Satellite Attitude Dynamics and Control
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Lecture - 61
Satellite Dynamics with Control Moment Gyro (Contd.)

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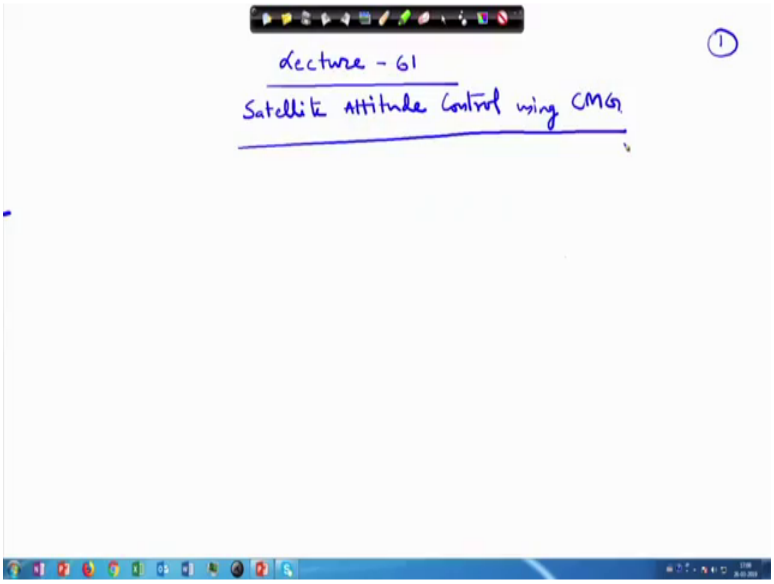

Welcome to the lecture number 61. So we have been discussing about the Control Moment Gyros. So, we will continue with that topic.

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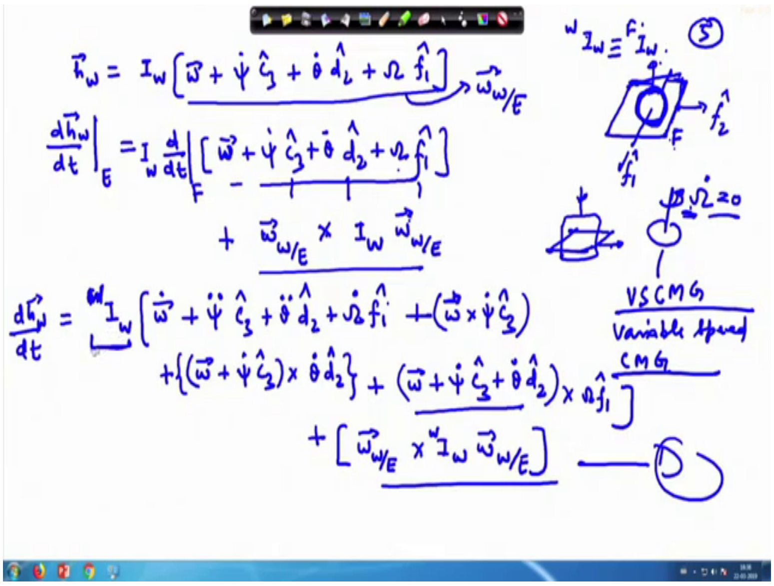

So, I w times this becomes omega dot and then for all of them first we write here psi dot time c 3 cap theta dot times theta double dot time; d 2 cap and capital omega times F 1 cap and plus then we have to differentiate this quantities for the c 3 d 2 and f 1 cap.

So, as per our earlier discussion this psi dot times c 3 cap will change at, c 3 cap is fixed in the body and therefore, it is changing at just omega. So, this is psi dot times c 3 cap plus theta dot also we have looked into the earlier part, this will be omega plus psi dot c 3 cap cross theta dot d 2 cap ok. Next you are the last part which is adjusting here. So, that will change because of the omega plus psi dot c 3 cap plus theta dot d 2 cap ok. This becomes the angular velocity of the wheel with respect to the satellite, this particular part and cross capital omega times f 1 cap. And once we close this bracket, the last part that remains is this one so, we copy it here omega w slash E plus I w omega W slash E.

So, these are the three equations of motion for the satellite which c m g, So this is your equation number earlier we have written c so, this becomes equation number d. So, this is the wheel, it is defined in the wheel frame or either in the wheel or w whichever you write F or W both are ok. What is if; now, you have to add all the four say you can consider that how complicated this equation becomes and then you have to take care of the other things here.

