Space Flight Mechanics

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Lecture – 29
3 - Body Problem

Okay, welcome to lecture number 29, so we have been discussing about the Kepler's equation, so there remains one problem to be done furthermore. So we will do that but let us start with the 3-body problem. In the 3-body problem if you remember we have already discussed this 2-body problem but in that I did not discuss about what are the forces acting on the system.

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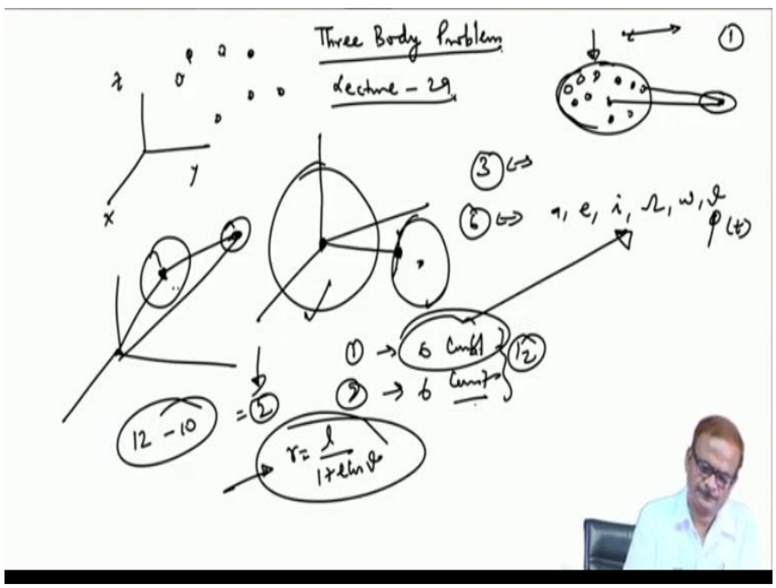

See in the case of 2-body problem it is possible that if your body is finite in size, so the basic gravitational term that you are taking that is not the only term available. There may be some extra; there are some extra terms also and as I told you that are infinite number terms related to gravity. Similarly, in the case of but if we consider only 2 particles, so safely we can describe it by the inverse square law and our job is done.

But that is not the case when the body is little oblate and also then mass is distributed over like this, so you can consider that these are the different elements in masses and then how it will act on some other body which is also finite in size. So we will come to this problem but not right now at this stage, we are going to consider that this these are particles and thereafter. So what we are assuming that we have this x, y, z reference frame and in this we will have different particles of different masses and how they are going to interact with themselves and what will be the consequent motion.

And in the case of the 2-body system or the 2-body problem which we have indeed written for the 2- particle system, we have observed that we can only get the 10 constants and what we have solve; either we can solve it for the relative motion of one particle with respect to the other or either if the one particle is massive like earth and other particle is satellite, so this is earth and say this is satellite.

So, well in this case because the centre of mass is as we know for this the system of particles moving under mutual gravitational acceleration, it is free from the external force and therefore centre of mass will not accelerate okay, so basically it moves with a constant velocity, so given the initial position and velocity, you can always transfer the centre of mass to the centre of the main body or the primary body.

And in the case; as here in this case we are taking the satellite, so safely your centre of mass can be centred here in this place and thereafter you get just 3 different equations, second order differential equation in x, y, and z coordinates and you can solve, that means you are getting a total of 6 constant. So, these are the 3; 6 parameters of the orbit. Now, the important part is that if this satellite is not a particle but this is also finite size like this, so we have 2 system; one here, another one here, we have the coordinate system here, one big massive body here, another massive body here.

So, in that case we get 2 equations; vector equations, so and that vector equation; the first vector equation will give us 6 constant and the second vector also equation gives 6 constant, so total of 12 constants we got and out of this 12 constants, only 10 we were able to identify and rest 2 we are not able to identify that means, only these are the 10 constants available to us. What we are looking for; we are looking for a closed form solution, okay.

Obviously, as I told you that if the initial values are given, you can always integrate the equation of motion and you can get the feature positions, so you know the how the system of particle is evolving but the close formed solution like the; closed form solution we have written

$$r = \frac{l}{1 + e \cos\theta}$$

for the conic section, we have derived for the 2- particle system, the motion of the other particle with respect to the first, second particle with respect to the first one, so this we got is close form solution.

But this we got for a relative motion, one particle with respect to other, in the case we have the one satellite, a small satellite and I have the earth, so safely I can assume that the centre of mass is lying at the centre of the earth and thereafter we can write the equation of motion because with the initial condition, your centre of mass velocity will be known, velocity and position and that can be transferred to the centre of the earth and thereafter you can work with only 6 constant.

