Satellite Attitude Dynamics and Control
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Lecture - 54
Gyrostat (Contd.)

Welcome to the lecture number 54, so we have been discussing about the Gyrostat.

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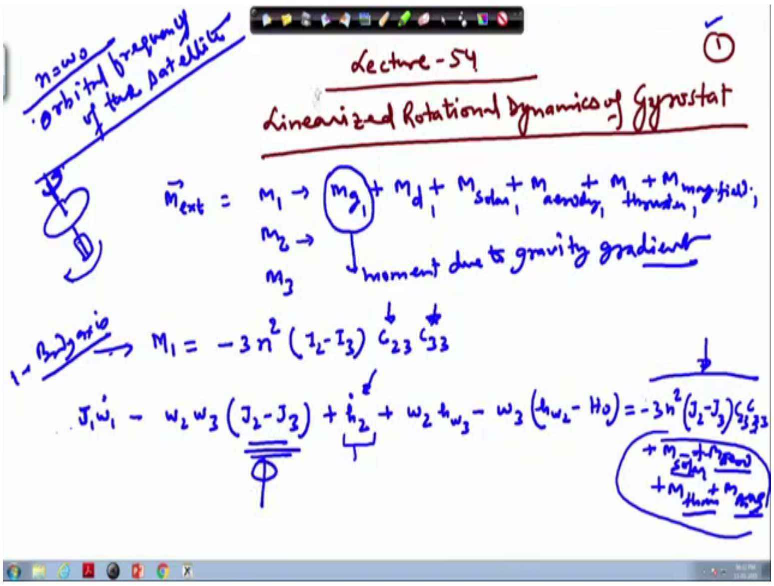

So, we derive the equation of motion for the Gyrostat in the expanded format and now we are going to linearize that system in order to get basically once you are trying to control the satellite and you are looking for the local control, means the small disturbances I have occurred and for that you are trying to control the satellite.

So, in that case you have to linearize it and get into the linearized equation of motion, we have done it so many times for the gravity gradient satellite for the spin stabilized satellite. So, for them we have linearize the equation of motion and we have written that so it is the same part only thing some extra added things are added here.

So, here the external torque see once we are writing the equation that M external and then we are writing here as M 1 and the other part as M 2 and the third part as M 3. So, this can consist of M gravity gradient plus M disturbance plus M torque due to other things like there may be the torque due to the solar radiation, torque may be due to the aerodynamic drive ok.

Then torque may be due to the thrusters then torque may be due to the magnetic move field and so on just keep adding ok. So, the torque this is long one direction, so this will be corresponding direction torque we have to write it like this. So, here what I am going to do that this part will be always present, the moment due to gravity gradient this is moment due to gravity gradient.

So, it is a customary to write this equation what we have derived earlier, so instead of just writing I will just write M 1 instead of M g 1 and this you know this quantity will be equal to minus 3 omega 0 square which we have written as omega n square in the gravity gradient equation.

So, n equal to omega 0 this is the orbital frequency orbital frequency of the satellite, this is the orbital frequency of the satellite and this times minus I 2 minus I 3 and then also we have written here C 23 C 33 ok. So, these are the coefficients from the direction cosine matrix and obviously this is along the body axis.

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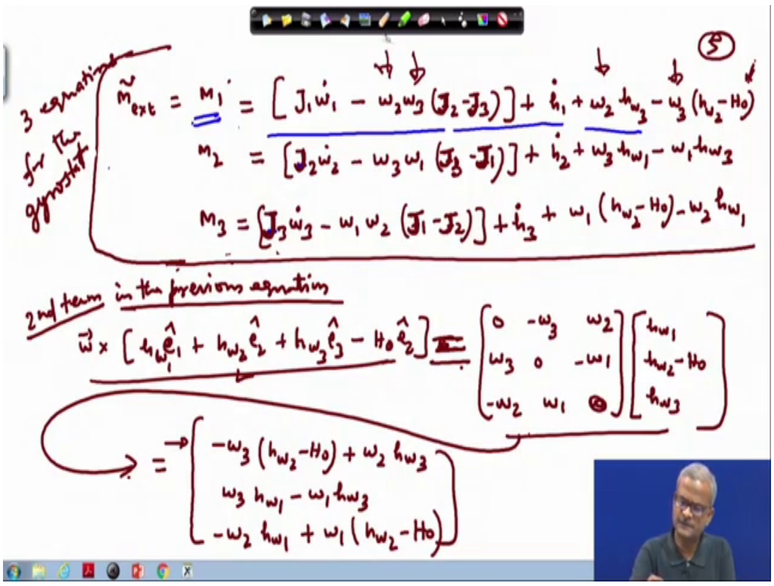

So, one body axis and if this linearized equation has to be modified, so simply on the right hand side say if I expand the previous equation here written this particular one. So, if I right here the gravitational term were the M g 1 and the other terms right now I will ignore and put that is 0 whenever required so you can just insert that value on the right hand side you will get the equation I will show you what I mean so far as I work so that will be much more clear.

So, in that context so we are start with the our equation, so M 1 this equal to S will write it on the right hand side let us first write the, I will copy this part first from this place this whole thing. So, J 1 times omega 1 dot here while writing everything has got into I 1 I 2 I 3 form. So, maybe I will modify this to I 1 I 2 and I 3 because, otherwise every were I have to change let me do that this is J 1 J 2 J 3 J 1 J 2.

