Satellite Attitude Dynamics and Control
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture – 71
Atmospheric Force and Moment on the Satellite

Welcome to the lecture number 71, we have been discussing about the Force and the torque due to the Atmosphere on the Satellite ok. So, today we will continue with that and the there are forces also due to various other factors. So, magnetic moment already we have considered. Due to solar radiation also, the force action the satellite and also it results in the aerodynamic torque, but the analysis for the solar radiation torque is along the same line as we are doing for the aerodynamic moment and the forces. So, I will do if very short discussion on the solar radiation pressure and rest of the materials will be uploaded because, it follows the same trend as the analysis for the aerodynamic force and the moment.

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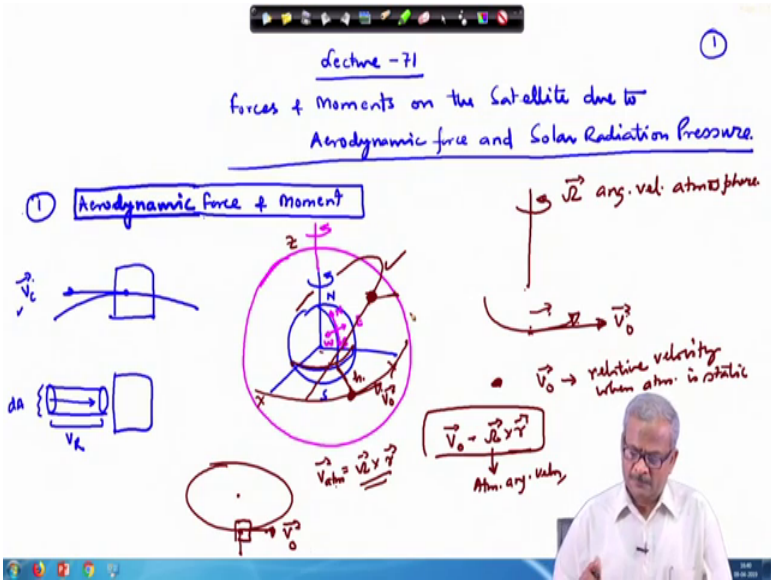

So, let us continue with what we have been discussing last time. So, what we did last time that, satellite is going in the orbit and let us say this is the centre of mass of the earth satellite it is a moving at the velocity of V C R in the orbit. And then we consider the atmosphere to be a static and that resulted, in the relative velocity which is the atmospheric particles will be moving toward the satellite ok. And we consider it in a small area d A of length V R and carried out the analysis.

Now, as we are aware of that with the rotation of the earth; earth is rotating on its axis from west to east ok. So, this is your north and ground side this is the south and this direction we show it the east I am sorry. So, this direction it will be taken as east and here, this direction will be taken as the north at any point and downward in this direction it will be the south and similarly here this direction will be the west.

So, as the earth is spinning on its axis, so does the atmosphere say this is the layer of atmosphere, I am showing it very thick this is not that much thick ok. So, atmosphere also rotates along with the earth almost same angular velocity ok. So, depending on the inclination of the orbit; this is the orbit inclined with the x y z plane. So, depending on this inclination of the orbit your what I want to say here that, if you have equatorial orbit so and satellite is here satellite velocity is V C R, which we have written V C R or V 0 I do not remember may be, so the V 0 we can keep V 0 is the centre of mass because we are choosing the centre of mass.

And we assume the atmosphere to be static now the atmosphere is also rotating along with the earth. So, with respect to the atmosphere this velocity of the satellite this relative velocity of the satellite will be different ok. That means, you have to accordingly workout say atmosphere is rotating along this suppose capital omega is the angular velocity of the atmosphere and earth is moving with respect to the inertial frame along this direction this is V 0.

So, if atmosphere is a static while the earth is moving so in that case we get the relative velocity of the atmosphere as V 0; so V 0 we have written this we have worked out this is the relative velocity; when atmosphere is static, but if this atmosphere is rotating. So, you can see that the atmospheric particle will also be moving along this direction and on at the altitude where you are considering and suppose this is the equatorial orbit. So, in the case of this is your equator rough calculation if we do.

So this is your altitude at the this is the altitude at the orbit you are considering here ok and here this is your if V 0 already it is a given. So, what will be the atmospheric velocity that will be omega cross the r where, r is the radius vector of this point; so this becomes V atmosphere.