The same thing also it is applicable to the 3- body system but here or the n body system which we in the celestial mechanics or in the basic mechanics we call the n particle system, so n particle system there is no more than 3; 3 or more than 3 particle system, it will not have any closed form solution, so we can only under some restricted condition, we can get certain solution to this; analytical solution or the analytical closed form solution.

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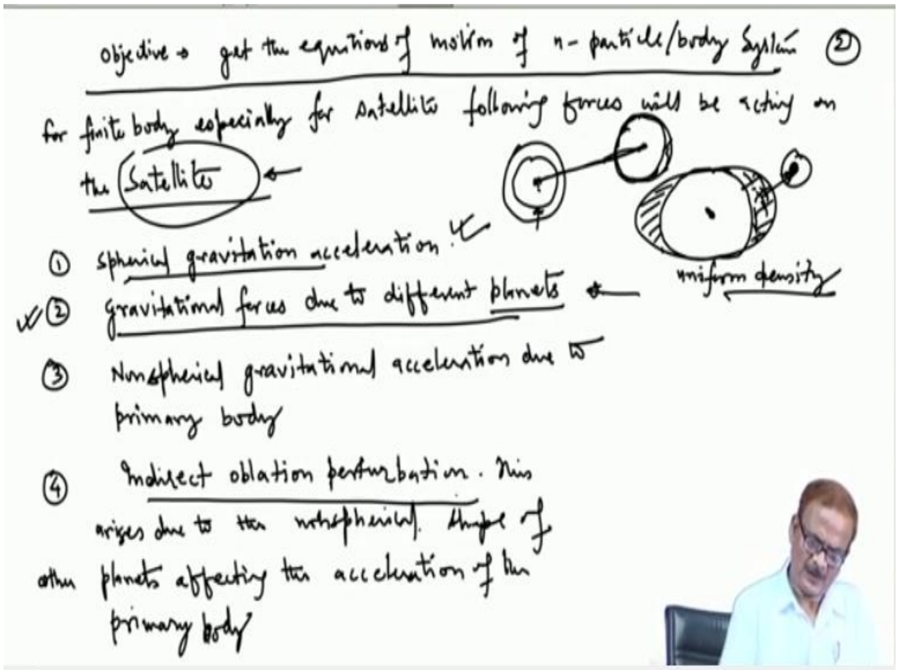

So, this is what we are going to learn here, so our first objective is to here get the equation of motion of n particle system or n body system, we are assuming that the whole mass is concentrated at the; at a point only under this, this will be valid. So, for finite body especially, in the case of the satellite, the following forces will be acting on the satellite. One is a spherical gravitational acceleration which all of you know.

So, this is especially valid for the satellite, gravitational force is due to different planets, remember that in a short run, this term will not be of any importance but if you are looking for a longer time, you are looking for a mission where over a longer period of time your orbit will change because of the perturbation from other planets, so then this becomes an important factor.

Then, we have non- spherical gravitational acceleration due to primary body that means, if your planet is not a spherical but oblate say, here is your satellite, so see if the planet is like this, the spherical, so it is a different issue, only the first one will be acting which is called the gravitational acceleration, in that case this gets oblate, so this will create an infinite number of terms in the on the gravitational; in the equation for gravitational acceleration.

Gravitational acceleration it is a vector, okay so you will get infinite number of terms in that and that was assuming that this is of uniform density, if you do not assume this to be of uniform density the situation is much more complex, then there are one more term is this, this is called the indirect oblation perturbation, this arises due to the non-spherical shape of other planets affect the; so this arises due to non-spherical shape of the other planets affecting the acceleration of the primary body.

That means, in the case of say if here is the earth and there is one Jupiter, okay big in size and if non-sphericity is taken into account or other planet you consider, if it is a non-spherical so this is going to also affect acceleration, already I have told that here in this case, this is non-spherical part of the earth it is affecting the satellite similarly, the non- spherical part of this also it will affect the main planet around which your satellite is moving.

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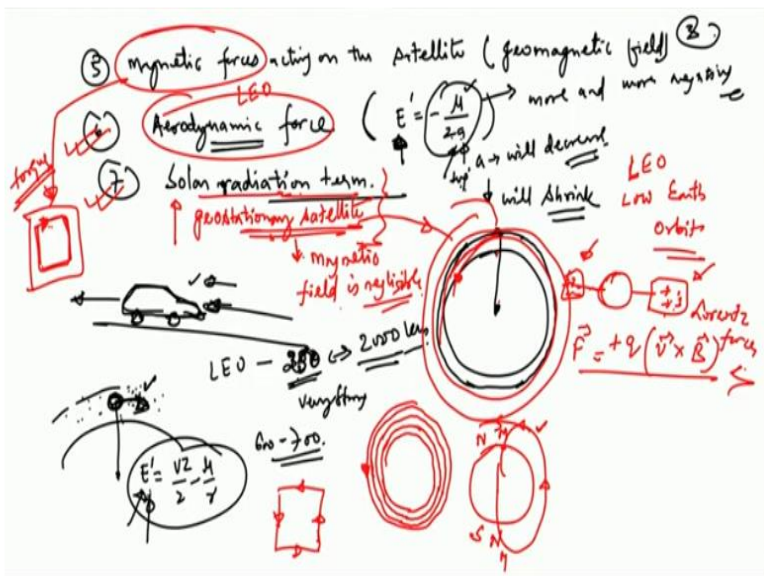