So, here then this becomes omega 2 times omega 3 J 2 minus J 3 and then pick up the other terms with is h 2 dot and plus omega 2 times h w 3 minus omega 3 times h w 2 and minus H 0 this is what is here h w 2 minus H 0 and we right here M 1 on the right hand side. So, this M 1 instead of writing here M 1 the gravity gradient term which I will pick up from this place and right here, oh ok.

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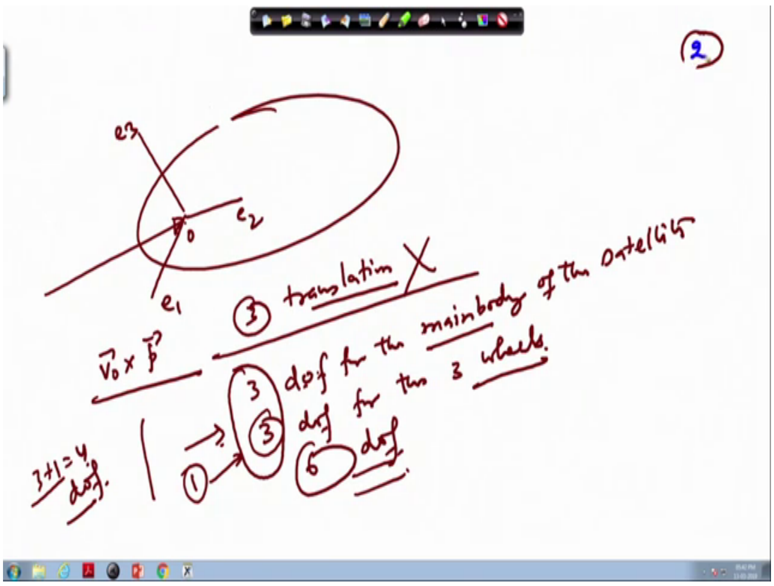

This some over other S 6 is written, this should be 2 this is 2 and then sum the 3 ok.

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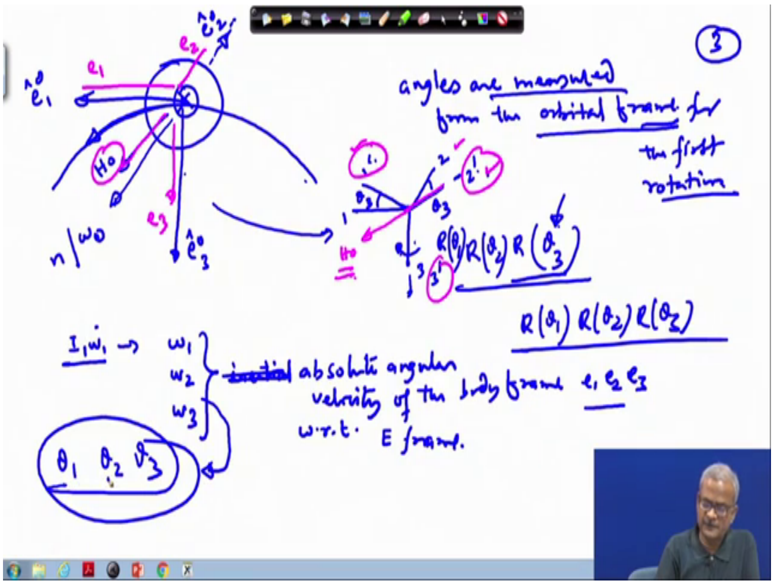

So, here we are writing the gravity gradient term which is J 2 minus J 3 times C 2 3 times C 3 3. Now what I was telling that this is only the gravity the gradient term and other terms right now I am assuming 0 ok, like the M solar radiation plus M aerodynamic torque plus movement due to the safe you have the thrusters ok, movement due to the magnetic movement or the magnetic field.

So, all these things I will assume this to be 0 right now and I will simplify this equation. So, whenever required on the right hand side of this equation if you put this quantities, so automatically you will get the linearized dynamics along with the other terms. Now this term here h 2 dot is present this term can be utilised for steering the satellite ok, inside if you change the angular momentum of the this wheel by actuating a motor ok.

So, by changing this you will produce a torque on has if you are a speeding it up, so the main body will also feel a torque in the opposite direction ok. So, along the 3 axis by using this h 1 h 2 h 3 we can steer the satellite and that is the internal control and the external control occurs like from the solar radiation from the magnetic moment from the aerodynamic torque and so on ok. So, therefore, it is h one here on the left hand side because, this is internal part and the external part can be shown on the right hand side.

So, this gravity gradient which is present here, so it has appeared on the right hand side but as we simplify this equation, so we will take it on the left hand side itself and right hand side will put to 0 and anytime you can put the other forces there and do the competition.

So, in this context we need to look into what will be the corresponding angular velocity in terms of theta 1, theta 2, theta 3. So, what exactly we have done we need to 2 and the 3 equation. So, your satellite is this is the e o 3 direction e 1 cap and the satellite is like this.

So, the 2 direction is going inside and so the orbital angular velocity it lies here in this direction or the orbital frequency it is a corresponding vector it will lie here because, it is the satellite is going here along this in the orbit and therefore it will lie here in this direction which we have written as omega 0 or the same thing as n we have written omega 0 or n the notation we have used.