So, now with respect to the atmosphere has the same rotation almost has the earth it is a rotation rate. So, in that case this is what we are writing so what will be the relative angular velocity in that case V 0 minus capital omega cross r ok. We are this is referring to atmospheric angular velocity so you have to subtract, so we are dealing here with the relative velocity that is the part you have to remember. If you take it in any other orbit just like I have shown here in this place any different point. So, accordingly you have to calculate how the V will get affected the three components of V will get affected ok. So, that so many transformation other things we have done so that can be calculated ok.

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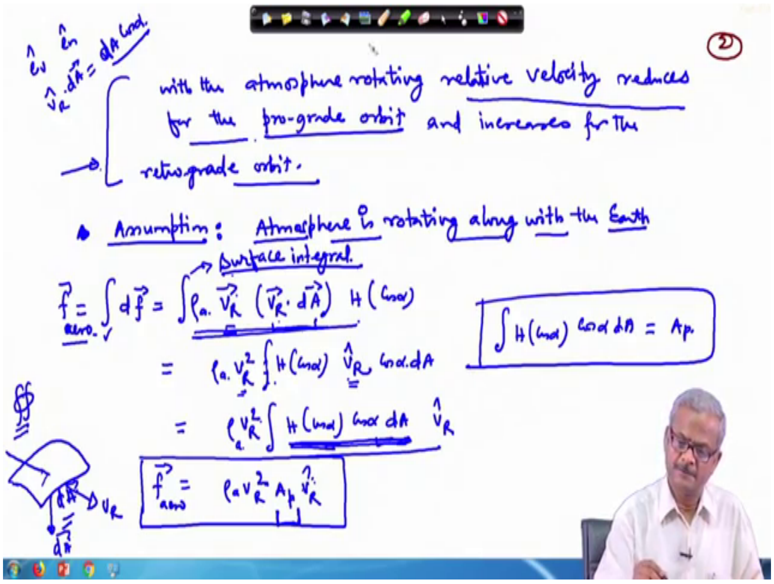

So, with the atmosphere rotating the scene is that the relative angular relative velocity reduces. So, with the atmosphere rotating relative velocity reduces for the pro-grade orbit. Pro-grade orbit and retrograde orbit already we have discussed retrograde has different the opposite sense of rotation of that of earth and pro-grade orbit have the same sense of rotation as that of the earth ok.

So, satellite in that orbit basically for that will be rotating in the same sense. So with the atmosphere rotating relative velocity reduces for the pro-grade orbit and increases for the retrograde orbit. So, for precise calculation all these things must be taken care of ok.

So, the next step now will be that we have to work out perhaps I have not a still calculated the torque acting on the satellite. So, that part I will finish today so assuming, so now, let us assume so the assumption I make here now the assumption atmosphere is rotating along with the earth.

So, in that case we are writing this expression and it has the it will also have the same expression, as we have derived earlier only think here the relative velocity will be changing.

So, earlier we have written d f equal to rho a times V R times V R dot d A and we have also taken into account the orientation of the surface by writing H cos alpha and then we integrated it and we have got the f aerodynamic. So, rho a can be taken outside V R square can we taken outside and this can be written as H cos alpha times V R for this part times V R cap V R we have already taken outside here. So, this is the V R cap or either you use e v cap and for the area you can use e n cap either of the notation can be followed.

So, for this V R writing here V R cap V R is taken outside and from this also one V R is taken outside. So, the scalar quantity comes outside V R cap dot d A. So, V R cap dot d A this we can write as d A cos alpha as discussed earlier. So, this is cos alpha times d A and then what we did that V R square we have written it this way H cos alpha times cos alpha d A and V R is independent of the integration. So, we took it outside and rho atmosphere is also independent of the integration sign. So, these are the surface integral and either you can put here like I can put here double integral to so that this is over the surface or single.

So, I will not carry this I will just go along with the single one unnecessarily this gets complicated ok. So, this stands here for I will write it is explicitly this is surface integral. This quantity we have written as a projected d A cos alpha is the projected area so element of the surface we are considering. So, d A is its area and V R is here in this direction ok. So, perpendicular to V R what will be the projected area d A, where d A we have taken it inside like this. So, A p that comes for this and then V R cap so, this is what we have derived last time.