So, it is appears to be a formidable task, but let us first do something. We are not going into the controls of the c m g in this long format. It has no way we can discuss it here because it is a very long and we will not get any benefit out of that. Once you learn this part, then you can start doing on your own ok.

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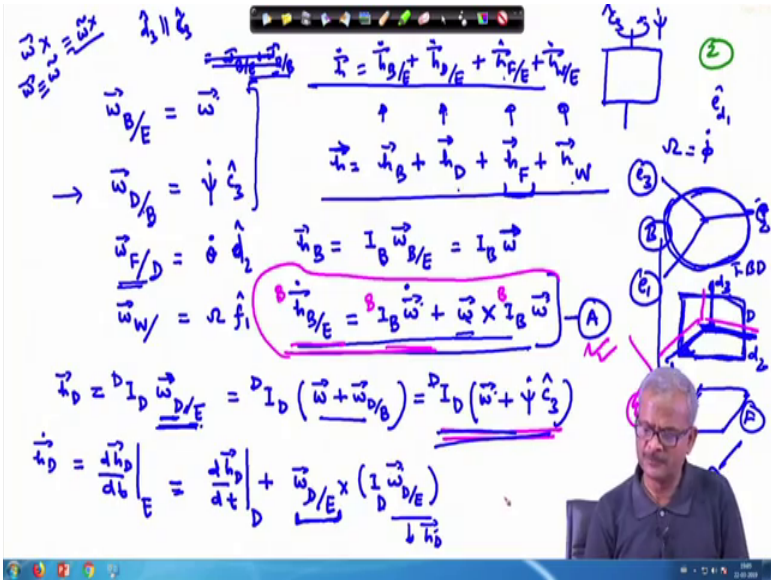

Thereafter we have to add all of them, but before this we do little bit of manipulation ok. So, this is the first equation we have written, equation number A. So, all these equations we have to write it in a little special way to get to the solution, but before this I want to discuss something with you. Now you can see that this is the moment of inertia of the wheel defined in the wheel plane or in the F plane.

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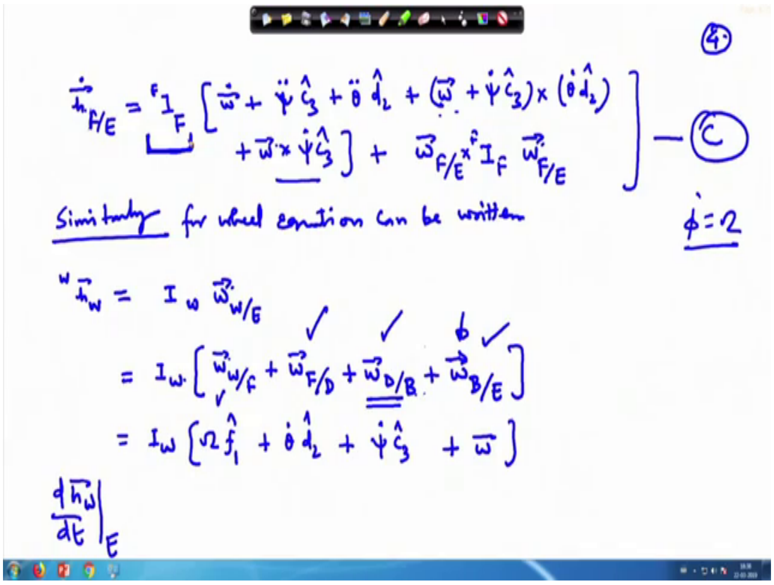

Similarly, this is the moment of inertia of the F frame defined in the F frame itself.

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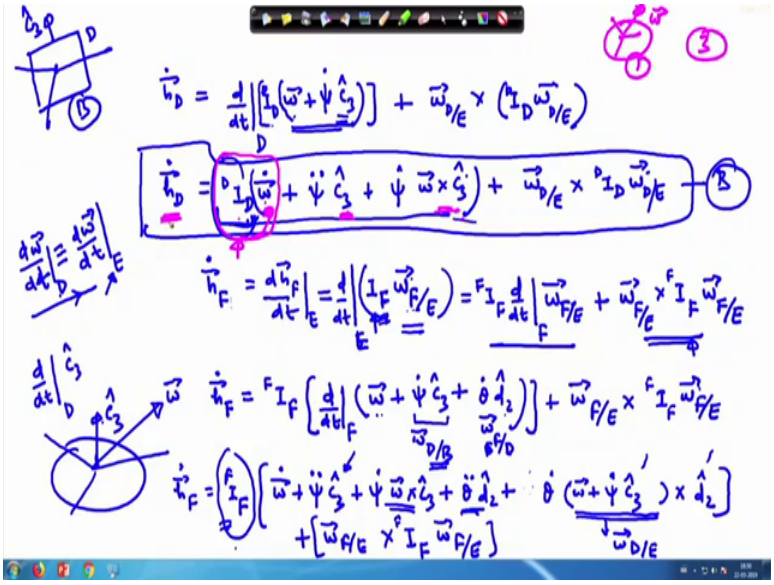

