Space Flight Mechanics
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Lecture – 18
Problem Related to Orbital Elements

Welcome to lecture number 18. So, we have been discussing about the vital parameters in that connection we will solve one more problem.

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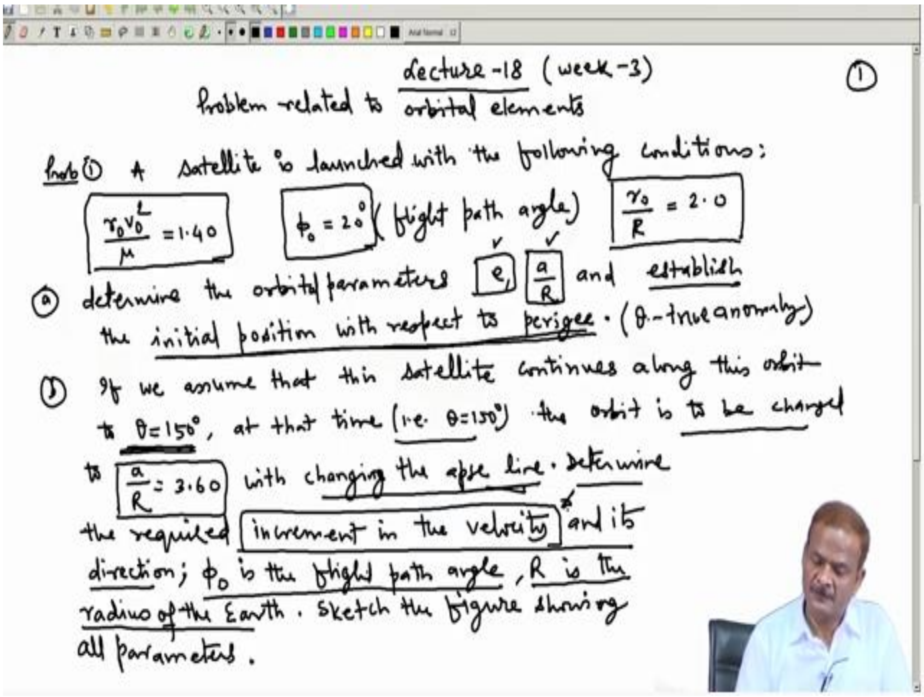

And see the problem is defined here basically it is a satellite launching problem also it is a trajectory change problem which we are going to do little later but because this is a very simple one so we can include it with our satellite orbit determination problem. So, a satellite is launched with the following conditions already you know this quantity we have discussed so many times this has given 50 this is a flight path angle. This is given and the initial position of the satellite not in terms of the vector but in terms of just a scalar quantity.

So, this is non-dimensional quantity r is here the radius of the earth as it is shown here r is the radius of the earth 50 is the flight path angle okay. So, what is the question? determine the orbital parameters a/r eccentricity and the semi major axis is to be determined and establish the initial position with respect to the perigee initial position how do we determine. This will be determined in terms of ‘θ’, the true anomaly.

So, this we need to work out and then if we assume that the satellite continues along this orbit. So, wherever the satellite has been launched okay. So, from there it continues till true anomaly. once the true anomaly reaches a value of 150° at that time the orbit is to be changed to a/r. Now the semi major axis is to be changed without changing. the axis line means the periapsis line you do not have to change where the perigee location is there from focus to the perigee location.

So, that line has not to be changed that should not rotate it will remain the same. So, determine the required increment in the velocity and its direction. So, how much velocity increment will be required? This is a trajectory transfer problem basically but this part because other parts it is related to orbit determination, so we are doing it here in this place and this is a very simple problem and it is a direction.

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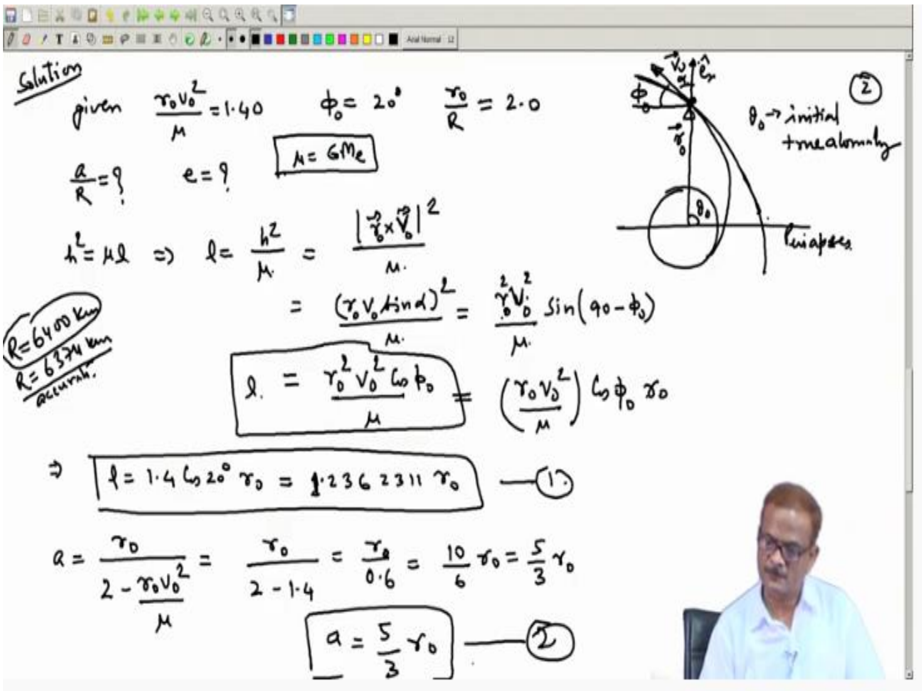

Okay so we will start with the problem this is given 1.40 and 50 is given which is a flight path angle = 20° and r_0/r this is given 2.0. So, we have to get the value of what is a/r and what is the eccentricity these two things we have to determine we will start with $h^2 = \mu \times l$ so this implies and we know if the orbit looks like this satellite has been launched and like this and this may be the injection point.

So, this is the velocity vector and from here to here this is the $\vec{r}$, and this is $\widehat{e_r}$ this is the velocity vector so this angle is α and here this angle is φ_0 and this becomes r_0.

If this is the periapsis line or perigee line so we have here this angle we have referred to as in this case we will write it as θ_0 means that initial true anomaly. So, h has to be known

$$l = h^2/\mu = |\overrightarrow{r_0} \times \overrightarrow{v_0}|^2/\mu$$

and in this case the μ will be the value for the earth which is G times mass of earth. this is velocity.

So, this be r times v $\sin^2 \alpha$ divided by μ . because α is not given but φ is given. So therefore, we can write it in terms of and here we will make it r_0 v_0. So, r_0, v_0 $\sin^2 \alpha$ divided by μ . So, r^2 v^2 divided by μ and what is the value for sin α so this will be sin 90° – 50. So, this gets reduced to

$$l = \frac{r_0^2 v_0^2 \cos \varphi_0}{\mu}$$

. So, we insert the value. this becomes 1.4 and cos 20° and r_0 is 2r. So that value we can insert here okay. So, for the time being let us keep it like this l = 1.4 cos 20 times r_0. we know the value of the r is around it depends on the if we want to take it simply this is 6400 km.

