Space Flight Mechanics
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Lecture – 02
Conic Section (Contd.)

Welcome to the lecture number 2, so today again we are going to continue with the conic section, I thought of winding up but there remain something I want to discuss and moreover before that starting the conic section, I will discuss about the satellite motion. So, let us get into the first the satellite motion because it will be useful to discuss this first.

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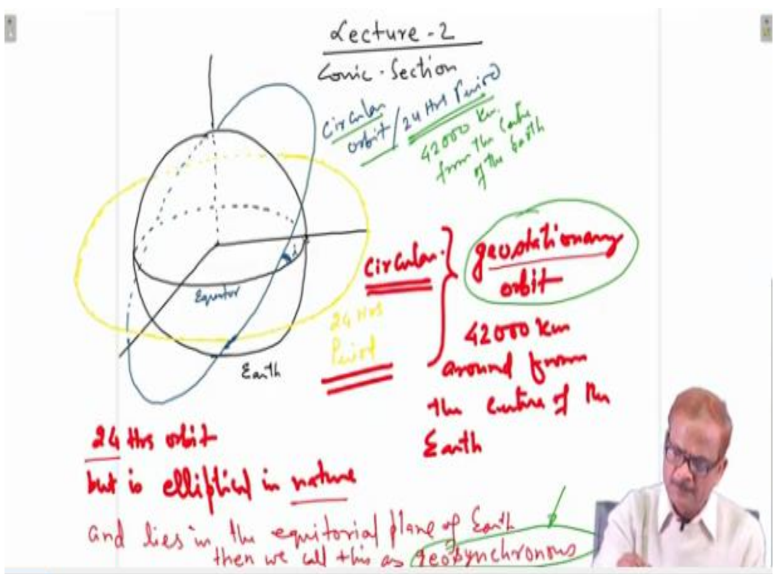

So, what I have shown here, this is say our earth and around the earth, we need to put satellite, so satellite can be put in various orbits around the earth, so if we have an orbit which is in the equatorial plane of the earth, so this is in the equatorial plane of the earth and if it has a period of 24 hours, 24 hours period then and the orbit is circular, so such arbitrary call as the geo stationary orbit.

Basically, this orbit; the geostationary orbit it is located at around 42,000 km from the centre of the earth, so on the other hand if we have 24 hours orbit but is elliptical in nature, if it is in elliptical in nature and lies in the equatorial plane of earth, then we call this as or term this as geo synchronous orbit, okay, this is geostationary and then we have geosynchronous, okay.

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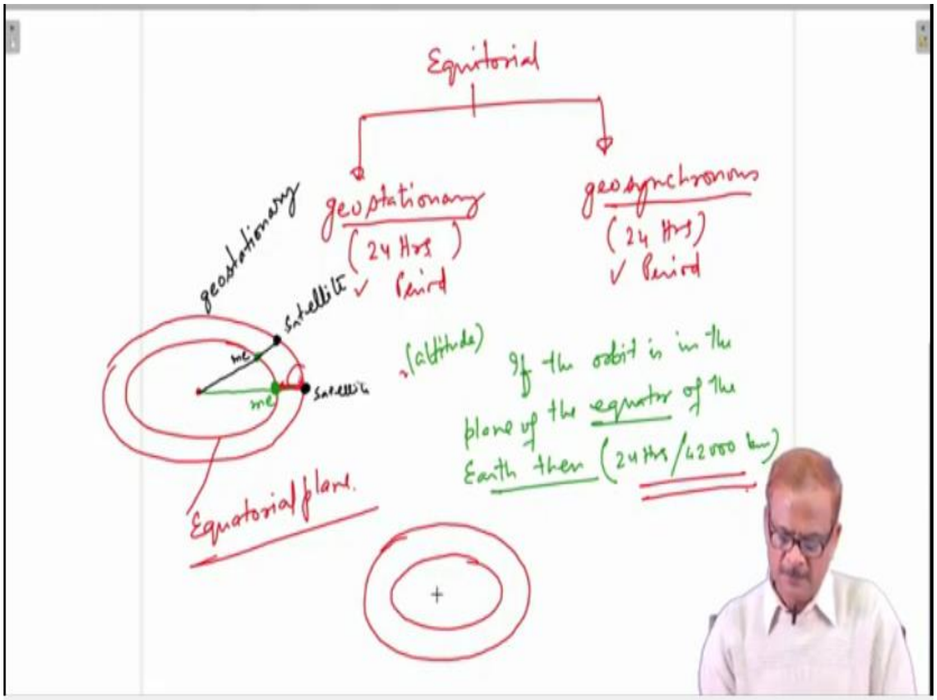