If we come to this place, here this is the moment of inertia of the D frame. It is a defined in the D frame.

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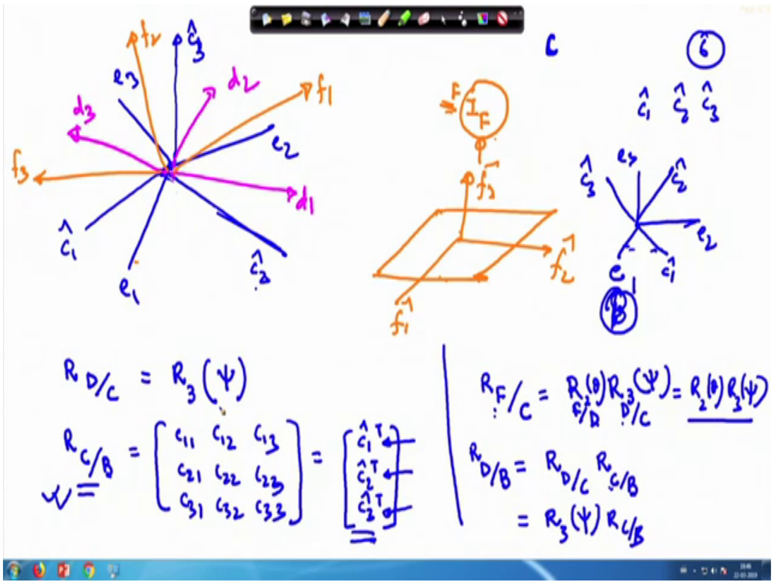

So, what we see that, my body frame is located here. This is e 1, e 2 and e 3. With respect to then we have the, somewhere the even frame here in this directions c 3 cap and c 1 cap, c 2 cap these are located ok. This center they are matching it is a little poor figure with respect to this c 1, c 2, c 3 your d frame is oriented, ok. So, d frame is oriented somewhere say for that we can draw it by other color. So, this is your say the d 1, this is d 2 and this is d 3 and then with respect to the d frame your f frame is rotating. So, with respect to this d 1 frame, then f 1 frame is defined. So, this is f 1 f 2 and f 3. So; basically we can see that the moment of inertia which we have written as I F F, the moment of inertia of the F frame in the F frame itself because in that frame only in the F frame only the moment of inertia of the F frame which is the horizontal frame which we have shown as f 1 cap, f 2 cap and f 3 cap.

So, as this frame rotates, so this frame is also rotating along with this. So, only in this frame its moment of inertia will not change other. It will change in the e frame ok. So, we cannot just simply add the whole thing and get the result. We have to do the conversion for that and for that we need a proper changes. So, we write here first we have the orientation of the D frame with respect to the C frame ok. So, C frame is a non-rotating frame and it is a, that will be given by rotation about the 3rd axis and this is done by psi. So, we write this R 3 psi.

And your C frame is oriented with respect to the B frame and this orientation you can write as R C B which we can write c 1 c 1 2 c 13 c 2 1 2 2 ok. So, c 1 frame it is basically if defined by your c 1 cap vector, c 2 cap vector and c 3 cap vector, so we can write in terms of this c 1 cap, c 2 cap, c 3 cap, these are the vectors. If this is your frame e frame and with respect to this somewhere your e 1 e 2 e 3, this is of your of the body B or with respect to this c 1, c 2, c 3 is located. So, this is the c 1 cap; so, c 1 cap has certain direction cosine. Isn't it? That comes here in this place. Similarly the c 2 cap it has certain direction cosines. So, that enters here. Similarly c 3 cap has certain direction cosines so, that will enter here in this place ok.