Otherwise more accurate value is 6374 km okay. So, this gets reduced to 1.2362311 r_0.some simple calculator I am not truncating or trying to present a systematic value because here if you truncate it, we do a lot of error in the estimate. So, on the computer the things are done in double precision so that you maintain the accuracy of your result. So, this is 1 r_0 v_0^2 divided by μ it is a given this is r_0 divided by 0.6 we will get 10/6 r_0 = 5/3 r_0 so

$$a = \frac{5}{3} r_0$$

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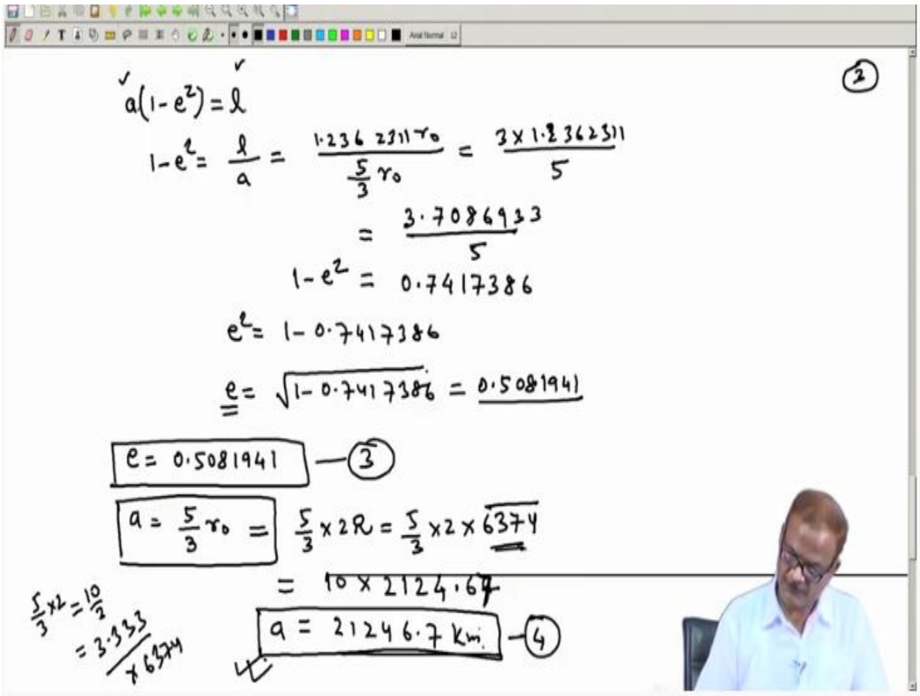

So, already we have written the expression for the l we utilize it to find out

$$a\,(1 - e^2) = l$$

So, from here the l we have written the expression for a we have written therefore 1 – e^2= l/a and from the previous 1 =1.236 like this 1.2362311 r_0 and a we have written as 5/3r_0, so r_0, r_0 cancels out. So, we do not have to again put the value of r_0 and work it out.

So, got in this just cancels out. this is 3 times 1.2362311/5. So, this is the value here and e square this is = 1- 0.7417386 and therefore e becomes 86 under root and this value will be 0.580 5081941 so

e = 0.5081941

and remember the a we have written as 5/3 times r_0.Though we have not written it explicitly so if you put r_0 is given to v_2 × r.

So, this is 2 r and if we insert, the value say 5/3 times 2 times 6374 so in that case if we use this value 6400. So, depending on the value used here 3 times this is 21 and so this becomes around 21246.7 km why? because this is 5/3 times 2 so this is 10/3 which is nothing but 3.333 like this.

Now this times 6374 so that will exceed your value of 20000. So, this is what we are getting okay so this is our result. Number (1),(2) this is (3) and write this as (4).

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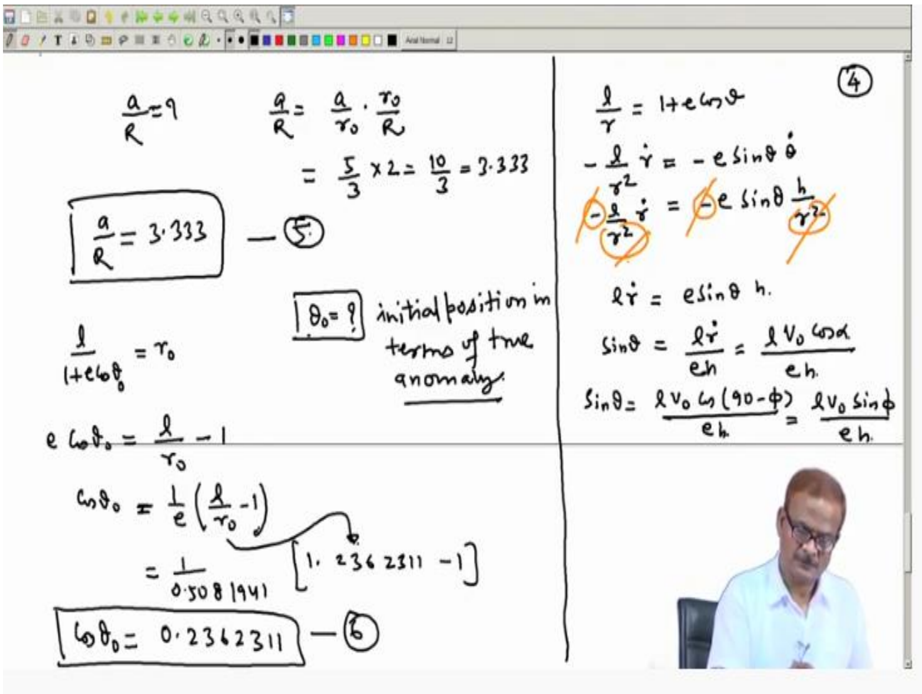

The question other things will determine a/R okay so the question was also finding a/R how much this will be. So, we have a/R already this a/r_0 we have written so a/r_0 we have bought as 5/3, and r_0/r is given to be 2 so this is 10/3 so 3.333. so, a/r so this is one part in the question this is okay what else is remaining determine the orbit parameters ear and determine the initial position with respect to perigee.

So, we have to determine θ also. So, we proceed in the usual way

$$\frac{l}{1 + e\cos\theta_0} = r_0$$

and this is θ_0 so here what is required θ_0 = what it is the initial position in terms of true anomaly this is required. So, cos θ we have to determine θ_0 we have to determine. So we start with writing

$$e\cos\theta_0 = \frac{1}{r_0} - 1$$

so this is cos θ_0 is therefore this is 1/e times l/r_0 – 1. already we have determined all these quantities.

So, here in this case 0.5081941 l/r_0 we have already determined here l = 1.23 so l/r_0 from here it becomes 1.22362311 -1. this is the l/r_0 and once we solve this so you get here cos θ_0 this = 0.2362311. So as earlier also I have mentioned that this al1 is not sufficient to determine the value of θ_0.

So, we also require sin θ_0 so that we can resolve the problem. Now therefore we utilize the relationship this is the way you can work out in the exam the way I am doing. so write it

$$l/r = 1 + e\cos\theta$$

everything you need not remember

$$\frac{l}{r^2}\dot{r} = -e\sin\theta \times \dot{\theta}$$

and – e sin θ and $\dot{\theta}$ is nothing h/r^2 so minus minus sin this drops out this will and this will they will cancel out this r^2 and r^2 this will cancel out. and then we are left with

$$l\dot{r} = e\sin\theta \times h$$

and e sin θ = l $\dot{r}$ /eh also we will remove from this place.

Now $\dot{r}$ is the original component of the velocity vector. So, if we go back here in this direction so this is $\dot{r}$ here in this direction so this will be v_0 cos α. So, $\dot{r}$ is v_0 cos α so l times r_0 v_0 l times v_0 cos α divided by e times h l times v_0. Now α is 90 – φ this is cos 90 – 5 is 20 ° it is a given e times h. So, therefore this gets reduced to lv_0 cos φ becomes sin φ cos 90 – φ that becomes sin φ divided by e times h.

$$\sin\theta_0 = \frac{l v_0 \sin\varphi}{eh}$$

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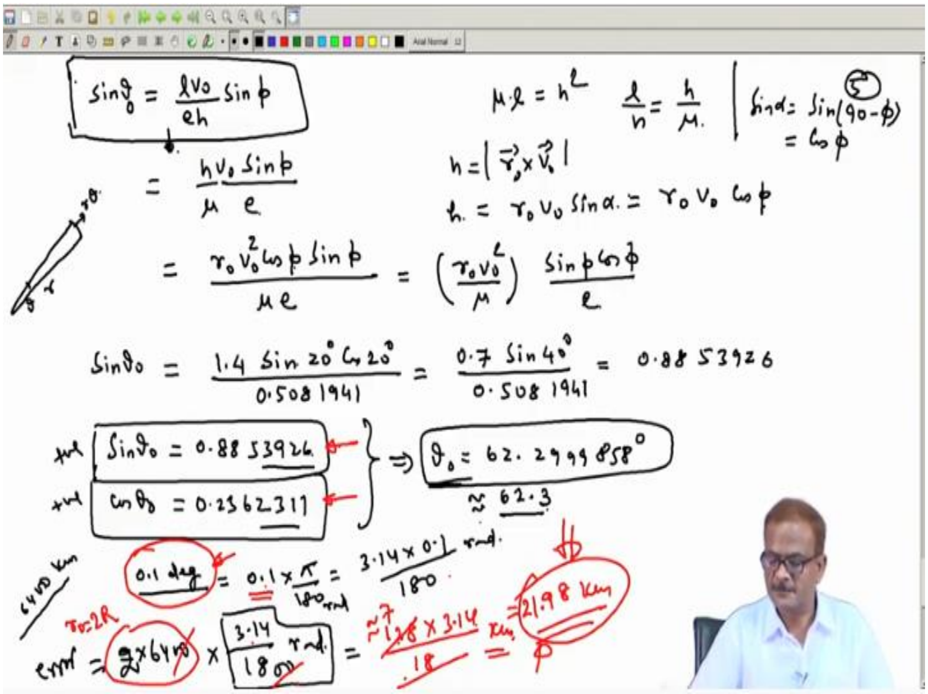

So, from we have sin θ; here we will write it sin θ_0 because we are looking for the initial value. So lv_0 divided by eh sin φ so we need to insert the values to get the solution. Now also we remember that here this l is written in terms of μ times l this is written as h^2 okay. So, we need to eliminate some of the things from this place so if we eliminate l/h if we write it like this l/h so you can see that this will be h/μ .