So, equatorial orbit we are dividing into 2 parts; one is geostationary, another one is geosynchronous, 24 hours period and also here 24 hours period, now it is; this is not the only orbit possible, if I have the earth equator here, so I can have satellite at the altitude of even 300 km, so from distance from the surface of the earth to this place, this is 300 km but then we will not call this as either geostationary or geosynchronous.

Because depending on the altitude from the surface of the earth or from the distance from the centre of the earth, the period of the satellite differs. If the period is 24 hours, so this will be either geostationary or either this will be geosynchronous and this would lie in the or equator of the earth, so this is equator of the earth; equatorial plane. Now, what if the orbit lies out of the equatorial plane?

So, in this case suppose, my orbit it lies something like this, if it is something like this that means with the equator it is inclined and suppose this is also circular orbit and period is 24 hours and let us say, I is the inclination here with respect to the equator, this is equator, then here the 24 hours period, okay an altitude assumed that this is 42,000 km from the centre of the earth, so will this orbit be categorised as geostationary orbit or will this be categorised as geosynchronous orbit which I have written here in this place.

So, what is mean by geosynchronous and what is mean by geostationary, this is a primary importance and geosynchronous orbit, it can be circular also and also elliptical but it depends on the inclination with respect to the orbit which we are going to discuss and thereafter I will take again the conic section. So, if the orbit is in the plane of equator of the earth, then and also this is 24 hours period, altitude 42,000 km around from the centre of the earth.

This is not altitude, but distance, altitude we call from the surface of the earth, this is altitude, so this is distance from the centre of the earth, so in this case it will so happened that because the period is 24 hours and you are and suppose you are sitting here in this place, so this is me, let us assume this is me and here this is satellite, satellite is located here in this place, so this is satellite and both are having 24 hours period, means both are moving with the same angular velocity.

So you will, what we will see that the satellite and me, both are always in the same line that means I will always see the satellite overhead, so this is me here and this is satellite. So, if I am at the equator so I will see always the satellite, it is a line over my head, so this kind of orbit we call this as the geostationary because it is a appearing as a stationary with respect to a person sitting on the equator, this is geostationary in that case.

Now, what about geosynchronous; so in the case of the geosynchronous orbit, this is the equator of the earth and again, so here then the distance will be; this distance we will have to modify in that case, this distance will not be then 300 km, see if we are looking for the geostationary satellite, so our distance will not be 300 km rather it is going to be 42,000 km from the centre of the earth, this is 42,000 km.

So, this we have to keep in mind, 300 km just I showed that that is also possible but it will not have period of 24 hours. Now, if we have an orbit which is here what I have drawn, so it is not looking like circular, so let us go to the next page and I will look into this.

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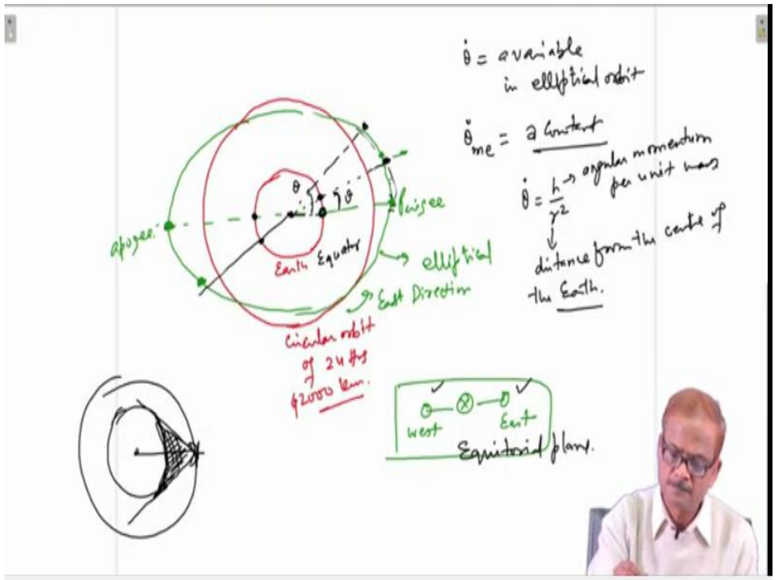