So, similarly we can write here the rotation of the F frame with respect to the C frame. C frame is a frame which is fixed in the body and so, your C frame orientation we have written with respect to the body which we have written here in this place, ok. So, with respect to the C frame the F frame rotation. So, that will be given by two rotations, one rotation first we have to do F with respect to the D frame and then, D with respect to the C frame. So, with respect to C we rotate the outer frame. So, this is the rotation and that rotation you are giving about the 3rd axis and thereafter from D frame you are going to the F frame. So, this rotation is about the two axis of the D frame. So, here you have this is R 2 and this you are rotating by theta so, R theta psi. So, this rotation is basically this you can write as R 2 theta and R 3 psi.So, this becomes R F C.

And R D slash B, the orientation of the D frame with respect to B frame. This can be written as R D orientation with respect to C frame and then R C orientation with respect to the B frame. So, this is your R D so, R C we already have written in this place, ok. So, that we will not change the notation, but his part R D slash C, this is R 3 psi. So, this way we can write the all the equations. .

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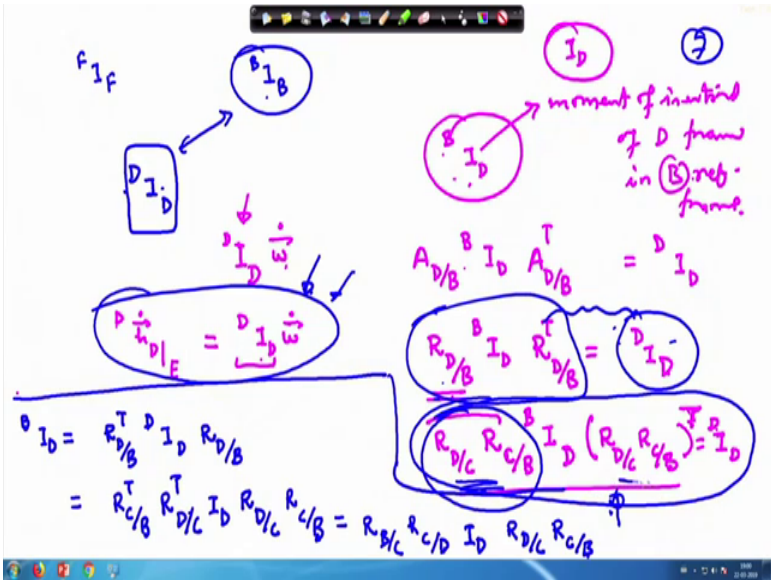

Now, the equations already we have derived, however as we have seen that the moment of inertia we are of the F frame we have defined in the F frame itself ok, while for the moment of inertia of the body we have written along the body axis. So, all this equation A, B, C and D this is in a different frame; this is in obviously this we have done this is in a different frame ok.

So, if they are in different frame, we cannot just add them. We need to convert them into the same frame and what is the idea that we convert them all of this moment of inertia and these vectors also to the D frame itself. So, this is here theta dot we write it together theta dot. So, this vector C 3, D 2 cap, C 3 cap they are appearing and this moment of inertia, they need to be converted into the B frame which is our here in this case, the main body which we have written by a small a 1, small a 2, small a 3 ok. So, this conversion, after doing this conversion only will be able to do something.

So, for converting that we need the following things say, we have the; somewhere we are getting the moment of inertia of the say D frame, in the D frame itself this is the term we have got ok. Now, this term because we have to make it compatible with this. So, I need to convert this particular term into this equation. So, for that reason I will copy just one part and show you how we do is I will pick up this equation. So, this is I D D these are all I D D, I will pick up this part this has been expanded later on and written here as this equation ok.

So, here the D I D what you have written so, this is the moment of inertia of the D frame in the D frame itself and we need to convert this into the D frame. So, I will take this particular term which I am marking here this one, rest other I will not consider because then it is a formidable task to do it. My idea is you have to explain the principle ok, but we cannot do this as a class room problem in a very extensive way to this extent we can do there are after we have to close, it is a matter of the research.

So, once you are doing the research problem we can you can take it out. So, if I pick up this term D I D so, here I D and this is D and then we have omega dot. Now also one thing you have to consider that this omega dot though we have written here using this notation, but this is also this the omega dot means the omega vector at which your this body is rotating, but omega dot component we are taking along the D frame; this is written in the D frame while you are writing here in this notation. So, that means, all these components you have to write along the D frame. And if you try to write in the B frame, then you have to convert all of them into the B frame, this is the issue here.