So, this can be rewritten as l/h = h/μ; v_0 sin φ and divided by e okay. Now h already we have written as $\vec{r} \times \vec{v}$ magnitude this is r_0 v_0 sin α and this we have written as r_0 v_0 sin α is nothing but sin 90 – φ = cos φ so this is here becomes cos φ this is your h so therefore if we insert the h here, so

$$= \frac{r_0\, v_0 \cos\varphi \sin\varphi}{\mu e}$$

So, 1 v_0^2 becomes now this is in a format where you can work it out sin φ cos φ divided by e. So, all the values are now known r_0 v_0^2 divided by μ this is going to be 1.4 sin 20° times cos 20° and divided by e which we have already determined e value we have written to be 0.5081 941.

So, more compactly one more step we can write at 0.7 sin 40° divided by 0.5081941× 0.8853926 so this is sin θ_0 0.8853926 and cos θ_0 we have got as 0.2362311 see in the actual calculation even as small angular difference though for our purpose which is so much of number of digit it is not required. but if you remember that a very if we are looking for 6400 km distance okay 6400 km distance and you do a mistake of say 0.1° in evaluation.

So, 0.1° this will be around the 0.1× π/180 means say approximately 3.14 × 0.1 divided by 180. So, 3.14 divided by 18 this much of radiant so it is converted into radiant. And if you are multiplying it by 6400 km, you have committed a mistake. this is just the radius of earth let us take double of that that where the initially the satellite is located.

So, this is the error done so see what will be the corresponding value. This is r and if this is θ in radiance so this distance becomes r times θ this is the principle we are using. So, this is your θ in terms of radiance sorry here this will be 0.314 this is once we okay this is fine 3.14 because he has oh sorry yeah this will be 1800 3.14 divided by 1800. So, if we cancel out this part. okay so this gets reduced to 128 × 3.14 divided by 18 km.

So, 0.1° of error, let us check once more this is 0.1°. So,0.1° times π/180 radiant it is a converted and 1 radiant is around 3 pi is 3.14× 0.1 here divided by 180. So we convert it 1/10 so this is 1800 3.14/1800 okay and this multiplied by at the distance we are looking for 2 times r_0 this is 2 times r_0 equal to distance how much error we will be doing? so 128 times 3.14 divided by 18 and if we approximate this by 7 let us say this is approximately 7.

So, this is 21.98 km so this much of difference if you do an error or 0.1° in measurement. So this much of error will be committing in positioning the satellite. And this is a very large error very large error okay. and suppose then you are dealing with an interplanetary satellite like your satellite is going to mars and other position. So, you require very accurate value in these positions.

So, therefore while dealing with doing the orbit determination for interplanetary satellite or even for the earth satellite where the precision is required. The angles are not involved because small error in angle it will result in large error in the position of the satellite. So, 0.1° results in 21 km if you make it 0.01°. So, this will be 2.198 km it is still it is a large if you are looking for some precision manoeuvres like the docking and these things of distance will not work simply.

So, if this is one learning that we need very precise value in terms of angles even and therefore the angles are quite avoided in the satellite orbit determination problem because it just creates a lot of problems. Okay now based on these two we need to find out the θ so we can see that both of them are positive this and this both are positive. So that means it is aligning in the first quadrant as we have discussed in the last lecture. and then this turns out to be 62.2999858°.

So, if we approximate this is 62.29 it is almost this is 62 point or better, we can write if we approximate to 62.3 and we are approximated to this place. So, this is the initial angular position of the satellite.

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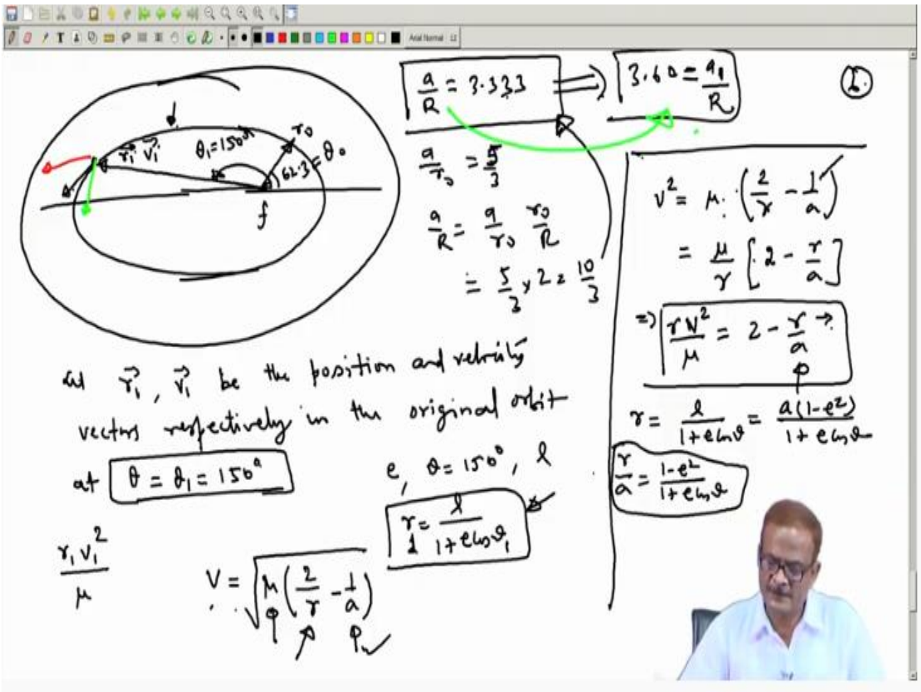

So in this problem this is over now we have to go to the part number b. So, it is a telling that the situation here this is the focus and a satellite is at true anomaly of 62.3° was 62.3. So, this is the initial position θ_0 and from there it continues till 150 °s. So, 150 °s here it is going from this place to this place this is $\theta_1 = 150°$. So, this is given to $\theta = 150°$. at which time the orbit is to be increased to a value of a/r = 3.6.

So, here the velocity vector is tangent to the eclipse this is the velocity vector. So, what it is asking that now the initially a/r this was we have obtained it a/r to be 5/3 or something a/r_0 we have got as 5/3 a/r was perhaps 3.33 something we have got. So a/r_0 we got as 5/3 and from there then a/r a/r_0 times r_0/R so this is 5/3 times r_0/r is given to be 2. So, 10/3 which is this quantity we have got.

So, from this value this has to be increased to a/r = 3.6 without rotating that periapsis. So, this will be your new a1/r so this is older one and this is new one we will integrate it by a/r. So, what are the things required? so in that trajectory transfer you will need all the velocities and other things but from where those things will come we have to do it from this place itself whatever we have worked out till now the theory part using that theory we have to determine what will be the velocity vector and where in which direction it has to be given.

Whether the change in velocity has to be in this direction or whether the change of velocity has to be in this direction. In which direction we have to give impulse. So that your a changes from this place to this value to this value 3.333 to 3.60. So, this is the exercise that we need to do here okay? so to work out this problem we need to find out $r_1 v_1$ so this was r_0 here now here this is the position that is let us say r_1 this is the position r_1 and the velocity vector at this place is v_1 okay in this orbit.

Let $r_1 v_1$ be the position and velocity vectors respectively in the original orbit at $\theta = \theta_1 = 150°$. So, we need to find out first $r_1\ v_1^2$ divided by μ at this position what this value will be. So, your eccentricity is known e is known and here θ is given to be 150°; l of the original orbit is known. So, you know that

$$r = \frac{l}{1 + e \cos \theta}$$

so from here your r_1 can be evaluated okay and this will be your θ_1.