So, this is your e 2 positive direction, so satellite is going in this orbit and then it is a perturbed from this condition ok. So, we are measuring the angles from the orbital reference frame, so angles are being measured this we have discussed earlier, but here I am again repeating for the sake of your convenience.

So, angles are measured from the orbital frame; which one? The first rotation whichever you choose ok, there after the things will change say this is your right now this is the corresponding axis which is shown here. So, if I give rotation about the first axis so this rotation about the first axis, so I should show in the positive direction positive direction will be toward this. So, this will go from this place to this place and this will go here so this is 1 this is 2, so this will become 1 prime and this will become 2 prime this is 3.

So, 3 prime will remain here, the next rotation I give about the 2 axis, so this rotation I am writing as theta 1 this is theta 2. So, all the rotations angles are measured from the orbital reference frame for the first rotation and there after it has changed because now it has become I 1 prime I 2 prime and a 1 prime 2 prime and 3 prime, so no longer this is the orbital reference frame. So, 3 you cancel rotation we can give say first we have given about the three. So, we are writing that as theta 3 not theta 1, so this is theta 3 which is equivalent to psi so this is theta 3.

So first rotation we give as theta 3, so this is the corresponding rotation matrix there after we give the next rotation as the theta 2 about the 2 axis and then about the 1 axis means

R theta 1 R theta 2 and R theta 3 is the corresponding set of rotation we are choosing and inside the wheel already your you are giving the bias momentum. So, you are giving the bias momentum along this direction I told you that it is along the e 2 direction of the body axis. So, write in the beginning your body axis is coinciding with this frame, so your body axis is like this is e 1 this is e 2 and vertically down is your e 3.

So, H 0 will be pointing write in the beginning here in this direction if you give disturbance, so this H 0 vector will be then in other direction. So, H 0 is bias momentum so I am not giving it along the positive e 2 direction ok, but this will always point opposite to e 2 direction. So, if your from this say in the next level ones your distorted by this so 1 will come to this position and then the body axis will be here this 2 prime position it will come your body axis 1 prime 2 prime and 3 prime ok.

So, your h vector then will be just opposite to this 2 prime direction, so this is what we are calling as the bias momentum. So, this is given to the wheel and this will help maintain the orientation of the satellite, if the wheel is large or the velocity of the wheel it is a angular velocity of the wheel is very high a small wheel very high angular velocity.

So we go about doing all this things, so in your equation of motion where I 1 times omega 1 dot it is appearing. So, we need to find out this omega 1 omega 2 and omega 3, these are your inertial or is simply say the absolute angular velocity of the body frame e 1 e 2 and e 3 means with respect to this is with respect to the e frame.

And therefore we need to, if we are work trying to work in terms of the theta 1 theta 2 how with respect to the orbital reference frame your system is getting disturbed what will be it is orientation with respect to the orbital reference frame that we will measure in terms of theta 1 theta 2 and theta 3 ok. So, this omega 1 omega 2 omega 3 must be converted in terms of theta 1 theta 2 theta 3 as we have done earlier also. So, let us convert that, so first I will simply a state those values and there after I will do that I have done it earlier also.

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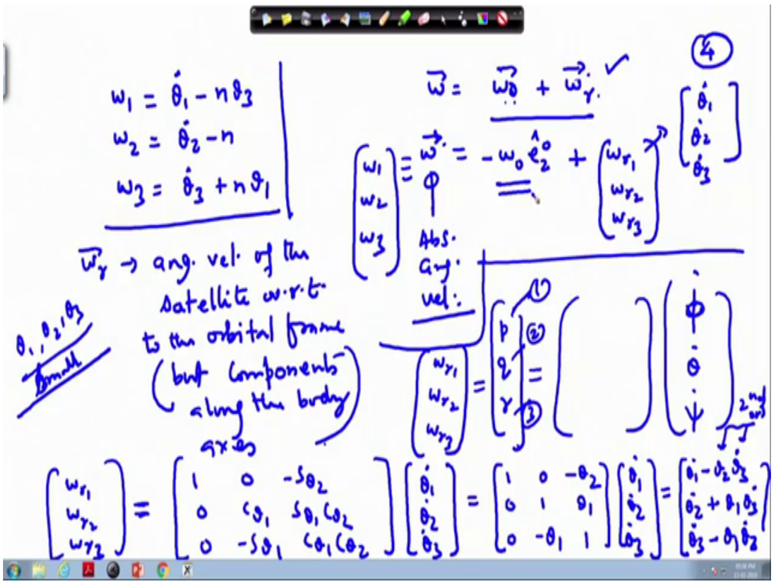

But again I will repeat theta 2 minus n omega 3 equal to theta 3 dot plus n times theta 1; and how we are getting? For this we are using the information that omega this will be equal to absolute angular velocity omega r plus omega 0, means this the orbital reference frame and or either we can write it this way, first we write omega 0 and with respect to orbital reference frame how your body is moving.

So, that gives it the absolute angular velocity; and what this quantity is? This is omega 0 times e 2 cap o with minus sign ok, here your angular velocity is directed along this direction which is the negative e 2 direction ok.