So, therefore, this all of them will be converting into the D frame. So, I will show it for one of them. So, say this is the particular part. So, you have here on the left hand side, we have h dot D this is with respect to the E frame, but all the components of this we have taken along the D frame itself. This is with respect to the e frame, but the components are taken along the D frame because this is the way your I D is appearing and because of that it will be in the D frame the components.

So, let us pick up only this term and leave the others, otherwise it will be too long to work here. Now, if I need to change this into the D frame, so what I need to do? So, say the I D B is the moment of inertia of that I D, this D frame moment of inertia, this is the moment of inertia of the D frame in D frame B reference frame.

So, how we are going to convert this into the D frame so, this is the moment of inertia of the D frame in the B frame. So, if I have to convert this so, how is we are going to convert as you remember that we are use the attitude matrix ok. So, for converting this from B frame to D frame so, that it will come to this so on the right hand side let us write this as I D D, ok. So, this is the attitude matrix or the rotation matrix.

So, we have to go from B frame to D frame and B frame to D frame this transpose or if you write simply in the rotation matrix terms, so this is RD slash B I D B this is transpose. So, this will be D I D, but this quantity we have to insert here. And for this, what we have written that first rotation which of the going from B frame to C frame and then going from C frame to D frame.

So, this is the equivalent rotation here and then I B D and similarly here we have R D C times R C B, this is transpose I D D. So, this way we have to convert the matrices for from one frame to other frame. So, I will write here in short and this we are going to utilize here in this place. This the only one term we have picked up. So, I will work on this, but before just let me write all those terms.

So, I D in the B frame, then will be written as if you write from this place. So, just you have to reverse the whole thing. So, this term you have to take it on the right hand side. So, the first term will appear as R D slash B transpose D slash B. If we multiply it by a transpose and here by R so on this side also. So, we get this equation and because it is a rotation matrix, so transpose times that the rotation matrix that becomes unity matrix.

So, we get like this and obviously, then you have to put here D slash B; already you know D slash B is this quantity R D slash B. So, you need to insert here in this place. So, if you put it, so that becomes R C slash B transpose times R D slash C transpose I D, then we have done we are I think I B D. So, this we have taken here this is fine. So, this is transpose and then simply we copy this R D slash C and R C slash B. And then, this we can rewrite as this can also you can write as this will be nothing but B slash C, R C slash B transpose is nothing but the reverse direction R C slash D ID and R D slash C and R C slash B.

Now, we pick up this particular equation and then work on this.

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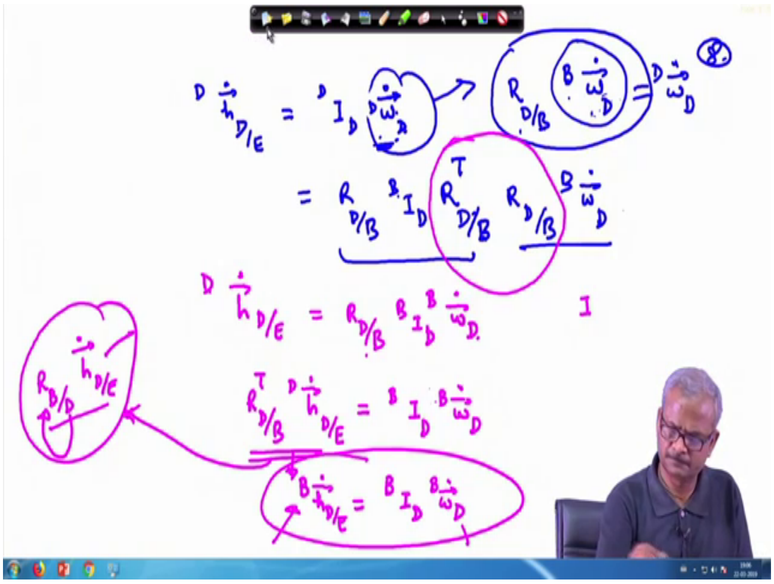

So, we have h dot D and this we have done with respect to E and this is in the D frame itself on the right hand side we have I D D and then omega dot, but it is taken along the D direction. So, we need to convert all of them into the B frame, only one term I am picking up as I have told you,. So, from the previous one you just pick up. So, this quantity is written here in this place which we have expanded here in this place. So, we just need to place it there. So, I will use a shorter notation, I will use this part; this part not this part because this is here two matrices are involved. So, it will not be that convenient. So, we will pick up and then write it there.