Let us draw a circle, so this is earth and this is your circular orbit of 24 hours, altitude 36,000 km around or say from the centre we have the distance of 42,000 km, later on we will calculate these values. Now, if we have an elliptical orbit of 24 hours means, let us say that it looks something like this, so this is the nearest approach which we call as the perigee or the periapsis and this we call as the apogee.

So, the satellite is here, so let us assume that while I was sitting here okay, now we are discussing about the elliptical orbit, so this is elliptical orbit. So, while I was sitting here on the equator and this is also equator, satellite was directly above my head, okay but will it remain always over my head? No, why? Because the angular velocity of the satellite the rate at which this theta will change what we call as the true anomaly which we are going to discuss in future.

So, this angle it will not change at the same rate as my angle will change, okay that means your theta dot this is a variable; variable in elliptical orbit while my theta dot, this is me, this is a constant, okay. So, therefore what we will see that the satellite is moving sometimes ahead and sometimes it is getting back, so we start from this place I am here and the satellite is here, okay, so this is perigee position, satellite will move faster, angular velocity of the satellite it will be given by

$$\dot{\theta} = \frac{h}{r^2}$$

Here this is angular momentum, this; all these things we will derive, momentum per unit mass and this is distance from the centre of the earth, therefore what we will see that the angular velocity here will be more of the satellite, so this will go ahead. So, I am here, by the time I reach here, so the satellite will be somewhere located here in this place, so it will not be here in this place rather it will be little ahead of this.

So, I will see that it has gone a little, it is not just overhead but it is a little ahead and slowly, slowly it will come and we will match here in this place, so while I reach here in this place, we will match here and again I will be going ahead, satellite will be lagging back, okay so my position will be here, so satellite will not be directly my overhead but it will be lagging somewhat say here in this place, okay.

And ultimately, once I reach here in this place, so at that time again the satellite will be overhead, so what is happening that this is your east direction, so you will see that if this is my position, so satellite will be sometimes on this side and other time it will be on the backside and sometimes it is east and sometime it is west, so it depends on the angle covered and depending on that your satellite will be either appearing ahead or either it will be appearing toward the back, okay.

So, to study this e, suppose you want to study this, so how you will calculate what is the position of the satellite with respect to you, okay and then this is very important in the case of communication system. If you are just trying to look at something overhead and whether that satellite is remaining overhead or not, so it matters a lot, okay little bit of division front and back, so for communication satellite it may not be that much of problem but say for some other purpose you are using.

So, then it is a really a great question, for the communication satellite, if I have a communication antennae here, so the wave can be sent like this okay and it will come under the purview of all, the maximum angle covered will be this one, so you can cover this area on the earth, this particular part on the earth through this communication system. So for that it is not a problem that there is a little bit of division back and forth east and west.

So, if this division is taking place it is okay but in the case where you just want it to be located overhead then there is a problem, okay. So, if the satellite is in the elliptical orbit it will appear to move east and west in the equatorial plane, okay.