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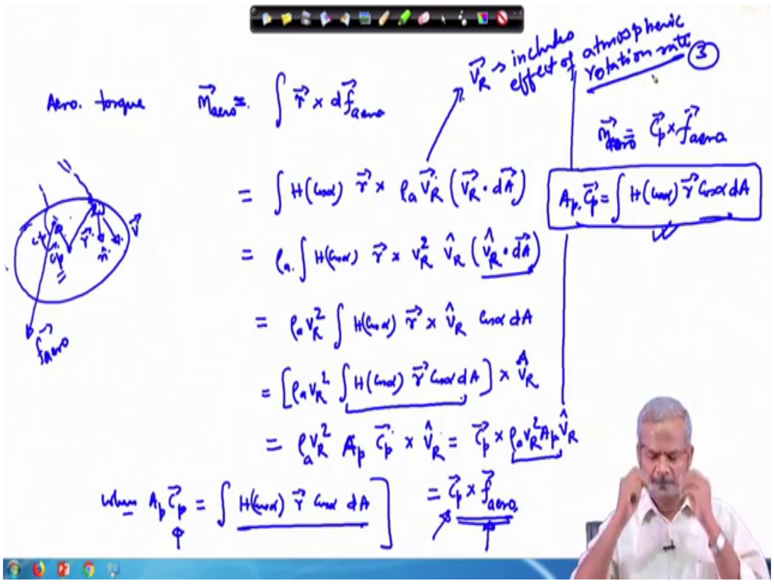

Using this only we can also find out the torque acting on the system. So, the torque aerodynamic torque we can write that as M aero and this will be given by r cross d f aero, where r is the distance from the centre of mass to the on this body we are calculating torque about the centre of mass ok. So, there can be any surface area like this so from here to here this is the radius vector r to that surface area and from somewhere your the relative velocity this way atmospheric particles are coming here in this direction n we have taken here as n cap or e n cap ok.

So, this same equation is still applicable. So, we can write here as H cos alpha which we have taken into account as you know this is Heaviside function. So, either it will be 0 or one depending on the value of alpha ok. So, H cos alpha times r cross and d f we have to copy from this place this is the quantity d f. So, rho a times V R dot d A ok rho a can be taken outside r cross we can write here as V R square times V R cap times V R cap dot d A.

So, rho a times V R square H cos alpha r cross V R cap and then this part we are writing as cos alpha times d A V R cap ok. So, we can write this as rho a times V R square and this quantity we can write as now look back here into the on the previous page this H cos alpha times d A ok. This is the quantity we have written as A p ok.

So if we will define a new quantity and in terms of that, then we can write the equation completely; so this will expresses rho a times V R square times the projected area C p cross V R cap where, C p A p time C p this quantity we can write as H cos alpha, r cos alpha, time d A. So, C p basically locate the centre of pressure means we are the effective forces the this is called the centre of pressure basically where the resultant force is passing through.

So, if you look here in this place C p cross this is cap not (Refer Time: 19:55) this is cap. So, we can rearrange it as C p cross rho a V R square A p times V R cap. So, this quantity then becomes C p cross this is nothing, but if you look here in the previous page this is f aero. So, this is the location of the centre of pressure and this is the point and this is the force the resultant force.

So, the resultant force at what distance is acting from the centre of mass that locate the centre of pressure and what is the say the centre of pressure may be; here this may be the centre of pressure and this is C p and f aero if it is acting here in this direction this is f aero. So, this is r cross f r vector is or the C p vector is lying here in this direction so from here to here.

Thus your M aero this can be written as C p cross f aero where C p equal to H cos alpha r cos alpha times d A. And another abbreviation that we have used this is H cos alpha so that also I will write here H cos alpha times cos alpha d A this quantity we are writing as A p this is the projected area sorry. Here this part a is missing A p times C p ok.

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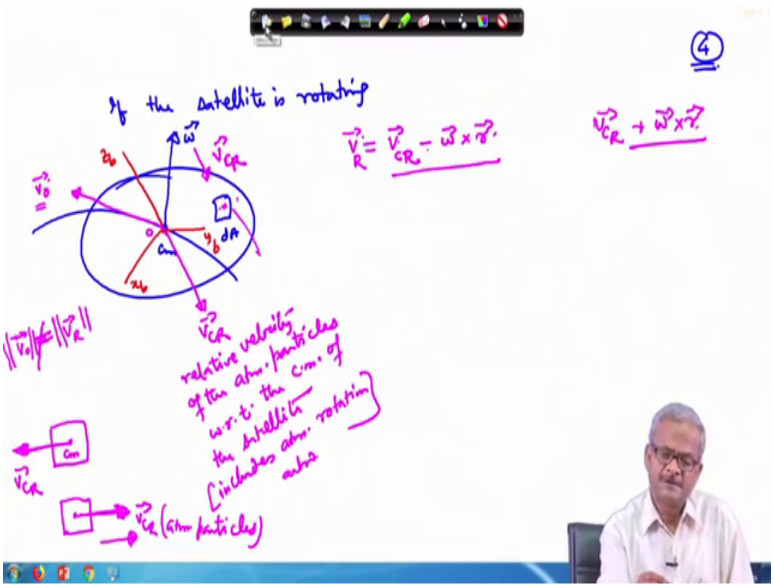