So, $r_1 = l/1 + e \cos \theta_1$ and how the v_1 will be evaluated so for evaluating v it is a we can use this relationship

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

so in the original orbit r position is known will be known from this place a will be known this already known μ is already known okay because it is related to earth. and therefore, v can also be known. But instead of doing that stretching this long simply we square this so this becomes $v^2 = \mu \times (2/r - 1/a)$ and we take outside r as a common okay this becomes 2- r/a see this relationship we have a squared it.

So, while you are squaring this under root has been removed okay. And thereafter we are taking from this place r as a common. So, this becomes μ /r and this r goes here in this place. So, this becomes 2 -r/a and therefore this implies r v^2 divided by μ = 2 – r/a so here look into this equation only r is required. Okay at the new position a is already known and therefore we can work out r v^2 divided by μ .

So, both the way we can do that is not a problem either you do it this way or that way it is all okay and moreover as we have written r = l/1 + e cos θ this = l = a times 1- e^2 divided by 1- e cos θ and therefore r/a becomes

$$\frac{r}{a} = \frac{1 - e^2}{1 + e\cos\theta}$$

so this is also r/a can be written this way. So, the same problem as I have told you cannot attempt in a number of ways and irrespective of your ways your results should be the same.

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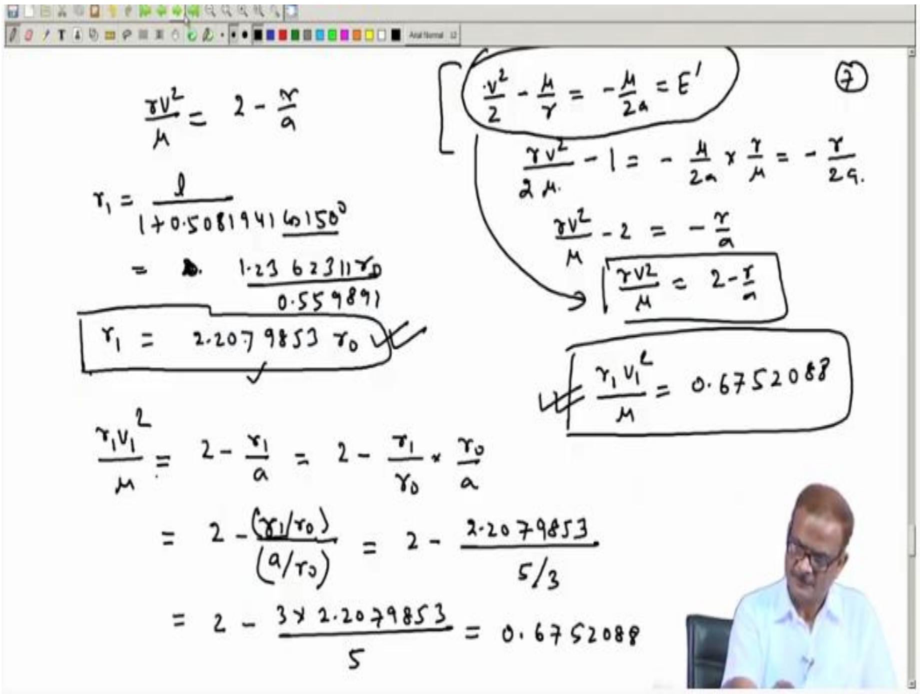

So, therefore v^2 r v^2/μ becomes and you remember this equation is nothing but our energy equation I will take up that issue again r v^2= 2- r/a this is nothing but your energy equation we have written v^2/2 – μ /r = -μ /2a which is nothing but your e', a specific energy. Once we have rearranged it so if we pull here r on this side v^2 divided by μ this 2, we are multiplying by r/μ .

So, this becomes – 1 and here on this side -2a this is r/μ μ, μ cancels out this is -r/2a and therefore r v^2/μ -2 this becomes -r/a and this is nothing but r v^2/μ = 2-r/a this is same equation energy equation we have used. Anytime you do not remember this equation so you can derive it always from the energy equation from this particular one using this particular request and you can always work out okay.

So, here r_1 is l/1 + 0.5081941 cos 150° and this gives us a value of r_1= 2. and l is also known and l is known to be 1.2362311 divided by cos 150°s. So, this will become an this will come with a minus sign and therefore this value will reduce and this value gets reduced to 0.559891 and therefore r_1 gets reduced to 2.2079853 r_0 so this is the value of r_1 so what we need here in this place r_1 v_1^2 divided by μ .

So this will be equal to 2 - r_1/a and this can be written as 2 – r_1/r_0 if it is known from this place times r_0/a we can also write it like this because a is known. So, 2 – r_1/r_0 and divided by a/r_0. So, a/r_0 is known r_1/r_0 is known from this place this is 2.2079853a and a/r_0 we have written as 5/3 so insert that 5/3 here. So, this is 5/3 so this 2 – 3 × 2.2079853 divided by 5 and this value will get reduced to 0.6752088. So, what we have got here r_1 v_1 square divided by μ = 0.6752088

So, we have forgot to number the things we have not numbered anywhere equation so you can follow as it is. okay. so now this is available. These are the things I am putting the things into a box so that it is easily traceable whatever we have the important values these are all placed in box. okay till this we have numbered after we have not done.

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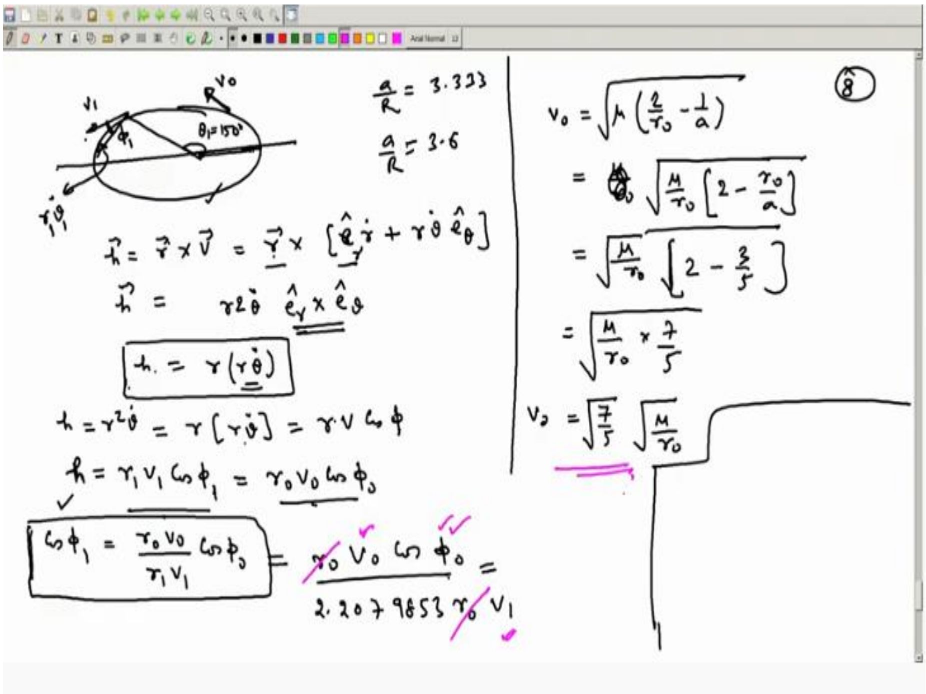

Okay once we have got this value then and what is required that at the point. This is the point at 150° θ_1 = 150° here the velocity is now v_1 initially somewhere here the initial velocity was v_0. So how much velocity changes is required so that without changing the periapsis a/r which was originally 3.333 so from that this gets reduced to a/r = 3.6. this is the question. we have to get it to 3.6.

See now here in whatever the direction we have to give the impulse okay. so we will required here the flight path angle and let us say the flight path angle here this is φ_1 without flight path angle this problem cannot be worked out. Okay so we need to determine this value. Now at this position the flight path angle if we are looking for h = $\vec{r} \times \vec{v}$ and as earlier also I have told you this can be written as r times $\dot{r}\,\hat{r}$ or $\widehat{e_r}$, + r $\dot{\theta}$ times $\widehat{e_\theta}$, and this being the same vector this cancels out.

And here you get this as r^2 times $\dot{\theta}$ times er cap cross $\widehat{e_\theta}$ and this vector will be nothing but perpendicular to the orbit which is shown here okay? And what this quantity is this is nothing what r times r $\dot{\theta}$ and what this quantity is r $\dot{\theta}$ is the velocity component along this direction. So, this is r times $\dot{\theta}$ at this position.

