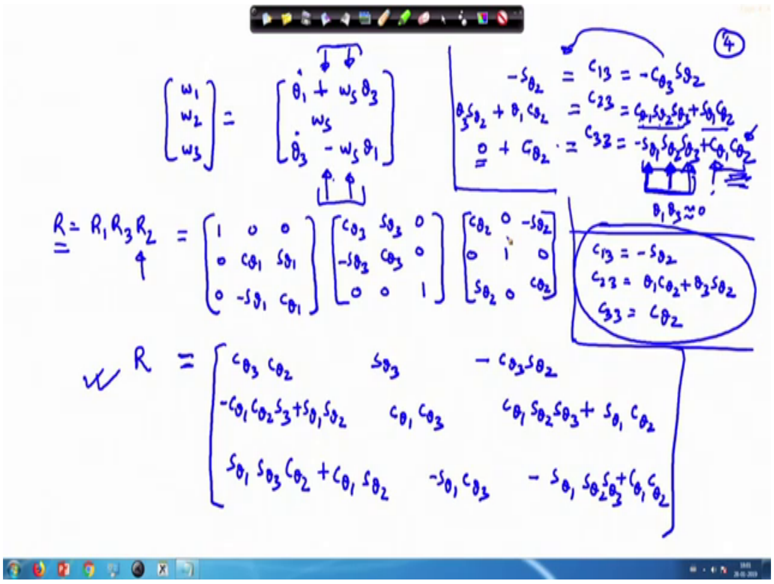

So, therefore, we have omega 1 omega 2 omega 3 we can write it like theta 1 dot plus omega S plan plus times theta 3, omega S and theta 3 dot minus omega S times theta 1. And what we see the sign here this omega x? Here the plus sign is appearing in this place and here the minus sign is appearing.

So, if your satellite is rotating in the opposite direction, say the omega S is not here in this direction as shown here, but rather here in this direction; if it is like this, then you just have need to change the sign. So, you need to change the sign for that. So, in that case sees of omega S omega theta 2 will be taking along see here, if it is rotating in this direction the or orbital frame; orbital frame is here and with respect to this you are showing the body axis rotation. So, theta 2 we have shown here in this direction and if omega S becomes here in this direction. So, they are opposite to each other so the sign change will take place.

So, I will point you again when the sign change will be required. Whenever such situation appears so, we if we do just the sign change the we will get the result for that, we need not derive the whole thing again ok. So, and R 1 R 3 R 2 this R we have written so, this also this matrix can be written as 0 0 0 C theta 1 S theta 1 minus S theta 1, and R 3 as usual we have 1 C theta 3 S theta 3 minus S theta 3 and C theta 3. And R 2 here in this case the angle involved with the 2 direction it is a large ok. So, I am not going to approximate that anywhere. So, this will be C theta 2 C theta 2 minus S theta 2 and S theta 2.

And then we multiply it; so, once we multiply, so again you can check the result I am writing here, this is C theta 3 C theta 2 minus C theta 1 C theta 2 S theta 3 plus S theta 1 S theta 2 C theta 1 times C theta 3. These are some of the mathematics that you cannot avoid in the attitude dynamics and we need to save some time here. So, every time I will not be computing this kind of things. So, this is your R matrix.

Now with all this information available, we are ready to do the dynamic analysis of this system. Now here, what is important that the gravity gradient terms involve C 1 3 C 2 3 and C 3 3. So, here in this case C 1 3 is minus C theta 3 times S theta 2, C 2 3 will be C theta 1 S theta 2 and S theta 3 plus S theta 1 C theta 2 and this will be equal to minus S theta 1 S theta 2 3 plus C theta 1 times C theta 2.

And once we are going to do the approximation so, I am writing here on this side. So, we can see that theta 2 cannot be approximated, but theta 3 can be approximated. So, therefore, this becomes minus S theta 2. Similarly for this particular one; this one we will have C theta 1 is there S theta 2 is there S theta 3 is there. And then, S theta 1 is there and C theta 2 is there. So, for this term we will have C theta 2, here this is theta 1 and for what about this term? This term is 1 and this is theta 2. So, S theta 2 will be present and then we have S theta 3 so, that we can approximate as theta 3. So, theta 3 times S theta 2 theta 1 times C theta 2. And in this particular one theta 1 theta 3 and S theta 2 this part will be just one, C theta 1 plus C theta 2 this gets reduced to 1; sorry, the theta 2 is large. So, C theta 1 is 1 times C this we can write as this particular part this is large theta 2 is large.

So, we are not going to approximate, but this part will get approximated. So, this because C theta 2. And the other part so this is your this part here it involves. In any case sine theta 2 is not going to exit plus minus 1, this is the value for sin theta it is not going to exit plus minus 1. But here we have this 2 terms, sine theta 3 and sine theta 1 this together makes if approximated theta 1 times theta 3. So, this is almost equal to 0 and this multiplied by sine theta 2 which will be lying between 1 and 2. So, this quantity becomes a small. So, therefore, that quantity we have neglected.

So, this part we have neglected here and only this part it is continues. So now with this information we are ready to work further. So, I will conclude here; so, we have C 1 3 this equal to minus S theta 2, C 2 3 this is theta 1 times C theta 2 plus theta 3 times S theta 2 and C 3 3 equal to C theta 2. So, this result will be required for our working ok. So, we will continue this in the next lecture ok.

Thank you very much.