So, this will come with minus omega 0 times e 2 cap it is a pointing along this direction and what this quantity is this quantity is your angular velocity with respect to the orbital frame. But we will take it along the body axis ok, with respect to the orbital frame this is omega r 1 omega r is the angular velocity angular velocity of the satellite with respect to the orbital frame.

But components taken along the later on we have to do, but components along the body axis. So obviously that we are trying to measure in terms of we will write this as first as omega r 1, see in the matrix notation we can write as omega r 1 omega r 2 omega r 3. So, this the left hand side in the matrix notation, this can be written as omega 1 omega 2 omega 3 this is the absolute angular velocity, this is the absolute angular velocity.

So, we have to get up to this equation now omega r omega r 2 omega r 3 these are the body rates and they are connected with as we have derived earlier, like if you remember we have written the equation in terms of the phi dot theta dot and the psi dot and this we multiplied by a matrix and converted it to the corresponding p q r or we have use the notation for this perhaps omega r 1 or omega 1, I do not remember exactly.

So, this is the notation means this is the angular velocity along the body axis x body axis y body axis z body axis or either the 1 other the 2 and this is along the third direction and this matrix we have written here ok. So, this derivation I will not be doing here, I will just re call from the previous one, so here your omega r 1 omega r 2 omega r 3 this can be written as 1 0 minus S theta 2 0 C theta 1, I am repeating it there was no need to repeat, but because it is a not a book where I am writing here. So, it will be difficult for you to immediately go back to the lectures and look into that, so therefore I am just repeating it, C theta 1 times C theta 2 and this phi dot theta dot psi dot then we are writing as theta 1 dot theta 2 dot and theta 3 dot this is what we have done.

So if we assume a small angle, so this gets reduced to one will remain 1 this will remain 0 and this will be minus theta 2 this remains 0 theta cos theta 1 this becomes one for a small theta 1 theta 2 theta 3. So, theta 1 theta 2 theta 3 are small, so that approximation can be done this are small. So, this one gets reduced to theta 1 and this part is 0 this is minus theta 1 and C theta 1 C theta 2 that becomes 1 times theta 1 dot theta 2 dot theta 3 dot so this gets reduced to.

Now, we can see that we can write this as theta 1 dot minus theta 2 times theta 3 dot; theta 2 times theta 3 dot and the other part this we can write as theta 2 dot plus theta 1 times theta 3 dot and this one as theta 3 dot minus theta 1 times theta 2 dot. So, this is your omega r this quantity then it becomes ok, thereafter if you see this the these are the terms which are together these are the small terms ok, so this is the second order terms so we ignore it.

So, conveniently you can remember that omega r 1 omega r 2 can be replaced by this because this term are second order term. So, the they can be ignored ok, therefore your this part gets reduced by theta 1 dot theta 2 dot theta 3 dot and this part we have to write just use it write it properly.

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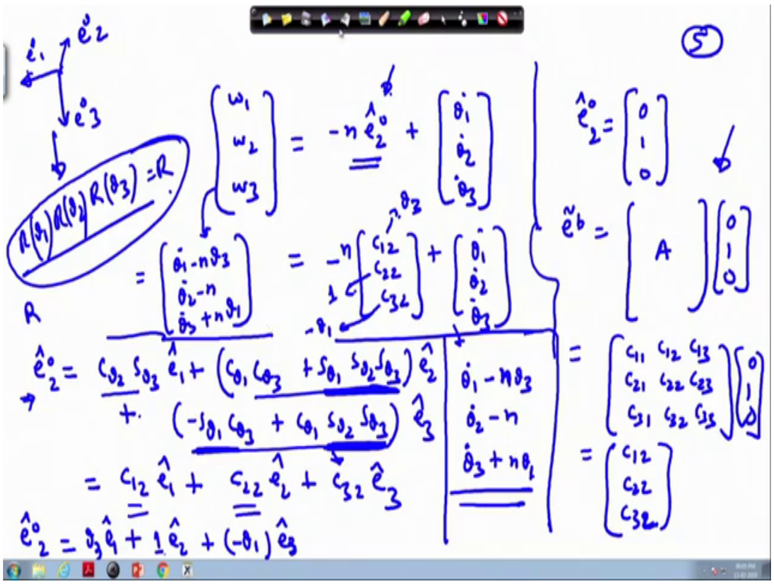

So, your then the omega 1 omega 2 omega 3 these vector can be written in terms of minus omega 0 times or we are writing in terms of n. So, this will write as n because n is much more convenient to write.

So, minus n times e 2 cap and then plus omega r 1 omega r 2 omega r 3 which is nothing but theta 1 dot theta 2 dot theta 3 dot, here e o 2 this is a vector 0 1 0 this a unit vector along the second axis of the orbital frame ok. So, e o 2 is vector is written like this, so that if we want to convert this into the body frame means what will be it is components along the body frame of this vector.

So, that will be can be written as along the body frame as here the attitude matrix times 0 1 0 and you know that attitude matrix it can be written in terms of C 1 1 C 1 2 C 1 3 C 2 1 C 2 2 C 3 2 times C 3 3 and then 0 1 0 ok. So, the corresponding components then will turn out as C 1 2 C 2 2 and C 3 2.