So, I D D can be replaced by; I D D This can be replaced by I D B and this multiplied by R D B slash B, R D slash B, D slash B and then R this one is D slash B transpose. Now, look into this part this is also in the D frame so, we need to convert into the B frame. So, let us say this omega dot has component in the B frame if I write like this I B. This is the component of B; D frame in component of the D frame in this notation we can chose to this I put here as I D T so, then it will be; so, this is omega dot D let us assume that.

This is represented as this vector in the B frame. So, from the B frame once we are converting, so how we have to write? We have to use the same thing R D slash B and here R D slash B transpose. So, for that we will insert this quantity here. So, R D slash B B omega dot D R ok only one part this is only this not a moment of inertia vector this is just a simple vector this not moment of inertia matrix. So, we do not need to do this exercise this is not required here, this is the only one part is required. So, this part we just remove it, ok. So, this converts it from, if this is the corresponding vector or the D vector in the B frame, so this we are converting into the D frame. And this then comes equivalent to or the equal to omega dot D in the D frame.

So, you replace this quantity here so, I have inserted here. Now, the quantity which is present here this is just a unit matrix ok. So, this becomes R D B slash B I D times omega dot D and on the left hand side you have h dot D slash E and this is D. Now, if I take this on the left hand side C multiply by R D T transpose; so, R D slash B transpose here and then H dot this is in the D frame, D slash E this becomes equal to I D B times omega dot D in the B frame, this is for omega D; this is in the B frame. What this quantity is? This quantity is we can simplify as we can write as R B slash D transpose that gets changed to this notation D slash E.

So, this is nothing but it is converting from D frame to B frame ok. So, that means this quantity is converted to D frame to B frame. So, left hand side then we can write as h D slash E dot it has been now converted to the B frame and then you can see the things within gets simplified so, this is the whole conversion. So, you can see that now the angular velocity of the D frame it is expressed in the B frame. Now, it is ready for addition. So, if you do this operation for first one does not require any change because that is for the main body. The first equation is here in this place A, this does not require any change because already this is in the B frame, these are already in the B frame, this is also in the B frame.

So, here I do not have to do anything with this one, but for rest of them you have to do these changes. In the; you have to convert in the B frame to get an equation which will be useful for your control purpose, control design purpose ok. Now once we have done this part, so the next part is to rewrite these equations.

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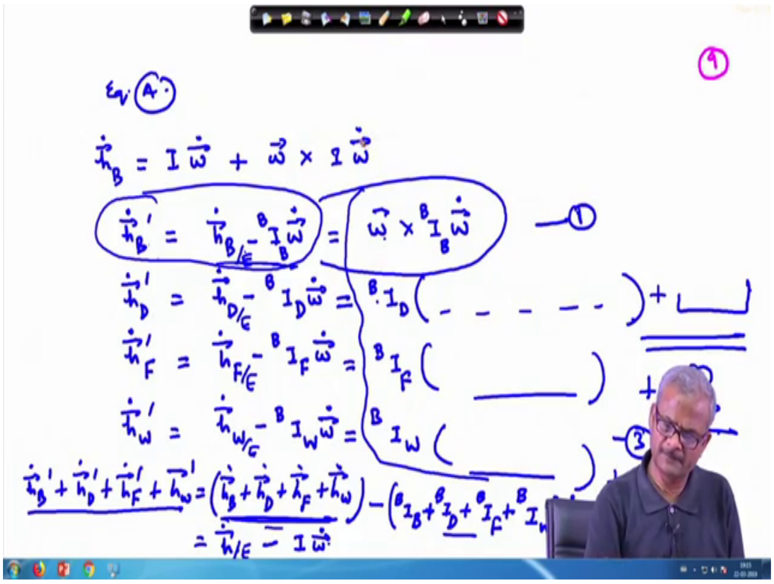

So, the equation A, now we can rewrite this as that was for h body, h dot body this equal to I times omega dot plus omega cross. We define a new quantity h B dot prime and write this as h B dot minus I omega dot and this equal to then right hand side becomes I times omega dot. Let us say this is equation 1.