So simply each vector once we are writing as $r^2\dot{\theta}$ so this is nothing but your r times $r\dot{\theta}$ = r times r $\dot{\theta}$ is how much v cos φ okay. So, therefore h is remaining because h is remaining constant it is a constant for the orbit. So on the right hand side we will have r_1 v_1 cos φ_1 so whatever the velocity is here. So, this we have to take so this equation we can utilize for finding out the value of the φ_1 this problem is bit long it uses just simple process.

But it is a stretching one and this also must be true that this should be = r_0 v_0 cos φ_0 which is at the initial position this is at the final position so h remains constant. This is also h but are two different positions so from this place we get cos φ_1 = r_0 v_0 divided by r_1 v_1 times cos φ_0, cos φ_0 is of course known to us okay so what will be the flight path angle in that place this becomes known to us.

Otherwise you can use whatever the equation I have developed earlier so there the flight path angle equation from there it can be written very easily but you need to remember those equations here without remembering anything you can do the work. In a sequential manner if you go you will be able to do it. Okay so we require now r_0 v_0 here in this place okay once we get this so our job is done.

So v_0 is the quantity

$$v_0 = \sqrt{\mu\left(\frac{2}{r_0} - \frac{1}{a}\right)}$$

r_0 we are taking outside 2-r_0 divided by a under root 2 - r_0/a we have written as 3/5 so this is μ /r_0 under root this is 7/5 so v_0 is 7/5 under root times μ /r_0. Of course μ is a known quantity for the earth so it is not a problem. So here in this place we insert the respective values what we have calculated r_1 we have calculated to be2.2079853 r_0 and r_0 is also given to us r_0 is given to be 2.

Okay r_0 this is already written in terms of r_0 so it is not a problem r_1 we have worked out here we have worked out r_1 so this we write here so r_0 r_0 will get cancelled out from here. this is v_1. We are remaining with v_0 cos φ_0 so this will drop out. we are left with it v_0 here cos φ_0 v_1 this quantity is known to us in this particular part okay so v_0 and v_1. So, already v_0 we have written here in terms of this so you will utilize it.

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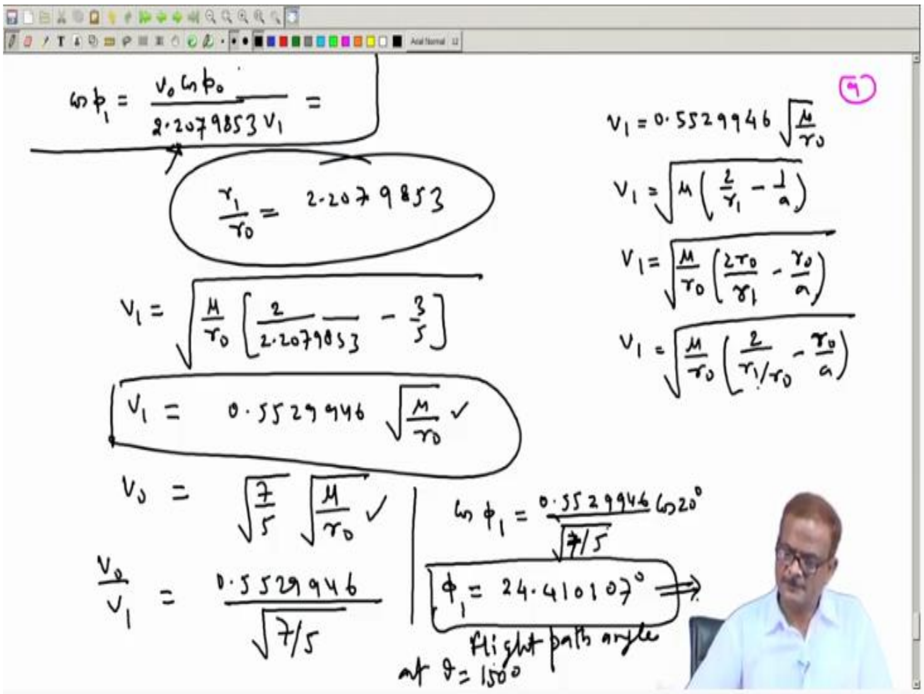

So cos φ_1 this is written as v_0 cos φ_0 divided by this quantity 2.2079853 times v_1 okay v_0 already we have determined this is our v_0 here so we should be able to work on this problem okay so v_1 we have written as 0.5529946 μ /r_0 okay this we have worked out and how we have got it we have used this equation as I told you this problem is little lengthy and with patience if we work we will be able to do it.

So this can be written as r_1 we need to replace so we take here μ /r_0 as common so 2r_0 divided by r_1 – r_0/a under root this is μ /r_0 times 2 divided by r_1/r_0 – a/r_0 perhaps we had the value we will just take the inverse of that we will get the result. So, r_1/r_0 already we have determined this is the quantity we are looking for but r_1/r_0 this quantity already. we have determined this is 2.207 as we have written here 79853 this value we are having.

So by inserting this we get v_1 in terms of terms of μ /r_0 times 2 divided by 2.2079853 and -3/5 r_0/a is and therefore this quantity we are getting as 0.5529946 divided by μ /r_0 this is it and v_0 is also known to us like the as same way v_0 we have written somewhere here 7/5 under root μ /r_0 under root.

So, everything is known here in this place. So, from here v_0 divided by v_1 this becomes 0.5529946 divided by 7/5 under root okay μ r_0/μ $r_0$these two terms will cancel out. So, once we get this and insert here in this place, we get cos φ_1 and obviously cos φ_1 = insert this value

0.5529946 divided by 7/5 under root times cos φ_0, so φ_0 was 20 °. So, if we insert this φ_1 and flight path angle it never exceeds 180 °. So, you get the correct value from here so φ_1 turns out to be24,410107 °.

So, this is the flight path angle at position 1 at the new position. So, this is a flight path angle at θ = 150°. Numbering could have made it easy but we have not numbered the things.

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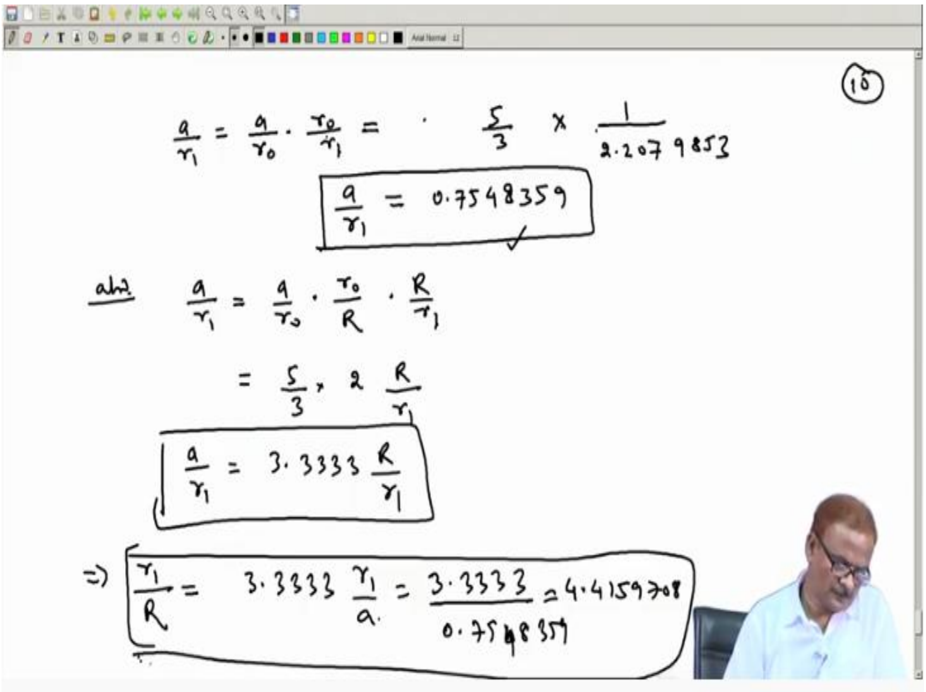

r_0/r_1 we have already determined. this is this path 2. r_0 divided by yr_1 this is 1 divided by we will have to look back and write all these values 2.2079853 and a/r_0 this we have written as a/r_0 5/3 so this value turns out to be 0.7548359. So, this is a/r_1 also we can write a/r_1 = a/r_0 times r_0/R R/r_1 a/r_0 is 5/3 r_0R was given to be 2. so this is R/r_1 so r_1/R will then be equal to we are looking for this value this is okay 3.3333 r_1/a so 3.3333 r_1/a is the quantity written here a/r_1 so we can divide it by 0.7548359 so this gets reduced to 4.4159708 so this is r_1/a.