So, this factor then becomes minus C 1 2 C 2 2 and C 3 2 of the attitude matrix and attitude matrix how you are getting from the orbital frame e o 1 e o 2 and e o 3 you are giving 3 rotations, the first rotation you have given by r theta 3 the second by r theta 2 and the third by r theta 1, so in the sequence you are giving this rotation ok. So, this r matrix this is equal equivalent to r and from this we have to choose this values ok, so this times n and plus theta 1 dot theta 2 dot and theta 3 dot and I have also ok.

If so e if you get time go back and look into the lectures I have written this C theta 2 S theta 3 e 1 cap plus C theta 1, this is the how the body frame is associated with the your orbital frame. So, C theta 1 C theta 3 plus S theta 1 S theta 2 S theta 3 times e 2 cap and plus minus S theta 1 C theta 3 plus C theta 1 S theta 2 times S theta 3.

So, this e 2 vector is nothing but the quantity which I have shown here, and this is written in terms of e 1 e 2 and e 3 and that how you can get you look back into the lectures I have done this in quite details ok. So, I am avoiding it here, so this quantity is basically your the first term is C 1 2 e 1 cap plus the second term is C 2 2 times e 2 cap and the third one is C 3 2 times this is e 3 cap ok.

So, this term is here this term is here and this term is here, in this place and once we do the approximation, so in the approximation you can see that the e o 2 term it can get reduced to C o 2 and this will be 1 and this will be theta 3 times theta 3 times e 1 cap theta 3 times e 1 cap, the second term will then appear as this will be 1 and this is all the this becomes a third order term this particular part and therefore that drops out.

So, this will be simply 1 times e 2 cap and here again this term is the second order term S 2 and S 3 S theta 2 S theta 3 or sin theta 2 sin theta 3, so this term also drops out we are left with this which is minus theta 1, so this is minus theta 1 times e 3 cap ok. So the corresponding term then this becomes theta 3 this becomes 1 and this becomes minus theta 1 and then obviously we can simplify all this things.

So, I will right here in this place itself this result so if we take this part. So, this becomes theta 1 dot minus n times theta 3 theta 2 dot minus n and theta 3 dot plus n times theta 1. So, this is what you get as omega 1 omega 2 omega 3, so this quantity is nothing but your, I will write face on the next page theta 2 dot minus n theta 3 dot plus n times theta 1.

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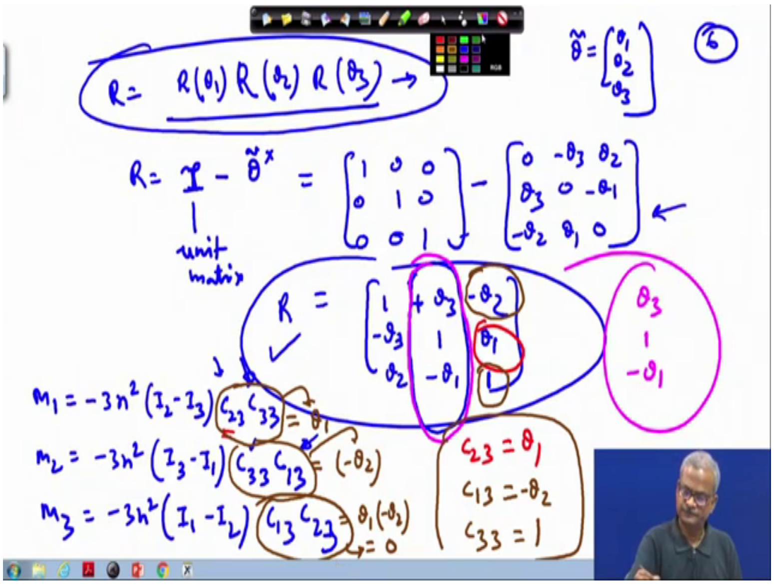

One thing also I will remind you that for a small rotation, so of the order does not matter, means the we have chosen the order choosing the order here r theta 3 we have use this order, but if the angles are small so this order is immaterial and in that case the r we have written as simply as I matrix or the I this is identity matrix unit matrix not the inertia matrix.

So, we can change the notation maybe and minus theta tilde cross where theta tilde is nothing but theta 1 theta 2 theta 3. So, instead of using that much of expansion we could have directly written in terms of this becomes 1 1 1 0 0 0 0 and minus theta tilde cross. So, that become 0 0 a this is the skew symmetric matrix so minus theta 3 theta 2 theta 1 theta 1 here minus theta 2 and this is theta 3.

So, this gets reduced to 1 minus theta 3 theta 2 with minus sign here this gets plus sign here and this is minus theta 3 and then this becomes 1 this will come with positive sign here this will come with a negative sign and this will become with a positive sign, so this is your R matrix. And this you can directly used to convert from the orbital frame to the body frame which we have already done here in this place ok.

So, this part we have already done here, n times theta 3 minus n times, so basically C 1 2 C 2 2 and C 3 2 where we are looking for which were nothing but the this part, this is your C 1 2 C 2 2 and C 3 2. So, this is the vector we have chosen here theta 3 1 and minus theta 1, so utilise it and write the equations; so, there after we can expand the equation.