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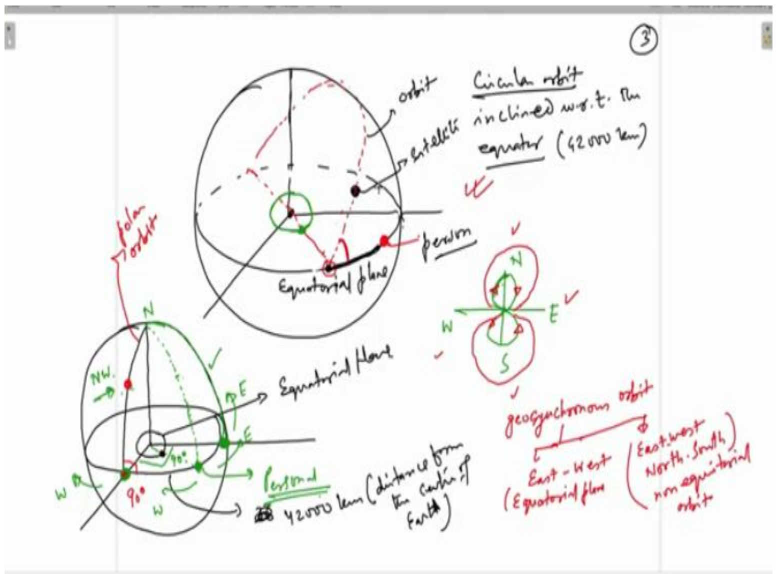

On the other hand, if the satellite is in circular orbit but it is inclined with respect to the equator, okay and again 42,000 km at a distance of 42,000 km from the centre of the earth that means, it is an in an orbit which looks like say which may look like this, so this is the equator and this is your orbit. Now, what happens in this case, suppose this is a circular orbit and or let us say, this is the equatorial plane instead of indicating equator because I am showing the orbit on the this sphere itself, so this is the equatorial plane.

Now, here what we can see that at the time; the starting time both the satellite and the person they are in this place, this is the angular position, now by the time satellite goes from this place to this place, so satellite I will show by some other colour, I will show by red colour, so this is your satellite. So, initially the satellite and the person they are in the same angular position, so satellite has move to this position, okay.

And what about the or say this is the person here, let us make this person because the person will be on the equatorial plane not on the in the inclined orbit, so if person is here and the satellite is going here in this place, so satellite will make by black colour, so this is your satellite. So, in the same time how much angle they will be covering? So, from this graph it will not be obvious to you whether they are going ahead or back of each other.

If I draw another figure, so this thing will be much more clear, now let us look into this figure, the person is here, this is person initially, the person was here in this place now, it has moved here, okay and both are having the same angular velocity because both are in the circular orbit, person is sitting on the earth surface which I have not shown, let us say that this is the equator of the earth and the person is sitting here in this place.

So, here also I can show by using the same kind of configuration, so this is your equatorial plane, okay, so the person is here in this place and on the corresponding 36,000 km altitude; 42,000 km altitude and this distance from the centre of earth okay, so the corresponding distance is here. Now, if we have start looking at this place, here the satellite is we will show by the red colour, so the satellite by the time it moves here, satellite moves here, okay.

And this is the polar orbit; polar orbit means it is making suppose here with respect to this 90 degree with the equatorial plane if it makes 90 degree, I will call this as the polar orbit. Now, by the time it reaches here, it reaches here in this place, let us draw a plane passing through the centre of the earth and the person who is sitting here, so this is person sitting here.

So, if we draw this plane, so this is your north direction and here in this direction we are taking this as the east, okay and towards this it will go as the west, from here this is west on this side, this side this is west, so this is east and this is west. So, what we can see that the person is here, so from this place where the satellite is located? You can see that this is located north west because you have to go north and then you have to go west to catch it, okay.

So, this location you will see that the satellite is drifting, so if the satellite is in the polar orbit with respect to the this the person here with respect to the earth, it is in polar orbit, so with respect to the person, you will see that it has gone north west, okay. So, here it is very much clear that if the person turns by 90 degree angle that means the person comes here, this is your 90 degree angle from this place to this place, okay.

It comes when 90 degree to this place, okay so in that case you will see that it is just lying north, okay here it is lying north west while here this is your north line, okay it is going toward the north, okay again, this will start here going ahead, so what you will see that the satellite is lying north west means, if I show it like this east, north, south and west, so somewhere the satellite it goes like this, okay.

And at this place it is directly north and then again the motion can be completed, satellite will go there and the person is going here in this direction, okay, so still it is a line north but now it is an east, so the motion is taking like this with respect to the person, the satellite motion will appear to be like this, so it will go, it will make a figure of ‘8’.