So, this term we are calling as the indirect oblation perturbation term, then we have other terms like the magnetic forces acting on the satellite, this is because of the earth's magnetic field, geo magnetic field acting on the satellite, this is the geomagnetic field. Then the aerodynamic forces; aerodynamic force, solar radiation term, okay. So, aerodynamic force this especially needs mention as you know for the ellipse, we write it like this as specific energy.

So, aerodynamic force this is a dissipative term, okay that means you see in the case of your car, it is moving on the road, so it is facing the drag, similarly in the while the satellite is moving in the space around the earth, so in the lower earth orbit; low earth orbit say 300 km especially at 250 km, where the satellite is launched, so 250 kms move onwards up to 2000 km, we can consider this to be a low earth orbit.

But the at higher altitude, this becomes the aerodynamic effect becomes small, if you consider 250 km, it is a very strong here in this case, very strong, at 600, 700 km, still it remains up rise sea level. So, here in this case there are small particles, atoms, others are present, ions, electrons. So, basically this is your atmosphere is rarefied here, okay, so while it passes through the atmosphere, if it is moving in the atmosphere, so just as the car fills the drag, the same way your satellite also fills the drag, though this magnitude is quite small, it is very, very small as compared to what we get in the case of a car.

So, this; so as you know that if you leave the accelerator in the car, you remove the your right foot from the accelerator, so that means you are no more giving in the input power, so in that case whatever the velocity is there, car will slowly start decelerating, only some this idle power; the idle power from the engine will be given as the input, so this car will start decelerating, the same way this also, the satellite also decelerates.

So, as a result of that that means, it is losing energy, so E' which is written as

$$E = \frac{V^2}{2} - \frac{\mu}{r}$$

so this E', the total energy of the system or the satellite it will disappear, so if this goes down, what will happen? So, if the left hand term it is a decreasing, so this indicates that E will decrease and thereby you can see that the right hand side; this side is positive for the ellipse, a is this is also positive and then this start decreasing.

So, this means this becomes more and more negative, and therefore as a result of your power dissipation, this total energy dissipation of the not power, energy dissipation of the satellite, it is semi major axis will shrink okay, so a will shrink, so this is has a very simple connotations. See, here if your satellite is like this, okay and say this orbit is now little elliptic and it is going like this, it is on this side, it is a smaller and this side it is a little longer, this is the centre of the earth.

So, if the energy decrease, semi major axis will shrink, so this way after certain time, it will start from the beginning itself, it will be decaying rather than showing here, I will show it here like this, this orbit will decay like this, though very small reduction but it will decay. In the case of the you know that India one shot down one satellite as an experiment using a missile, so and it was told that and it was in the low earth orbit.

And it was told that those debris created while the satellite is broken, so debris is created, so these debris will decaying or it will enter the atmosphere and burn out, okay, it will be clean from the space of, so this way it decays, okay, it is going here on this side, so it is a decaying. So, if it is decaying and some orbit if it start decaying like this, so what does it mean; slowly it will come and even if it does not burn not completely, it will hit the surface of the earth.

And the same thing is also applicable in the case of the say, your meteoroids and other particles, if the meteoroids of course, if the it so happens that the trajectory is like this, so it will go and hit directly but in the case of the satellite this decay it takes place in this way, so for the semi major axis shrinks and it will come and hit the surface of the earth or either it will get into the dense atmosphere and it will burn out.

Because of the friction, very high temperature is generated and that temperature will burn out the satellite, so this aerodynamic term is very important, the solar radiation pressure is also very good, it is a factor which is important for geostationary satellite, in the low earth orbit it is not so dominant but in the geostationary orbit it is a very dominant, here this is very dominant in the LEO; low earth orbit, LEO we use for low earth orbit, it is a very dominant there.

And magnetic force is say, if you have on the satellite, the solar panels are there and the solar panels some charges get generated because of the bombardment of the ions from the atmosphere or the from the those coming from the space and they are going to hit and if so this is a basically, a charge say the plus q and this is moving through the atmosphere, so this will produce a force on the satellite which is given by this equation, okay.