Similarly for the once you have done this conversion, you have converted all of them into the B frame ok. So, this is I B in the B frame itself, this is I B in the B frame itself. So, with this conversion then there after we take the D frame and there also we can write in the same way the left hand side was and these are with respect to; this is with respect to the E. This is the absolute angular momentum we have written. So, I just want to drop this E to not to carry this thing. It is complicating the whole issue ok. So, this way it looks little simple to work with otherwise to many subscripts are appearing.

So, this also we write as I D and once you have converted into the B frame times omega dot, already I have shown you that it can be converted and rest of the terms then appear on the right hand side. So, we have to go to the equation B and look into those terms.

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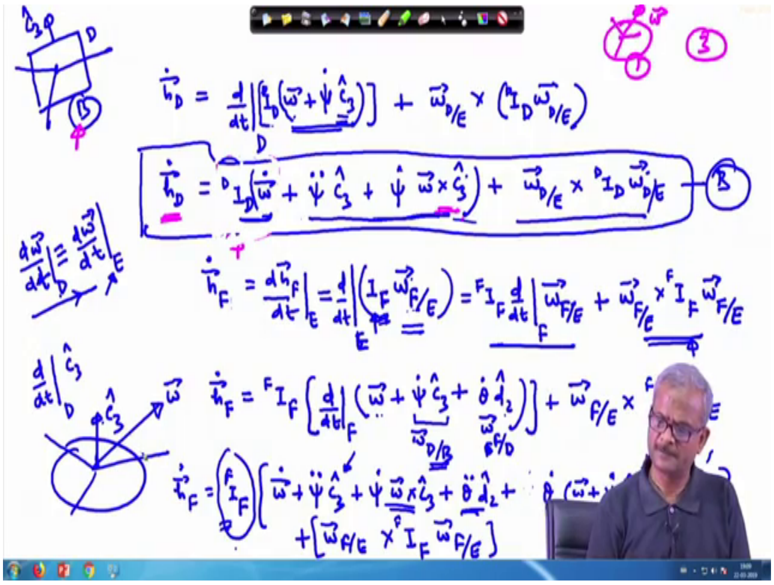

So, here this part, here from this place we have taken this part only to the left hand side. On the right hand side all this terms will be present. So, the other terms then can be written as. So, that we can write here as I D and already we have converted to the B frame so, this is I D times rest other terms and then your the cross product term is there. So, all of the terms will come here in this place and this I term as equation number 2, I am not writing very long and it is not required here what I am going to do.

Similarly, h dot prime then we have the F frame that also we write as h F dot and then I h F this has been converted to the B frame by the operation we have written and then omega dot; means, we go to the equation number C, this the equation number C. So, here this is the omega dot term. So, this will come on the left hand side, rest other terms will remain on the right hand side and that we are defining as h F prime. So, here then you have the I B F and all other terms and plus the cross product on which is there. So, this is equation number your 3.

And lastly you have the wheel equation. So, this also you write as h dot wheel minus body I wheel omega dot and along the same line, this is body and same way the other terms here and this is your 4th equation. So, once we add all the four, what we get here h B dot prime plus h D dot prime plus h F dot prime plus h w dot prime this equal to the quantity which are here h B dot plus h D dot plus h F dot plus h w dot minus the quantities which are present here. So, this is simply I B B times plus I B D plus I B F plus I B W times omega dot, ok.

This term we are leaving it here, we are just adding this. This particular; this forms one equation, once we have taken here it on this sides. So, we are defining h B prime equal to this quantity. So, we are adding this and the right hand side so, right hand side is here. This quantity is nothing but your total h dot because these are the with respect to the inertial frame; these are with respect to inertial frame. So, this is your with respect to the inertial frame total h and rest this quantity is I times omega dot.

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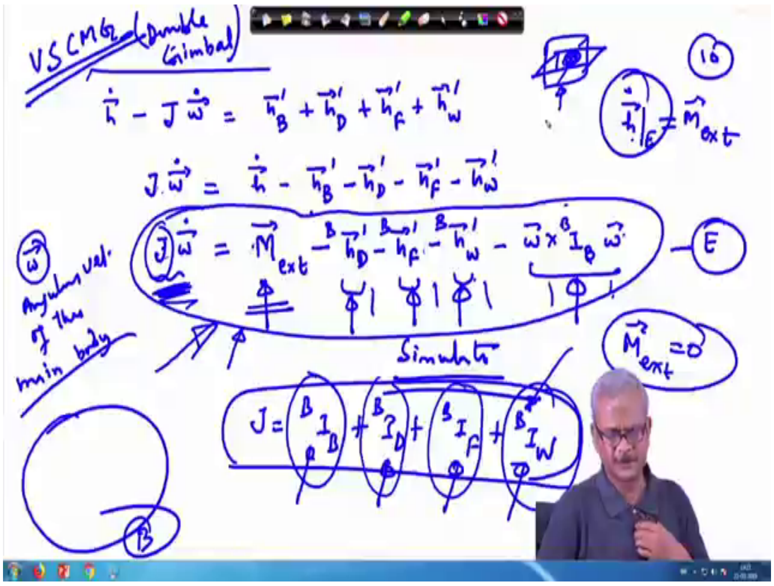