So, you can see that how it is moving; let us say, starting like this going like this, then it is a crossing here, starting like this and going like this, so it is making a figure of ‘8’, so there is east drift, and also the north south drift and west drift. In the case of the geosynchronous satellite in the equatorial plane only the east west drift was there, so for geosynchronous only east west drift was seen, this geosynchronous in the equatorial plane, this is in the equatorial plane.

While in the case of the non-equatorial plane geosynchronous orbit, east west as well north south this is non-equatorial orbit, so east-west, north-south drift will be visible, so by taking the polar orbit I am made it much more explicit, if you take a small inclination orbit, so in that case, this will not be visible to you, okay. So, and for that reason I have taken the polar orbit and the equatorial orbit, a person is on the equatorial plane and the satellite is moving in the polar orbit.

If we take this orbit, so you can see that in the same time it moves here and it moves here, so it is a very difficult to decipher what is happening whether it is an east-west, for that I need to take the globe, put the angles properly, everything we have to measure and then we will be able to locate on this simple figure, it is not possible to show it so clearly or either we have to do animation and through that it can be shown.

So, this makes it perfectly clear that if the satellite is moving in a circular orbit in equatorial plane then it will be always overhead, if the satellite is moving in the elliptical orbit in equatorial plane, then it will be drifting east and west, if the satellite is inclined with respect to the equator but still it is in circular orbit, you will see both east west drift and north west drift together, it is making a figure of 8.

And if it is an inclined orbit and simultaneously, the orbit is elliptical, so both of them that means the east-west as we have seen for the circular orbit, so the same kind of result you will see. So, for that reason you see that the satellite will not or be always moving in the circular orbit it will be an elliptical orbit in most of the cases, so and from time to time may be if the orbit is also drifting, so you have made it circular orbit.

But it will drift and it will go into the elliptical orbit, so in that case what we need to do; we need to correct it and bring it back to the circular orbit, so we need to know the conic section properly and though we are not doing very advance thing here with respect to the conic section but some elementary knowledge about this it helps in solving some of the problems. So, I have given the background to you that how the satellite moves in the orbit.

And how it appears drifting north-east, east west, so these things will be useful later on, so with this background another lecture, I will continue with the conic section some elementary things and or may be in this lecture, let me complete this conic section part quickly I will go and complete this conic section because this lecture was meant for the conic section, so I will finish it and then we will go to the third lecture.

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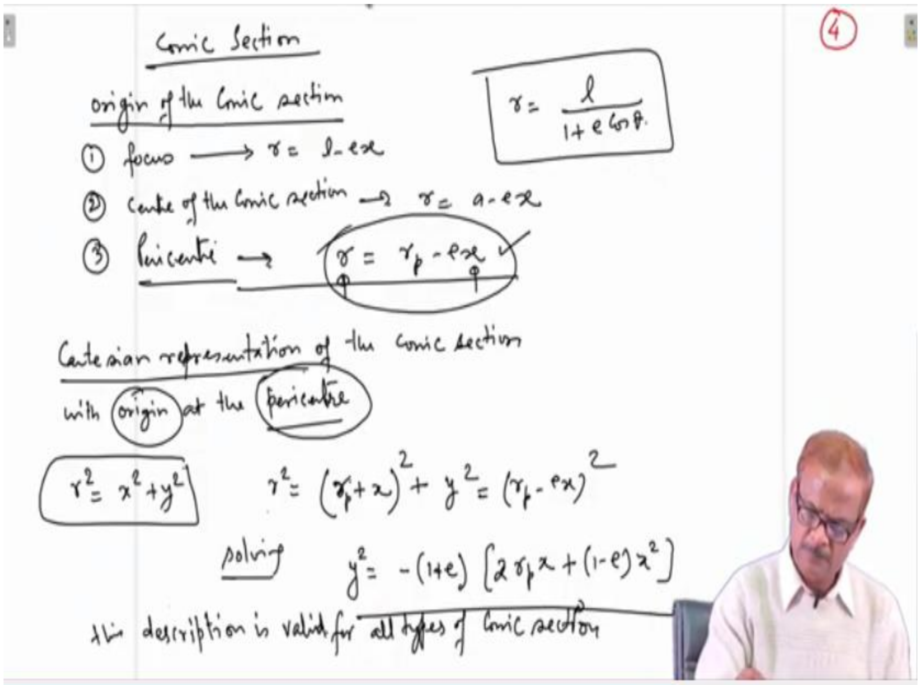