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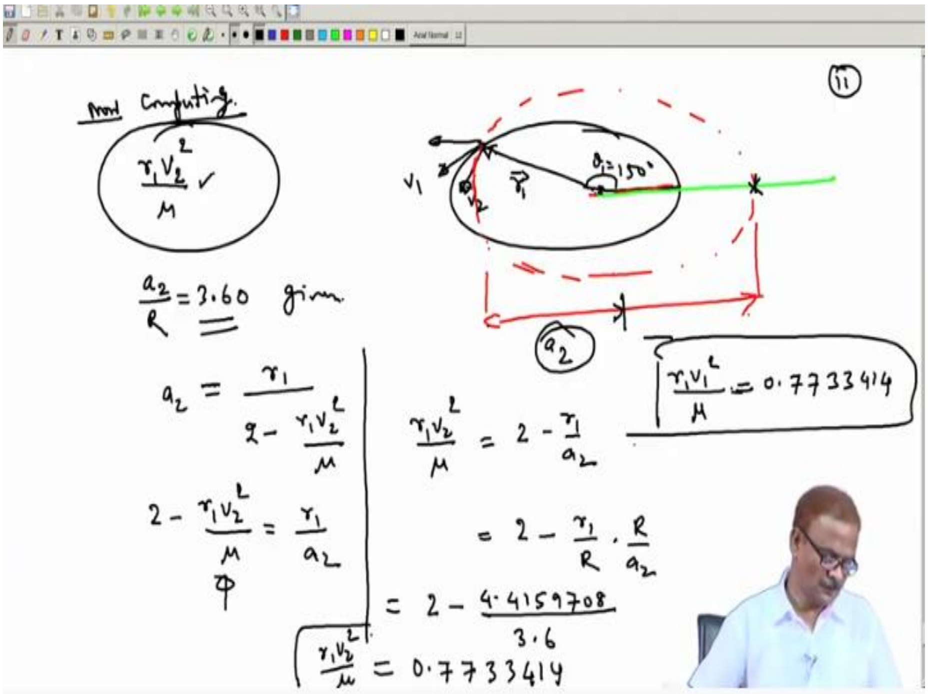

Now computing r_1 times v_2^2 divided by μ what this quantity is in the orbit at θ = 150° this is v_1 but either you are giving impulse here in this direction or here it maybe in this direction say v_2 so we need this quantity also what will be the in our computation this will be required. So, r_1 v_2^2 divided by μ this quantity we have to compute and given a_2/r is 3.60 this is given.

this is your r_1 position at this position you have to change. the orbit means it may look like if you are changing. so it may look like this without changing the periapsis, periapsis is same periapsis is here in this direction but the orbit you are changing maybe if you are giving here in this direction so it will look like this. So, this is your new orbit and for that then you have to calculate this distance divided by 2 that gives semi major axis.

So, this is the thing being written here so this is a_2/r is given to you. So half of this measuring from the obviously we have to this the focus remains unchanged okay? so total from one end to another end from this perigee position to that distance is 2 here and divided by 2 that gives you say this is the half distance here. So, this half distance from here to here that becomes a_2 so this is given to us. okay so we utilize this information.

So, a_2 = r_1 position is not changing so this remains r_1 / 2- r_1 v_2^2 divided by μ, and therefore we can rearrange it and write it as r v_2^2 divided by μ = r_1/a_2 and this is the quantity we are looking for so r_1 v_2^2 divided by μ this becomes 2- r_1/a_2. So, 2-r_1/r times r/a_2 = 2 – r_1/R. So,

r_1/R these quantities are known to us actually. So, we can directly substitute all these quantities here rather than pulling it r/a this is a_2/r this is given 3.6.

So, this is a minus sign and first we will r_1/R how much is this we will look for that r_1/R is here 4.415. So, this value we need to insert this value is 4.4159708 and a_2/r is given to be 3.6 so divide bring it in the denominator. So, this becomes 3.6 and finally then this l value this is equal to so $r_1\ v_2^2$ divided by μ this gets reduced to 0.7733414. So, we have here $r_1\ v_1^2$ divided by μ = 0.7733414. So, v_2 we can calculate r_1 is known to us and therefore v_2 can be calculated this is v_2^2. Therefore, v_2 we can compute from this equation.

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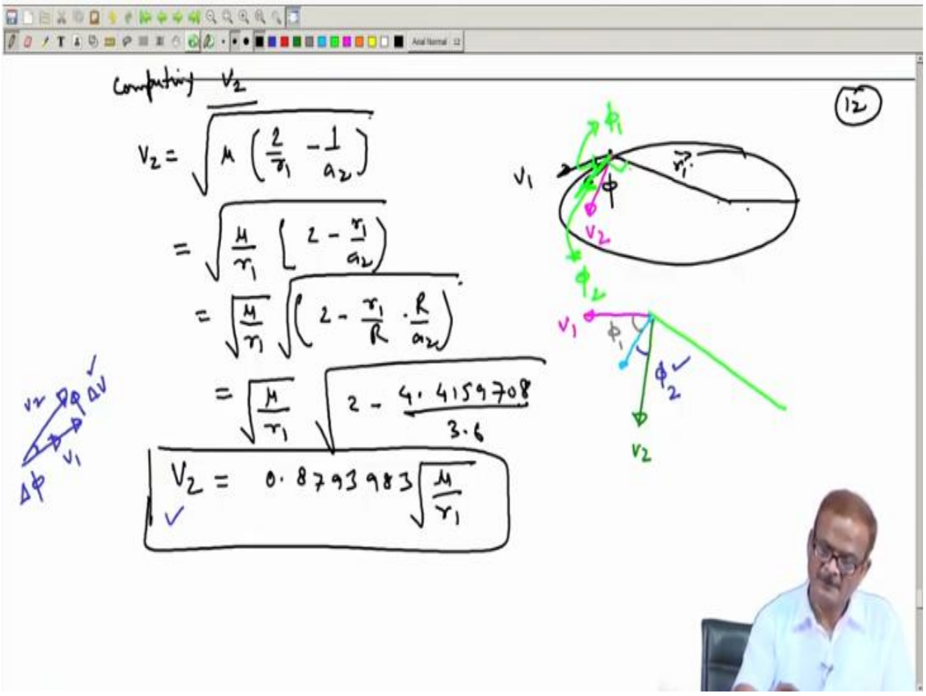

So, computing v_2 so either use that equation or either we can also utilize this equation μ times 2/r_1 -1/a_2, a_2 is already given here. So just I am telling this equation can be worked out in a number of ways. there is not only one way of doing it computing v_2. So v_2 is just quantity and then insert μ /r_1 2 – r_1/a_2 under root v_2/r_1 is given there so this is μ /r_1

$$v_2 = 0.8793983\sqrt{\frac{\mu}{r_1}}$$

so velocity we have got we need the flight path angle.

So, what exactly the situation is right now at this place the v_1 was known the flight path angle this φ_1 was known okay. Now the v_2 is also known at this point say the v_2 maybe here in this direction if this is a v_2 direction. Now we have to calculate the flight path angle. so flight path angle will be calculated from this horizontal. So this is your local horizontal which is perpendicular to this.

So then this becomes your this angle will be $\varphi_2$and this angle is your φ_1 guessing the situation I will draw again. So this is the line here and v_1 is in this direction the local horizontal is like this it makes 90° with this place and therefore this is your φ_1 the flight path angle the other angle v_2 maybe here in this direction. So the corresponding angle this is φ_2 so v_2 is known to us.

We have to determine this φ_2 also and if we get this then we will be able to know what is the separation between these two vectors say this is the v_1 vector and this is the v_2 vector. So, what will be the change? this is Δv how we will know if this we know the difference between v_1 and v_2 vector which will come in terms of Δ v can be calculated using the parallelogram rule.