So, we go on the next space. So, what we have got here, I times omega dot and then h dot minus I times omega dot, this quantity is equal to h B prime h D prime all these things. So, this is h B prime plus h D prime plus h and then F prime, little bit of rearrangement of this is required. This is the angular velocity of the satellite as we have said earlier omega is the angular velocity of the main body which is nothing but your satellite bus or the outer portion which we have indicated by B.

So, if we rearrange it, so we can write this as h dot minus h B prime minus h D prime minus h F prime minus h W prime and what this quantity is, as you know this is the total rate of change of the angular momentum of the satellite with respect to the E frame. So, this quantity is nothing but the M external. So, we can write here this part as M external and minus, then we write h D prime minus h F prime minus h W prime and then h B prime is there. So, h B prime we have to take care of that, your h B prime if you look here in this part.

So, this is just quantity omega cross I B B omega dot. This is not dot here, this part we have done the wrong thing written this dot is only here not here in this place.So, h B prime then we replace by omega cross I, this is the main body; this is the main body here I B B omega.

Now in this format say, this is equation number we write as E. So, if your M external is the force acting on the satellite and this quantity is as we have defined here, they are all converted in the terms of the, these are all converted ones expressed in the body frame. So, once you have converted here in this format and this I we have written here, this is the total I B we have used for that. So, this I will use the notation J for that soo, I will use notation here J instead of I so, I will write here J. This is the total moment of inertia of the whole body.

So, this case here this also I will change

Student: sir

Ha.

Student: [FL]

[FL].

Student: [FL]

[FL] wind up [FL]

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Ha ha [FL]

Student: Continue.

So, we have changed this. Now you can simulate this using the matlab you can write your equation. So, these are all converted in the body frames. So, these you will know from your numerical calculation ok, these quantities are known to you of the main satellite body this quantity is known and this is the total moment of inertia which I have written just now. The I B B plus I D B plus B I F and plus B, I here the wheel. So, this quantity will be known to you by conversion.

All this is known from conversion remember, these quantities are known from the matrix conversion. This part is already known to you, but these are coming from the matrix conversion. They are not in their own frame. Remember we are converting and therefore we are getting this.

So, put here in this and then you can do the simulation and this gives you the exact equation of motion for the satellite for simulating it, provided you calculate all the terms and put it here. All these terms are to be calculated and this is the external moment acting on the satellite if M external this quantity is 0. So, just set it to 0 and then you will get the result out of this. Say it is a very long exercise and there after you go for the control. So, the real trouble will start, then you have to add up all the equation and work with them, simplify them and then write in a particular format, then dividing the controls.

So, you can imagine that how complicated the whole equation will become, but these are the real things which goes into the actual engineering. They if you large satellite like the I S S, International Space Station and you are trying to control that you need the gyros and not one you will need many. There are four gyros over that; double gimbal gyros. And here this is coming little more complicated because we have taken the variable split control moment gyros.

So, this whole exercise we are doing for variable split control moment gyros V S C M G and this is double gimbal;. obviously double gimbal and not single gimbal; double gimbal. So, this completes the equation of motion simulation and here at least this part if you want you can code in the matlab you can check your things and some remaining part like the kinetic energy calculation, how to verify the your the simulation your doing its s running fine means you have done the correct simulation.

So, for that I will cover in the next class, but I am not going to cover any more controls on this. For the simple thing that we have discussed that the outer frame and the inner frame they had the 0 mass and only the wheel is having certain mass. So, for that case I will take and that for the control design we will not do. I will show you the control block diagram and if you are more interested you can look into the book by Marshall JCM and another book by (Refer Time: 46:28) which already I have given you the name in the last class. So, thus the today work it is completed and will continue in the next lecture.

Thank you very much.