So, in the last lecture what we have seen that origin can be taken at; origin of the conic section can be taken at the focus or at the centre of the conic section or at the pericentre or periapsis, so these are the 3 places at which the origin of the conic section can be chosen and accordingly, we have observed that the corresponding equation will the representation was; this r was written as $l - e\,x$ with the origin at the focus.

And for this it was written as

$$r = a - ex$$

and for the with the pericentre, it was written

$$r = r_p - ex$$

and a general description for the in terms of the r and θ, it was written like this;

$$r = \frac{\mathrm{l}}{1 + \mathrm{e}\cos\theta}$$

So, few simple things we will just have a look here, here your r and x, they are mixed, if you want to just express the whole thing in terms of x and y, so this will call as the Cartesian representation.

So, this can be written as

$$r^2 = x^2 + y^2$$

this is a general description, so here in this case, we have to write it in a proper way, so for our case if the, we are taking origin at the pericentre, so in that case we write

$$r^2 = (r_p + x)^2 + y^2$$

because this is your x as we have written in the last lecture because of the shift of the origin and plus y^2, so this r must be equal to this quantity.

So, therefore this is $(r_p - ex)^2$ and if we solve it, solving this gives y^2, this equal to minus; if you expand it and solve it, so this will be the result; $2\, r_p x + (1 - e\, x)^2$ and this description is valid for all types of conic section.

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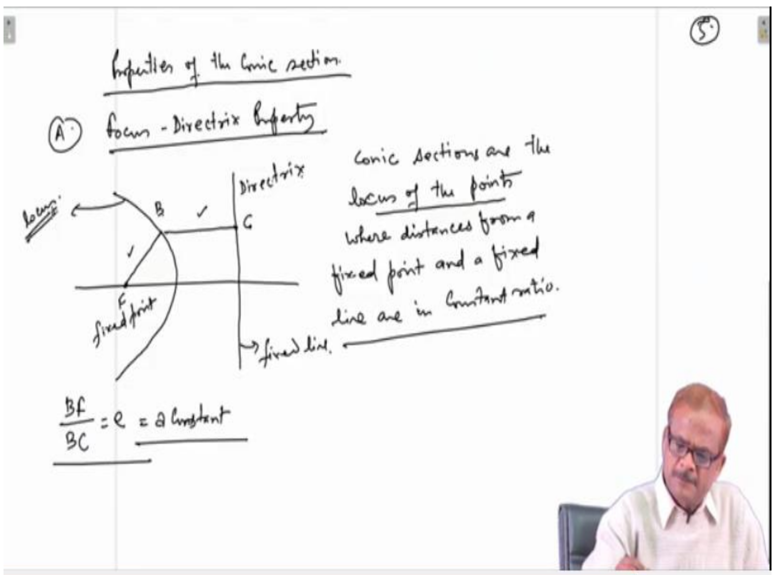

Later on, I will give you type version of all these materials while the course starts, so properties of the conic section. Focus directrix property already we have discussed that is if this is directrix which is a fixed line, this is my conic section, so from the focus this we have written as F and this point as B and here we located C, okay. So, the ratio of this and this distance, this is a constant, so the conic section, so what we have written, $\frac{BF}{BC}$, this equal to e which is a constant.

$$\frac{BF}{BC} = \text{Constant ratio} = \text{e}$$

So, conic sections are locus of the points where distances from a fixed point and a fixed line are in constant ratio, so this is your fixed point, this is fixed point focus and this is fixed line and this is the locus of conic sections are the locus of the point and this is the locus and this already we have discussed in the last lecture.