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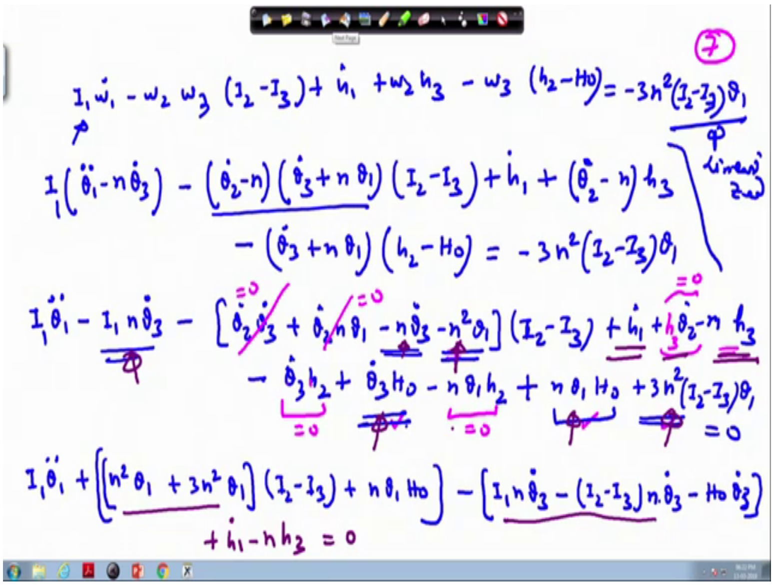

Once we have got this the rest of the things will be straight forward and also for the small angles similarly a in the inertia term this gravity gradient term will be getting terms like M 1 equal to 3 n square I 2 minus I 3 then C 2 3 times C 3 3 and M 2 equal to minus 3 n a square I 3 minus I 1 C 1 C 3 C 3 3 times C 1 3 and M 3 we get as minus 3 n square I 1 minus I 2. So, this will be C 1 3 times C 2 3 so this we have already done ok.

Now, we also need this quantities ok, so these quantities you can choose from this matrix here for simplicity because the angles are a small so C 2 3 it refers to your the terms we can choose directly from this place C 2 3. So, C 2 3 is nothing but your theta 1 ok, so C 2 3 this becomes theta 1 and here of course one term is missing, here there is 1 one once we add this we have added with minus sign this is got this term has got subtracted from this term. So, we are getting this so one is here ok.

So, this is this term is C 2 3 is theta 1 similarly we have C 1 3 C 1 3 C 1 the first row third one so this is your C 1 3 ok. So, C 1 3 is minus theta 2 and similarly C 2 3 we have already done C 3 3, so C 3 3 is nothing but your term which is present here C 3 3.

So, C 3 3 this equal to 1 and we can insert it here and rewrite our equation, so here itself let us reduce this part. So, this part will be C 1 3 times C 2 3 you can see C 1 C 2 3 times

C 1 3 this will be of second order. So, this quantity will be theta 1 minus theta 2 so this will be equal to 0 this is second order term. So, will neglect it C 3 3 is 1 C 1 3 is minus theta 2, so this becomes equal to minus theta 2 and this term C 2 3 time C 3 3 that becomes theta 1. So, this term is theta 1, this term is minus theta 2 and this term is 0.

So next we start rewriting our equation, so we have I 1 times omega 1 dot minus omega 2 h 1 dot plus omega 2 h 3 minus omega 3 h 2 minus H 0 on the right hand side we have minus 3 n square I 2 minus I 3 and the other term this is theta 1 this term ok. So, we pick up this and here we get as theta 1 ok, left hand side we have to rewrite because we have not linearized it so omega 1 is theta 1 dot. So, that becomes theta 1 double dot and then minus n times theta 3 ok, so that becomes n is a constant so we will not differentiate this this is theta 3 dot minus omega 2. Similarly this is theta 2 dot minus n and this becomes theta 3 dot plus n times theta 1 which is omega 3 I 2 minus I 3 times this plus h 1 dot and plus omega 2.

So, omega 2 again this is theta 2 theta dot 2 n times h 3 minus omega 3 which is we have to expand it, so omega 3 is theta 3 dot plus n times theta 1 times h 2 minus H 0 this equal to minus 3 n square I 2 minus I 3 times theta 1, already we have linearized this is the linearized part; is the linearized part ok. So, this quantity we have written here so breakup and do the approximation this type of work already we have done this is I 1 ok.

So, I 1 times n times theta 3 dot here this part we will expand it, so minus theta 2 dot theta 3 dot n times theta 1 minus n times theta 3 dot minus n square times theta 1 times I 2 minus I 3. So, this terms are old terms what we have discussed earlier they are the same term, only thing this h related term they are now appearing as the extra one, theta 2 minus n times h 3.

Here also we can break and if we break it so this will be theta dot 3 times h 2 plus theta dot 3 times H 0 minus n times theta 1 h 2 and this makes it minus n times theta 1 H 0 and this minus minus that gets plus sign. And then this term also we bring on the left hand side I 2 minus I 3 times theta 1 this will be equal to 0.

So, here the second order terms will be dropped out, so this term is 0, this being second order this term is dropped out being second order and the terms like this will be equal to 0 why because h 2 is initially 0 and then you are speeding of the wheel.

So, this quantity turns out to be a small so this we set to 0 similarly theta 1 times h 2 will set it to 0 and here H 0 is large quantity. So, this quantity will stay they this will stay here in this one also we have to break up. So, theta 2 dot times h 3 so I will break and write it here in this place. So, theta 2 dot or h 3 times theta 2 dot and then n times h 3 ok. So, similarly this term this is 0 this particular term this term will stay.