So, for the simple case we have calculated the how much we have calculated, what is the expression for the force and the torque acting on the system. Now if we can take into account if the satellite is rotating then what will happen? So, remember that on the previous page once we have written this in this case this V R this includes effect of atmospheric rotation rate that we have taken into account. Because initially we are started with a static model of the atmosphere and then just by little argument we have accept extended it to the case where the atmosphere is rotating. Now we will extended to the case where the satellite is rotating with angular velocity omega.

Now, if the satellite is rotating; that means, in this orbit the satellite is there, this is the centre of mass and there is certain element here whose area is d A and the satellite is having angular velocity omega. And the body axis also we can fix here this is the body axis x b, y b and z b and V o is the velocity of the centre of mass..However, we are discussing about the relative velocity between the atmosphere and the satellite. So, if in that case V o is or V o magnitude this is not equal to the previously we are using here V R not equal to V R this is you should keep in mind ok.

Now, if the satellite is rotating so the angular velocity of this will be added to the velocity. So, the V then we can write as or the relative velocity we can write as V C R minus omega cross r. Also if we can write here V C R plus omega cross r. It depends on in which direction the atmosphere is moving and then how you are modelling it.

So, if the relative velocity we have taken as this is the earlier we have written this as let us say this is the V C R, means the relative velocity is the V C R relative velocity of the atmospheric particles with respect to the centre of mass of the satellite. So, it is including this includes atmospheric rotation also. So, if this is V C R here in this direction as we are showing here and this area it moves here in this direction.

So, we can see that there will be reduction in relative velocity so in that case we can write as V minus omega cross r. On the other hand if it is moving here in this direction so, this will be just written as omega cross r, but irrespective of that whether you write here plus or minus it does not matter ultimately if you put the proper signs you will get the proper result. And one thing you should remember that if you are writing this V C R as the centre of mass of the satellite is moving here in this direction. So, atmosphere or atmospheric particle you are showing along this direction ok. So, it is a moving along this direction V C R for the atmospheric particle particles.

So, drag is along this direction, this is the velocity of the centre of mass with respect to this atmosphere. So, drag will be along V C R direction and it is a way of modelling in few places you will find this quantity written with minus sign like the f aero we have deducted here, if you write with a minus sign here in this place suppose I put a minus sign. So, this minus sign will referred that you are showing V C R here, in this direction which is in this case not velocity is not that of the particle, but rather of the satellite centre of mass.

So, that I have not done here so I have assume that V C R is the relative velocity of the particles with respect to the satellite. So, they are coming and impinging on the satellite. So and therefore this minus sign we are not using here ok; so if with this expression we can work out the torque and the force acting on the system while the satellite is rotating.

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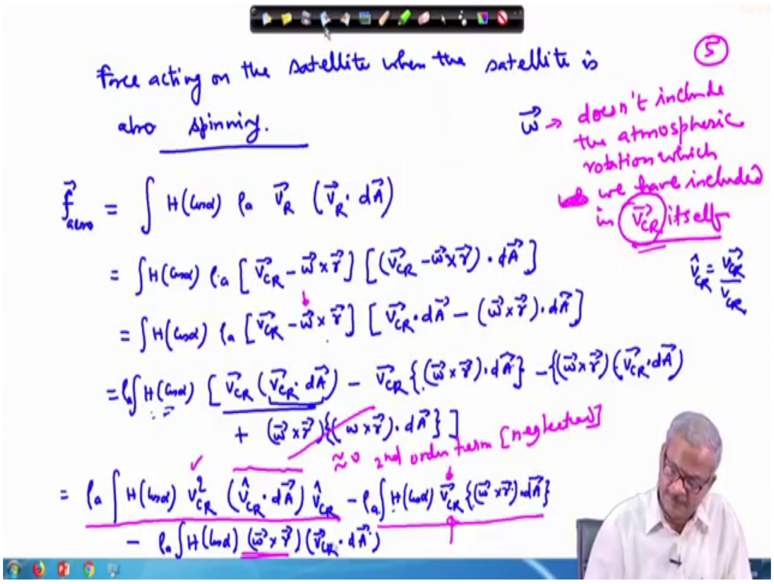

So now the force acting on the satellite when the satellite is also spinning. So, in that case we can write here then the f aerodynamic force it can be written as H cos alpha the same way we have written here rho a times V R times V R dot d A this is the basic equation we are using ok, it remains unchanged. However, here in this case we have to insert the expression for V R.