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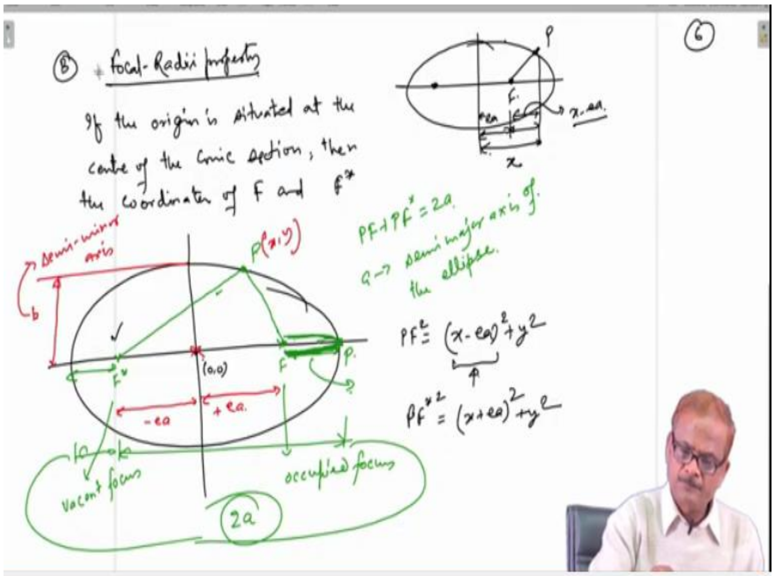

Focal radii property:

if the origin is situated at the centre of the conic section, all these things will be helpful to you while solving certain problems, centre of the conic section, then *F* and *F**, say if we take the case of an ellipse this is your centre, this is (0,0) you are taking this as the origin, okay, focus is lying somewhere here, so this is your *F* and this is *F**, this we call as the occupied focus and this we call as the vacant focus or empty focus.

So, if we have any point *P* on this orbit; $PF + PF^*$, this will be equal to $2a$, where a is the semi major axis of the ellipse and that you can verify very easily through the figure itself while this point comes to this point, so distance from here to here will be; this distance will be this point will be coming here, so *P* comes here, so you will see that it will be overlapping twice, this length will go somewhere here and this length will go from here and again this back.

So, the total will be same thing and back to this point, so this distance will be covered twice from here to here, okay and therefore this distance and this distance from this place to this place, this is also the distance from this place to this place, so this distance added and plus this distance added this in nothing but your total as a whole, this is your *2a*. So, figuratively it is a very simple to see but some of the other conclusions we can also write.

So, this distance we write as *ea* and this is positive and on this side, we write this as -*ea* because this is on the negative side you are taking origin here in this place, this vertical distance from this place to this place, we write this as b and this is semi minor axis and *l* and other things the semi latus rectum we have already shown, I will not shown on this figure, it will get much more complicated, so this is the *P*, its coordinate is *P (x, y).*

So, PF^2, this distance PF^2 this will be nothing but $(x - ea)^2 + y^2$, if you remember the last lecture figure, so if point *P* is line here, this is your focus, okay and so this is *F*, so you know that this distance you are taking this as x and this distance is *ea* from here to here, therefore this distance will be $x - ea$, so therefore here $x - ea$ has been written, so whether this point is here or here, this always remains valid.

So,

$$PF^2 = (x - ea)^2 + y^2$$

$$PF^{*2} = (x + ea)^2 + y^2$$

similarly, this will be $(x + ea)^2$ because it is lying on this the left side and plus y^2.