So, those you can see that this n square we are not neglecting why because this is not a variable term, your variable terms are theta 1 theta 2 theta 3 h 1 h 2 all those things not the n term neither the H 0 term ok. So therefore, if we rearrange the equation so this will be I 1 times theta 1 double dot and then we are start collecting the terms related to theta 1. So, there after the theta 1 dot term we will collect here; so theta 1 dot it appears theta 1 dot term is not here. So, we have the theta 1 term is there so theta 1 term will collect. So, this is your n square minus n square theta dot and this is minus sign here so that gets it to plus sign n square theta 1 times I 2 minus I 3.

So, we will take it this gets the plus sign and then from this place we have then this term plus 3 n a square 3 n a square times theta 1 and outside the bracket we have I 2 minus I 3 and this will turn up with a plus sign here in this place and somewhere. So, rest we have the theta 1 term available here the theta 1 term is one more term is in this place. So, this is, so will put it together n times theta 1 H 0 ok, then the theta 3 dot term which are present here that we will collect, this one.

So, we get the terms like we have the minus sign if we keep it outside, so this becomes I 1 times n times theta 3 dot and then from this place we have theta 3 dot term. So, minus minus that makes it plus so plus and minus sign we take it outside. So, this will come with a minus sign I 2 minus I 3 times n times theta 3 dot and then we have theta 3 dot and theta 3 dot is also here, so this comes with a minus H 0 because this is plus here H 0 times theta 3 dot theta 3 dot. So, this way we have to rearrange the terms.

So, we have taken care of the following terms, we have use this term and n times theta 1 H 0 we have used this we have used this, this one we have used this also we have used and this also we have used. So, we are left only with h 1 dot this term is already 0 this part this one we are left with this part is not included because they are 0. So, we are left with plus h 1 dot minus n times h 3 and this equal to 0. So we need to rewrite this equation in a compact format, so we will do it on the next page.

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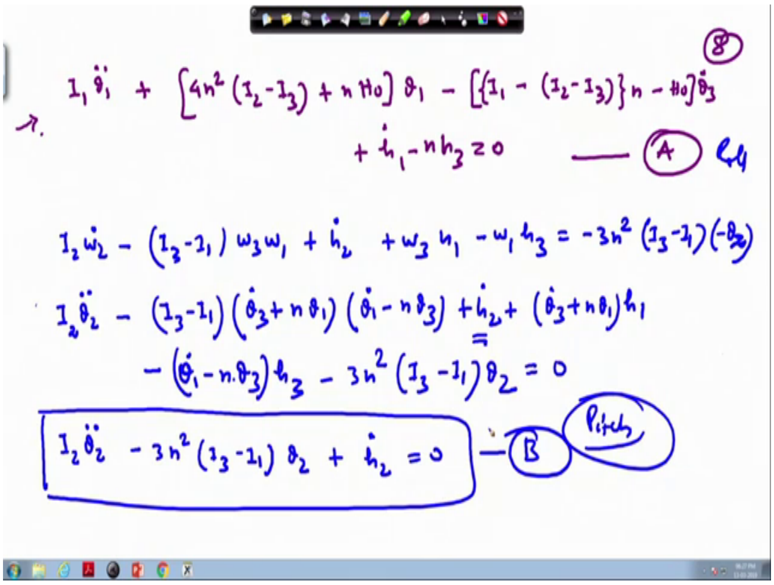

So, the first term is I 1 times will have to go back and look into all these equations, this term we will take it outside the n a square term from this place and then write as this basically becomes 4 n a square theta 1, so theta 1 we can take it outside ok. So, 4 n a square so plus 4 n a square I 2 minus I 3 times theta 1 going outside I 2 minus I 3 then n times H 0 plus n times H 0 theta 1.

So, this is the second term and then the third term we pick up from this place theta 3 dot we can flatly take it outside and inside we have I 1 n so this term we can combine together. So, that will give me I 1 minus I 2 minus I 3 times n times n and then minus h 0, minus H 0 times theta 3 dot and the other term we have h 1 dot minus n h 3, so plus h 1 dot minus n h 3 this equal to 0.

So this constitutes our equation for the related to theta 1 ok, the same way we have to write for the other one also ok. So, this equation is fine, now similarly we pick up the other also. So, I 2 times omega 2 dot if you follow this notation and expand and work like this, so the you can derive all this things this is the circular equation we have written earlier h 2 dot plus omega 3 h 1 minus omega 1 h 3 this equal to minus 3 n a square I 3 minus I 1 minus theta 2. So, same way insert, I will take some shortcut step and leave it to you to work out this things. So, omega 2 dot will be theta 2 double dot minus n dot. So, n dot term will become 0 so this gets lost from this place.

So, we just get here I 2 times theta 2 double dot and then of course we have the other terms theta 3 dot plus n times theta 1 and this is theta 1 dot minus n times theta 3 plus h 2 dot plus omega 3 is theta 3 dot plus n times theta 1 times h 1 minus omega 1 h theta 1 dot minus n times theta 3 and then multiplied by h 3 plus 3 n a square and this is theta 2.