So, V R here in this case this becomes V C R minus omega cross r and then V C R minus omega cross r dot d A. This omega one more thing I would like to point out here omega does not include the atmospheric rotation, which we have included in V C R itself.

So, atmospheric rotation has been taken care in V C R itself. So, we do not have to think about that separately here we are just dealing with the angular velocity of the satellite. So, this part we expand here V C R dot d A minus omega cross r dot d A, H cos alpha rho a we can take it outside and then expand this. So, this is V C R times V C R dot d A minus V C R times r dot d A and then this part minus omega cross r times V C R dot d A and plus omega cross r times omega cross r dot d A. So, this quantity is quite a small this is negligible this is 2nd order term hence neglected ok.

So, we are left with this three terms we can terms which we can work out now ok. So, at the next level we have H cos alpha and then V C R square we can write this term separately and V C R this particular term we can write as V C R cap this V C R we are already taken into account here in this place dot d A and times V C R cap this is the unit vector V C R. V C R is the unit vector which we are defining as ok, this minus then the quantity here. So, this is V C R so rho a times H cos alpha, which is here in this place its a multiplying to all the terms.

So, this then becomes V C R omega cross r dot d A minus rho a times H cos alpha omega cross r times V C R dot d A. So, these are the three integrals which we need to evaluate. So, now we evaluate the first integral, V C R square this is a constant so it can be taken outside.

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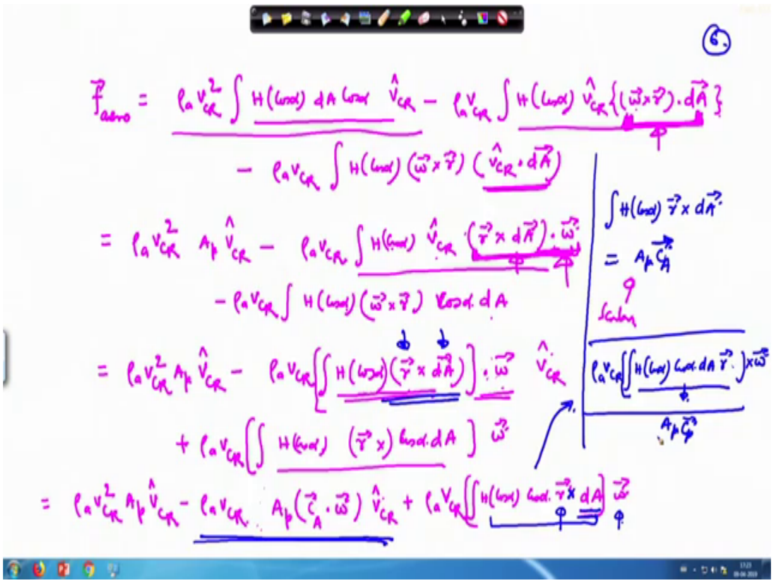

And the first term then we can write as so, therefore, f aero this becomes rho a times V C R square and in the bracket H cos alpha and then we have this quantity which we will write as d A cos alpha V C R is a unit vector. So, this we can write as d A cos alpha and then V C R cap. So, V C R cap this is the unit vector along the centre of mass velocity ok, therefore it can be taken outside the integration sign and this quantity already we know what this quantities, but let us write all the terms. Then rho a this quantity again this V C R is there. So, we can take it outside ok. So, this becomes rho a times V C R then H cos alpha and for this we will introduce the unit vector. So, we will write this as the V C R cap and there after the rest of the term comes we have this is omega cross r dot d A.

Similarly, for the lower one we can write this as minus rho a times V C R H cos alpha which omega cross r and times V C R cap dot d A times V C R cap dot d A. So, rho a times V C R square and this quantity already we have discuss about this quantity is nothing, but the projected area. So, a projected area times V C R cap this is the first term we are having in this place then the other one rho a times V C R this we need to little bit rearrange; in a particular way so, that we get a nice solution. We can write this quantity as r cross d A dot omega this is the dot triple dot product. So, we can use this property. So, omega we can separate out and this can be written as r cross d A dot omega the third term this quantity we can expand as V C R there is a unit vector therefore, we can write as cos alpha times d A.