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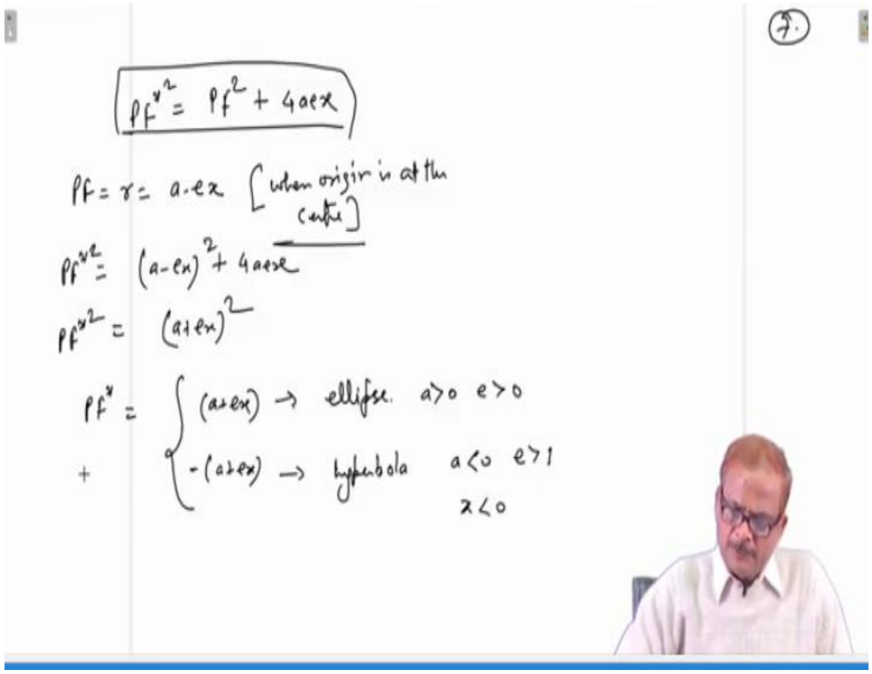

So, PF if you solve it little bit, you can check it yourself, I am skipping those small things that can be done by you, so it can be written here in this way but $PF = r$, this $= a - ex$, this is when this when origin is at the centre and this we have done in the last lecture, so I am not repeating it, therefore the PF^{*2} this becomes $(a - ex)^2 + 4aex$ and this will get reduced to $(a + ex)^2$.

$$PF^{*2} = (a + ex)^2$$

This implies PF* this can be written as $a + ex$ or either $- a + ex$, so this is the case of an ellipse, $a > 0$, $e > 0$, this is the case of an hyperbola, where $a < 0$ and $e > 1$ and $x < 0$, So, these are some of the conclusions which is which will be helpful while working with the problem of the orbits or problems related to the orbits okay.

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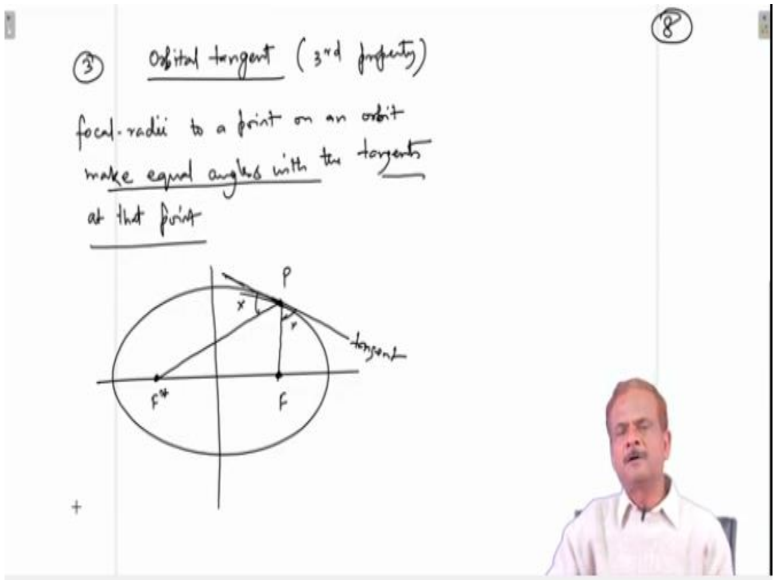

Finally, the third property we will write here, orbital tangent; this is F*, F, so at any point if I take a tangent, suppose this is the tangent here, this is point P here in this place and I joined this point P to F and F*, then this angle and this angle both are equal that is what it says, then make equal angles with the tangent at that point and all these can be proved in geometry, okay. There are few some more properties that we have to look into.

So, this lecture I will conclude here and going to the next lecture, there the conic section; we will continue with the conic section but we will also consider the one body problem basically, what we have written; if we look into the syllabus there we have written as a central force motion, so that we will taking as the one body problem, where the other bodies assume to be fixed, so thank you very much for listening.