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

so this way; this is basically a Lorentz force, so this also produces force on the satellite. The current inside the satellite, then you have the inside the circuitry is there inside the satellite and in that circuitry, you have the current following, there may be residual current, even if something is not operating but there may be some residual current remaining.

So that interact with the magnetic force to produce torque, okay, if it is a closed circuit, here you know that in a magnetic field, if I have a coil, current carrying coil, so it produces a torque in a uniform magnetic field, in a non-uniform of course it will also experience force but in the uniform magnetic field, it will experience only torque, so it depends on the size of the satellite.

If the size is very large and this earth magnetic field it is a something like, on the north side this is the south magnetic pole and the south side this is the north magnetic pole, so from the north side field emits like this and it goes like this. So, depending on the altitude then this starts decaying, as you gain the altitude, so here you see in the case of the geostationary satellite, the magnetic field will be negligible or magnetic field is almost absent, you can write, field is negligible. Here itself, it is a very small in the LEO orbit, so there almost its vanishes, so these are some of the issues related to the satellite.

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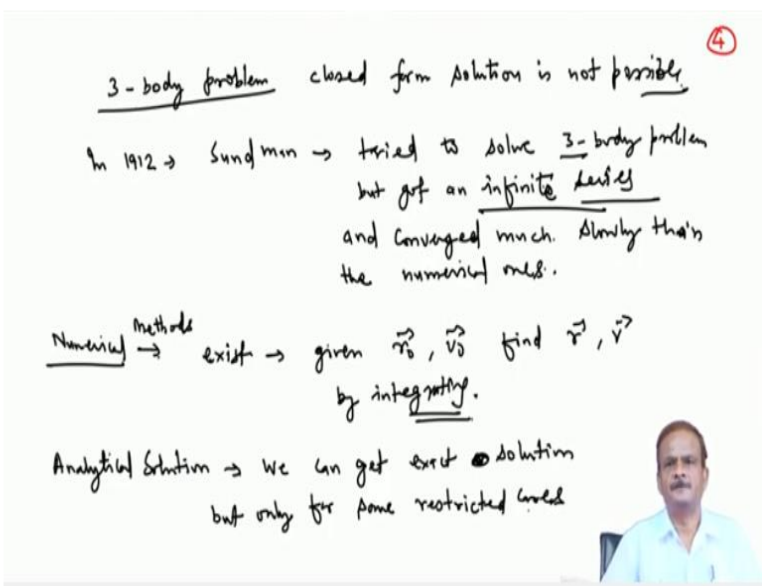

So, we now look into the 3-body problem but we have been discussing about; so for 3-body problem, closed form solution is not possible. In 1912, Sundmann try to solve 3- body problems but got an infinite series instead of a closed form, closed form solution means you are representing the equation in terms of finite number of terms, okay. If there are infinite numbers of terms, so it is not a closed form solution.

This is the basic concept, so he tried to solve it but he got an infinite number of; he got a solution which has infinite, which was in the form of an infinite series and converge much slowly than the numerical ones and hence loss is importance, so there is no benefit of doing that, if you have to solve an infinite series say, you have infinite series in terms of sine, cosine and other things, so by doing that much operation, you will do simply by doing the numerical integration, which will be much more easier and it will take very little amount of effort not in terms of computation or but also working with, okay handling it.

So, why to use than the infinite series; so it has not been successful, so 3-particle system or more than 3-particle system, it cannot be solved, only in the case of especially, the restricted 3-body problem under certain conditions, under certain assumptions, the 3-particle system can be solved and that too in the restricted condition as I am telling and these are the issues which we are going to tackle.

And this is very important, the 3-body problem very important for our further discussion of the variation of parameter which is the, what we term as the part of general perturbation theory, a special perturbation theory as I told earlier also we are not going to discuss because it is a numerical method and numerical method it does not constitute part of this course, okay until and unless you really implement it on the computer, you are working with this, there is no meaning in discussing all those things.

You can find the numerical methods in some standard books like the Vallado, Astrodynamics by Vallado, so that book is very good in terms of many other things, it is a comprehensive book, you can look into that and for regarding the mathematics in astrodynamics, you can look into Bacon, okay. I have already given the list and another book by Danby, it contains a number of problem solved, not unsolved problems and it is a very challenging problems also.

So, you can attempt solving those problems also, so Danby is also a very good book and the text book already I have defined by Curtis, so you can refer to Curtis and there are number of solved problem also and unsolved problems also, so that will help you build this course. So, in the solving this in the numerical methods; numerical methods exist which works by; so given r_0, the initial position and velocity, find r, v by integrating.

Now, like 2-body problem, 3-body problem will not be as simple as what we have got in the Kepler's equation, we can get exact solution but only for some restricted cases as we will look later on, so we will stop here and continue in the next lecture, thank you very much.