So, few more steps are required to finally, get this solution; now this quantity again can be rearrange in a particular way r cross d A V C R is not dependent on the integration. So, we can take it outside and if you remember this is the dot product here omega. So, this omega also we can push here on this side or either on the right hand side also we can keep it does not matter.

So, let us keep it on the right hand side, so this is V R cos alpha r d A times V C R cap this dot product is over only this part we mark it here. We have to be very particular about on which the dot product and the cross product are operating and it depends on the way we have write written the equation here this is very important. So, this bracket must be maintained if we mix it up so all the results will be wrong.

So, this V C R because it is a this dot product is with this and this so we can write here, dot omega and V C R we can keep it outside. And this term then rho a V C R H cos alpha we can change the sequential and write this as with a plus sign and here we can write as, r cross and cos alpha d A; d A is not a vector here, we have already changed that and outside we can write this as omega, so this is the way this is working.

Now, the quantity we are having here r if cross d A, so of this we are going to define as H cos alpha r cross d A. So, this we are going to write as A p times C p, A p times C A. So, if we do that so this particular equation then reduces to a format then this becomes A p; A p is a scalar this is a scalar, A p times C A this is not the centre of pressure as we have defined earlier this is C A, A p times C A and then if we look into this part dot omega. So, this is dot omega so these are together and then V C R cap and plus then we have the quantity which is given here r cross d A. And this will operate on d A is a scalar here on omega ok.

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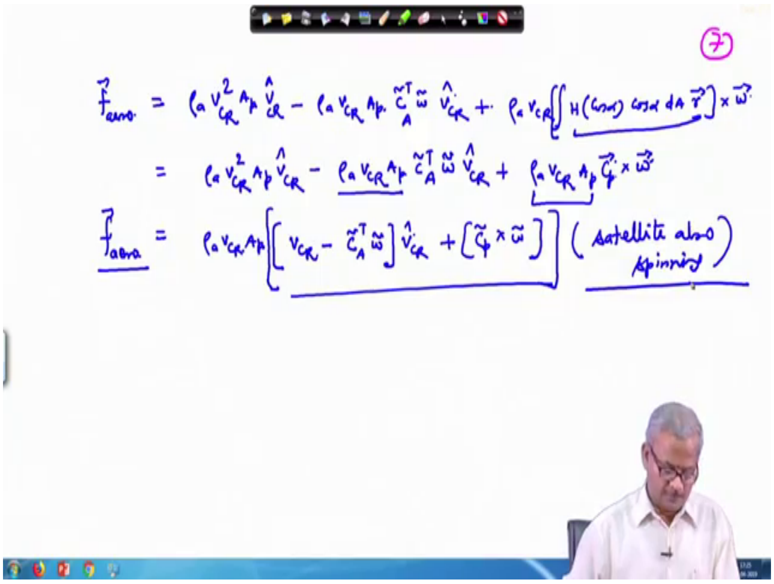

So, now we can wind up the thing so we can write here f aero this becomes rho a V C R times square, A p times V R cap or V C R cap we are writing so this is V C R cap. And then the second term we pick up minus rho a V C R times A p and this we can write as C tilde transpose times omega times V C R cap and the other term we have here plus with plus sign; so that gives us plus here also rho a times V C R and the term which is present here cos alpha r cross d A.

So, see the difference between this term here this is r cross d A and here this is r cross and d A this is a scalar. So, this cross is working on this omega not on d A d A as will we can write here on this side. So, what it implies that, we can write this term as rho a V C R H cos alpha and then, cos alpha d A r cross and this is working on omega, so this is the operation we have to do.

So, while we have written it like this omega this cross the cross term is there, so this cross term is not dependent on the integral sign. So, it will be convenient if we fit it on the outside the bracket and then you will look here in this term. So, this becomes A p times C p projected area times the centre of pressure distance.

So, using this then we can write as rho a V C R then H cos alpha we write it this way H cos alpha cos alpha d A r cos alpha d A times r and put it in bracket and then put cross here cross omega. So, the equation gets pretty simplified and comes in an elegant format rho a V C R this quantities A p times C p and then cross omega ok.

So, rho a V C R times A p can be taken outside the bracket and what we get here is V C R minus rho a V C R A p will go from this side. So, C tilde transpose a times omega tilde ok. And then V C R cap this also goes outside and we get here C p tilde cross omega. So, this is the aerodynamic torque acting on the system force acting on the system if the satellite is also spinning. So, we will continue in the next lecture.

Thank you very much for listening.