So, minus minus that gets plus so we will have a minus sign here in this place and this will be equal to 0. If you have any extra moment like the magnetic moment, so will remove this we will simply put here the magnetic M magnetic plus M aerodynamic and so on and this also you need to linearize, but what function they will appear it depends on that, so you need to rewrite in this way ok.

But for as for as the gyrostats are concerned, so here in this case the gravity gradient obviously they will appear and we are these terms will act as the actuating term, while you we try to do carry out the control ok. But currently we are going to write in this way whatever is written here so if we rearrange it ok, so I hope that you will be able to do the rearrangement I am skipping those steps and follow the approximation we have done this here the variable term theta 3 times h 2 and so on those terms will be 0.

So, those kind of terms you can delete so if you do that, so this will get reduced to minus 3 n a square I 3 minus I 1 times theta 2 plus h 2 dot. So, this is your equation number b so this is related to your pitch dynamics ok, the first one was this one is related to the role dynamics this is related to the pitch, in the same way the third equation can also be worked out.

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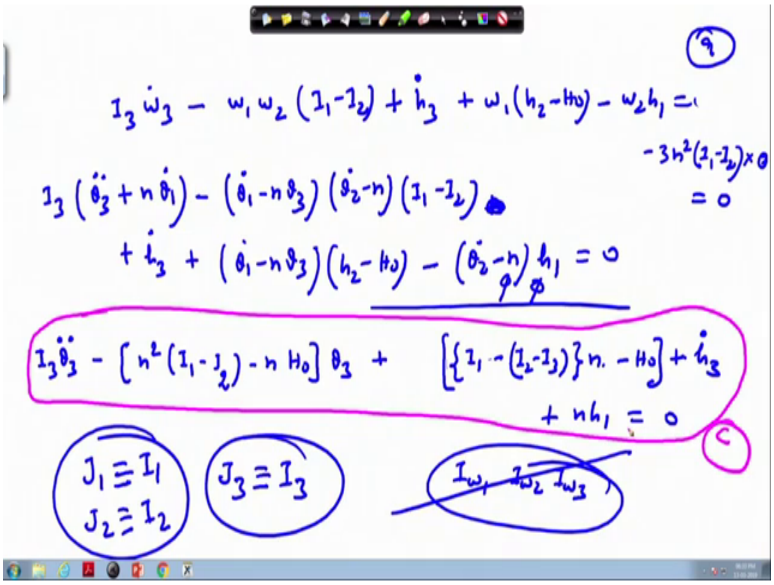

So, the third equation is I 3 times omega 3 dot minus omega 1 times omega 2 and then we have I 1 minus I 2 always right like this. So, that it is convenient to work with h 3 dot and then ok. So, for in this equation the other terms also we have to write. So, those terms here omega 1 times h 2 minus H 0 minus omega times 2 times h 1 and on the right hand side we have the gravity term.

So, the gravity term we need to write which is minus 3 n a square I 1 minus I 2 and then this is multiplied by 0, so this equal to 0. So, right hand side here in this case it become 0. So, with this simplification so this term here it is a getting out ok, so rest we need to insert the values for.

So, here this will be theta 3 dot plus n times theta 1 dot, then omega 1 we are replacing by theta 1 dot minus n times theta 3 and this is theta 2 dot minus n I 1 minus I 2 plus plus h 3 dot plus omega 1 is theta 1 dot minus n times theta 3 times h 2 minus H 0 theta 2 dot minus n times h 1 this equal to 0. So, this is the equation we get here in this case.

So if we again expand it and rewrite this will get reduced to theta 3 double dot theta 3 plus n times I 1 minus I 2 or I 1 minus I 2 minus I 3 this multiplied by n minus H 0 plus h 3 dot plus n h 1 which is the last term here this and this will be equal to 0 ok. So, usually ones you are writing this equation so many of the books they will write it this way, the minus sign will taken out of this and here it will be made as a plus sign ok.

So, instead of using here like the minus sign if you write in terms of I 1 minus I 3 so the this gets in terms of a plus sign. So, these are the some of the common changes it is a done, so what you can do that I 2 this is I 2 ok. We are started with somewhere J 1 J 2 J 3, but we have ultimately converts to I 1 I 2 I 3, now I cannot change it because I forgot while writing ok.

So, there is no way of rewriting all those things like here we were writing in terms of J 2 J 3, so for I will mention here explicitly that J 1 is identical to I 1 J 2 is identical to I 2 and J 3 is identical to I 3 for the wheels you can use the notation like I w 1 I w 2 I w 3, so this is your the third equation ok.

So this is the way we get all the linearized equation and this linearized equations are done for the local control not for the global control. So, will continue in the next lecture so we have finished the Gyrostat and something I wanted to cover I told you that in the moment angular momentum term, the last term there was perhaps I have included that in my lecture or not I have done that or I do not remember that.

So therefore, I will include that as a either as a supplementary material or either I will give as the assignment, so that once the solution to the assignment comes you can look into that solution. So, next time we are going to a start with the control moment Gyros, so here reaction wheels a in the further gyrostats this reaction wheals are being used for controlling the satellite.

Similarly the control movement gyros can be used to control the satellite and they have their own adventures, so if you so we are going to discuss that in the next lecture not today, because it will take time we will continue in the next lecture.

Thank you very much for listening.