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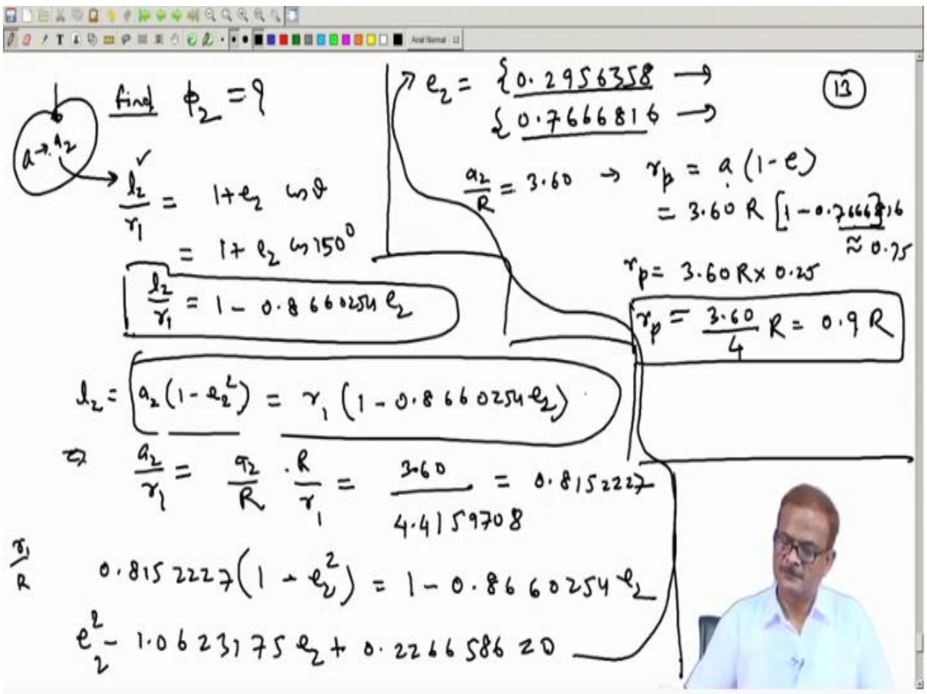

Okay so next step in this is to find out now find φ_2. this is the objective. this is equal to what if we get this our job will be done but still we are away from the final result. At the position μ position here in this place. this is $r_1$okay.and once we are looking for the new orbit that the new orbit it may look like your new orbit may look like this. So for this new orbit you have a_2 value which is different. So for this new orbit we are writing this l_2 because once your a is changing to a_2 or we have used here a_2 we have used here a_2 for the new semi major axis so corresponding to this, this is l_2.

So, this equation we write as 1 + e_2 cos θ so 1 + e_2 so now cos θ = 150°. So, this is l_2/r_1 this becomes = 1 – 0.8660254 e_2 this is r_1, so r_1 times 1- 0.8660254 e_2 and therefore from here we get this as so we have to solve okay a_2/r_1 this quantity is a_2/r times R/r_1 a. So, inserting these values a_2/r is given to be 3.60 and then we will have r_1/R value this is known to us this is 3.6 and we have already calculated. and this was 4.4159708.

So, this turns out to be 0.8152227 and once we use this a_2/r in this place if you use this here so you will have a_2 0.8152227 times 1 – e_2^2 = 1 – 0.8660254 e_2 and now you know this is the quadratic equation okay. So, if you solve it this comes in this format

$$e_2^2 - 1.0623175\, e_2 + 0.2266586 \;= 0$$

okay the solution of the e_2 is then 0.2956358 because it is a quadratic equation.

So, the solution to this comes we will have$_2$ values for this. another one comes out to be 0.7666816 so these are the two values of e_2 which comes from this equation. Now which one is correct. and which one we should use and which one we should not use this is the situation condition right now e_2 is known to you e_2 is known a_2/r this is given to be 3.60 okay. So, let us explore what will be the periapsis position r_p.

So,

$$r_p \;=\; a\,(\,1 - e)$$

this equation we can utilise so here a in this case is 3.60 r and if we use this value or that value. so depending on here you can see that what will the periapsis value? so we need to compute this. So, if we put here 1 – 0.7666816. let us say this I approximate this has a 0.75 for convenience. This has nothing to do with our computation 3.60 R times this becomes 0.25 3.60 divided by 4 R means this becomes 0.9 r that means the periapsis is less than the radius of the earth.

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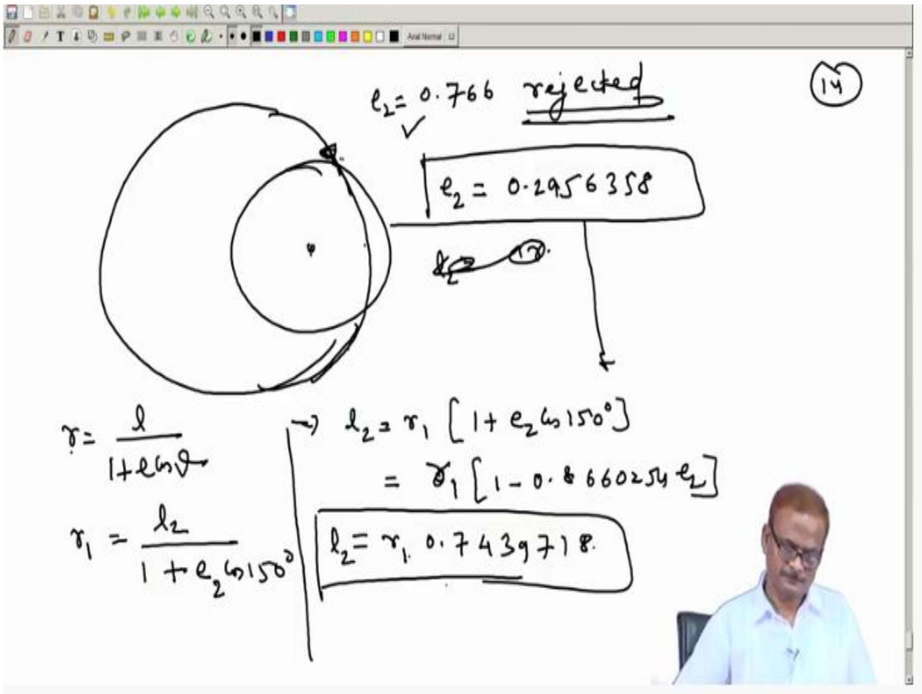

So, this simply implies that if you are taking the solution this solution okay 0.766 for the eccentricity. So, in that case your periapsis it goes inside the earth means the orbit becomes something like this. So, this is not a possible orbit because it is just striking the surface of the earth if the satellite goes in this path so it will simply go and hit the surface of the earth and it will get destroyed.

So this is not a possibility therefore e_2 = 766 this one is the dejected we do not work with this. So, what is our option e_2 = 0.2956358 this is the possibility and we will work with this. So, this is our e_2 so corresponding l2 then becomes denote this quantity r by we know that

$$r = \frac{l}{1 + e \cos \theta}$$

so here if we write r_1 is not changing r is not changing the position the position where we are giving the impulse which is here this point okay.

So, we keep this as r_1 while l will change to l_2 and 1 e_2 will become e will become e_2 and cos θ = 150° and plus and therefore this implies $l_2 = r_1$ times 1 + e_2 cos 150° and this will turn out to be if you use this value. So this will turn out to be r_1 times 1- 0.8660254 e_2 insert the value of the e_2 from this place. So this gets as r_1 times 0.7439718 so this is your l_2.

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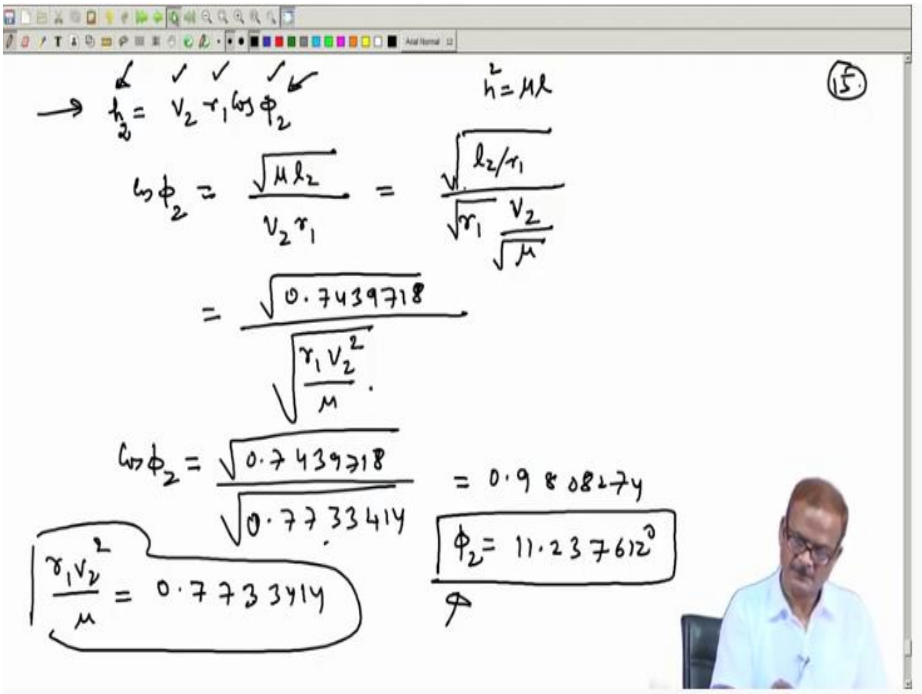

Now we are close to our work end but it is still it will take some 10, 15 minutes and we will wind up this. So h = v_2 r_1 cos φ_2 and h we will write as h_2 because once you are giving impulse so the orbit will change it will not remain the same orbit therefore cos φ_2 from this place as earlier we have used the equation the same equation we are using here only thing with the variables changed here.

Instead of h we are writing here h_2 okay so at the new position this is the $\vec{r} \times \vec{v}$ the specific angular momentum which we have called and this is the v_2 position velocity at the position r_1 this is r_1 itself and this is the corresponding flight path angle. So, cos φ_2 then becomes

$$cos\ \varphi_2 = \frac{\sqrt{\frac{l_2}{r_1}}}{\sqrt{r_1}\frac{v_2}{\sqrt{\mu}}}$$

so from there we are putting it v_2 r_1.

So all of these quantities already we have calculated so once we insert those values we will get the solution l_2/r_1 under root so we will write it like this and insert the values l_2/r_1 is 0.7439718 under root divided by this quantity this will be r_1 v_2^2 divided by μ under root.

So, this quantity is also known to us. So, we just inserting those values 7439718 under root divided by 0.7733414 so this turns out to be 0.9808274 so this is cos φ_2. So, r_1 v_2^2 divided by μ and see from here we can write this φ_2 value, φ_2 will turn out to be 11.237612° this is

φ_2 flight path angle and this quantity we will similarly you can write as this is 77 already we have used it here. So, this is 7733414 so now this flight path angle is available to us.

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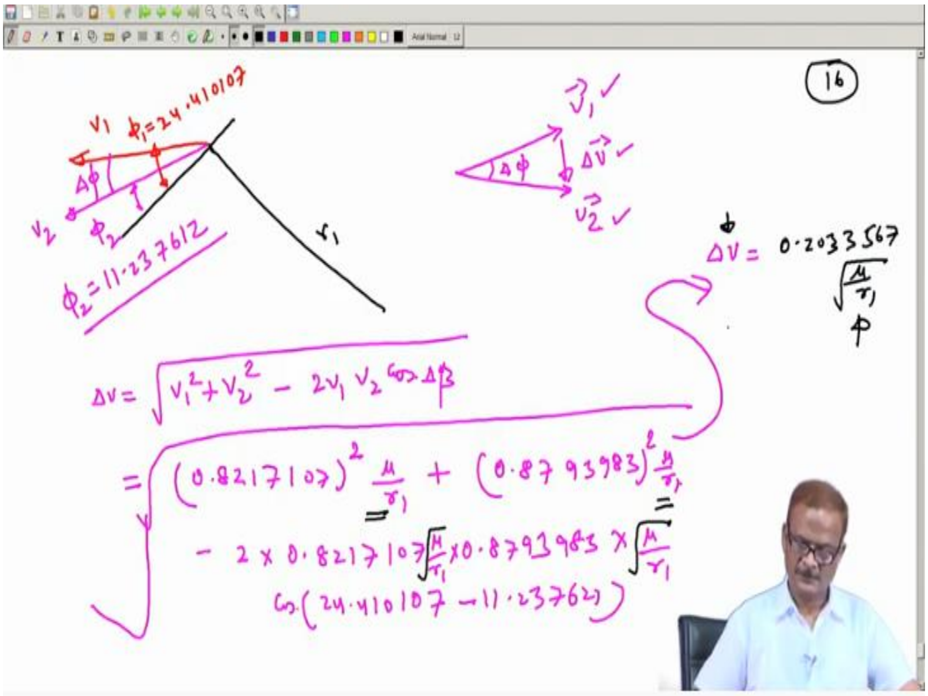

So coming to the final version. so this is r_1 here okay? this is the local horizontal and v_1 is here in this direction. and v_2 is here in this direction. So v_1 angle this angle from this place to this place this is around 24 something we have calculated this is 24.410107 we calculated this is φ 1. The angle. this one right now we have calculated this is φ_2 so $\varphi_2$is turning out to be φ_2= 11.237612.

So, this is your Δφ so if your one vector is here. another vector is here this is your v_1 and this is your v_2. So, you need to give this Δ v impulse while this is Δ φ is given here. So, these two vectors are available to us magnitude wise it is available and we just need to compute this Δv. So, Δv is the impulse magnitude just we require. So

$$\Delta v = v_1^2 + v_2^2 - 2v_1$$

you can look from this triangle here using parallelogram, rule you can construct it..

This will come again in that trajectory transfer because the most of the things were from the orbit part. So, I have done this problem here so delta v you put the values here 0.8217107 square this will be μ / r_1 + 0.8793983^2 divided by μ / r_1 these are the corresponding values we are inserting - 2 times r_1, r_2 where this v_1, v_2 value we have to insert here. So, 0.8217107 times

0.8793983 and times cos $\Delta\beta$. So, cos $\Delta\beta$ is 24.410107 – 11.237621. So, you compute this value this is all under root sign.

So, this will turn out to be Δv equal to okay thereafter μ 1 and μ /r_1 and μ /r_1 is associated with this and this we should also write here this is μ /r_1 times. So, μ /r_1 we can take it outside and this should be a square here. So, instead of μ /r_1 this is square after coming. So therefore we will put here under root sign to this, this is under root this is coming because of square if we look back all these values I am concluding it so this will turn out to be 0.2033567 μ /r_1 under root.

So, this value is known to us so therefore Δv so how much impulse is required in which direction the impulse is required this we have computed from this place.

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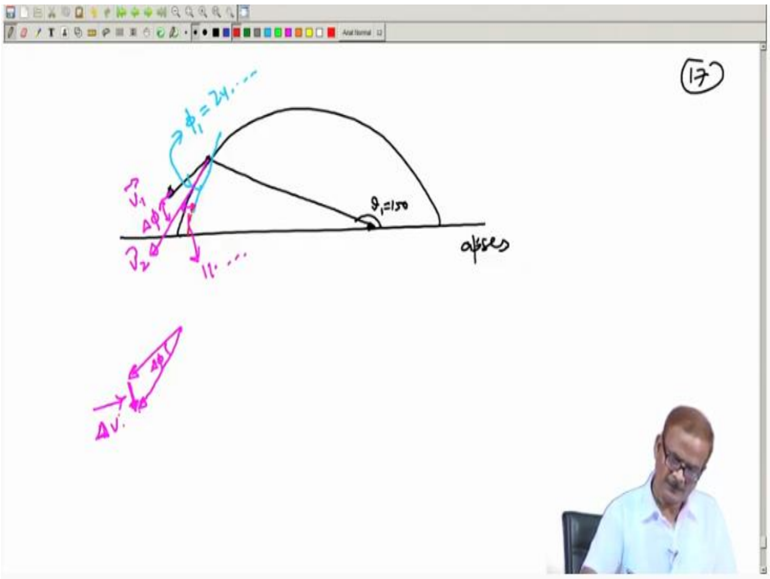

As a final observation this is your apses line from where the focus is located here this was the original orbit 150° lies somewhere here this is θ_1 = 150° okay at this point the v_0 is like this and this is the local horizontal. So, this angle is φ_1 = 24. something here and the new velocity vector is somewhere here this is the original I am not showing the magnitude wise so this is v_1 and this is v_2.

So, the new angle is from this place to this place so this angle is 11. something okay so this is your Δφ. So, in which direction you have to give impulse if this is one vector okay and this is another vector okay and whatever the magnitude is there accordingly depending on this Δφ value you will know in which direction you have to give this impulse and this is Δv is available to us. So, this is the situation.

So, with this new thing your orbit will change and orbit will look like the orbit will look like this. So, overall your a changes without change of the apses. So, the centre of perception remains same but the total from this place to this place this distance becomes your $2a_2$ and half of this will be the semi major axis okay. So, I feel this is enough for today so we have done one major problem here which requires a number of skills and integrating together different situations and solving the problem we conclude here. Thank you very much.