Space Flight Mechanics

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Lecture – 03
Conic Section (Contd.)

Welcome to the lecture number 3, so we have been discussing about the conic section, so some more material is remaining to be discussed, so we will finish that and thereafter, we will start with the central force motion.

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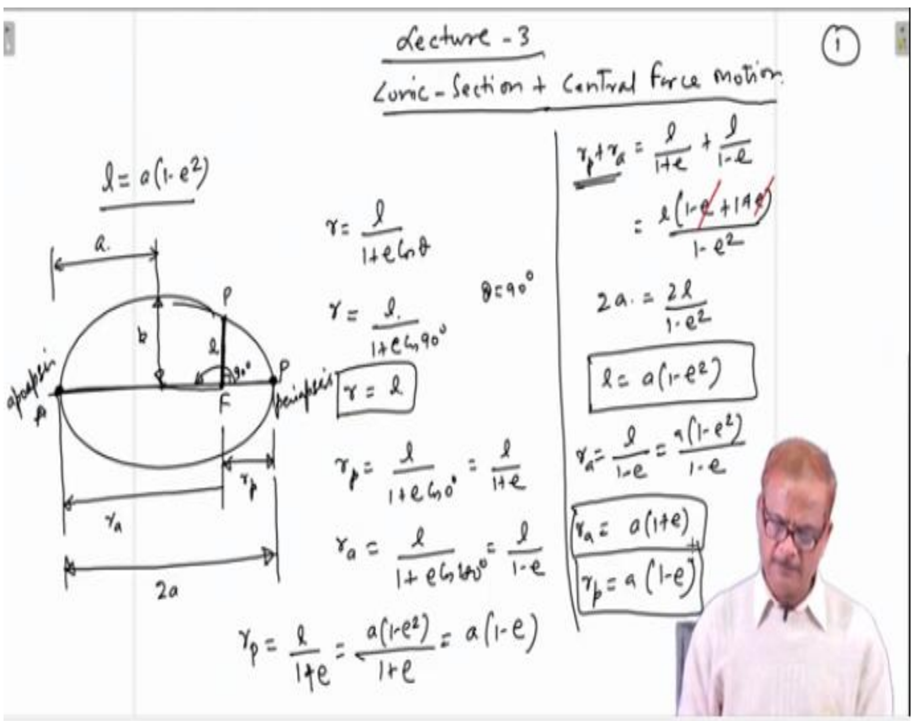

So earlier if you remember I have written

$$l = a\,(1 - e)^2$$

but I have not shown you how this is coming okay, so let us consider the case of an ellipse, this is the focus, so through the focus we draw a line which is perpendicular to the axis, so this is 90 degree here, okay so this distance we have taken as l, the centre is lying here in this place, this is your F and let us say this point we write as P.

Now, the conic section equation we have written as

$$l = 1 + e\, cos\theta$$

so we know that if θ equal to 90 degree, so we get this as $1 + e\, cos\, 90$ that means, this is l, so $r = l$, so this distance this is your r, now from here to here this becomes your equal to l, this distance from the distance from this place to this place this is your written as rp and distance from here to here this is written as ra, so this is your apogee; this is apogee and this is perigee.

Let us write this as apoapsis and periapsis, okay this total distance this is *2a*, the half distance we have taken as a, which is the semi minor axis, so semi major axis, this vertical distance half we have taken as this distance as *b*, so this total distance is *2a*, so r_p this equal to $\frac{1}{e}$ cos 0 degree because this angle if we are taking this position, this is perigee and let us say this is apogee, so $\frac{l}{1 + e \cos 0}$, this equal to $1/e$.

Similarly, *r* apogee this equal to $l\ /\ (1 + e\ cos\ 180)$ so we are looking at this position, from here to here, so this angle then it is 180 degree, so in that case this becomes $1 - e$,

now add both of them so, if we add both of them; $r_p + r_a$, 1 + e + l by 1 – e, so a goes as common, 1 – e + 1 + e, this, this cancels out and what we get here this is

$$2a = \frac{2l}{1 - e^2}$$

and on the left hand side this quantity is nothing but 2a.So, therefore

$$l = a\,(1 - e^2)$$

and this is one of the result we have used, so you should remember

$$r_p = \frac{l}{1 + e}$$

and if I substitute here

$$l = a\frac{1 - e^2}{1 + e}$$

so this becomes $a(1 - e)$ and similarly, *r* apogee then this gets reduced to $l/1 - e$ equal to so

$$a\frac{1-e^2}{1-e} = a\,(1 + e)$$

this is r apogee and r perigee is a times 1 – e, so these are some of the elementary results quite often we required to work out the problems.

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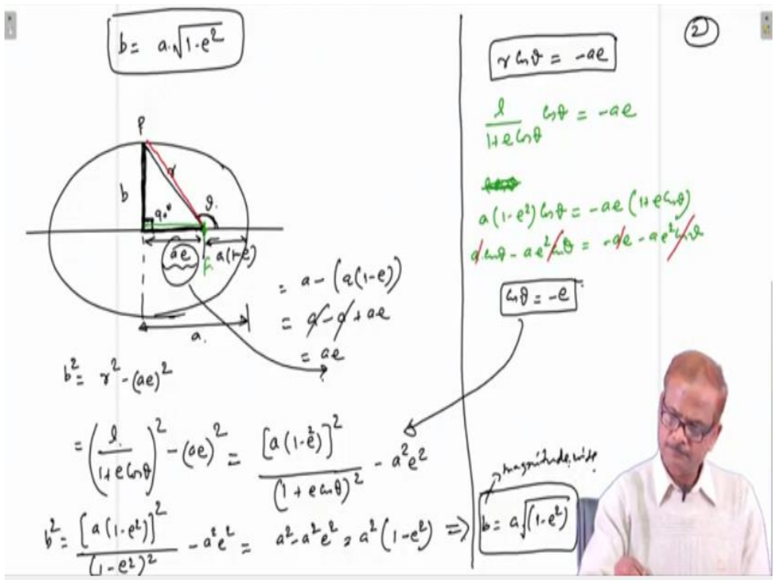

So, the next result is $b = a\sqrt{1 - e^2}$ and this we need to prove and this is also a very useful result which you will need quite often to work out the problems. So, again take the case of an ellipse, here this is the focus, this distance we are writing as *b*, from here to here this is our *b*, let us take this as *r*, this is your point *P* now, this angle is θ, this distance we know, this is *a* times *e* and this distance from here to here, this is *a* times *1 - e*.

So, this configuration we will be able to work out this problem, so we can write

$$b^2 = r^2 - (ae)^2$$

because why this is ae; you can see it easily, this distance from here to here this is *a*, so from *a* if you subtract, a – a(1 – e), so that gives you a – a + ae, so these 2 cancel out and what we get, this is ae, so this is what is we are getting here this as the ae.

So, here b^2 using the this right triangle we can write here b square equal to, this is 90 degree here,

$$b^2 = r^2 - (ae)^2$$

now this equation I am sort here in this place and sort the value for the l, so $a\ (1 - e^2)$. Now, you can also see that a is the projection of *r*, so r $\cos\theta$; θ is here greater than 90 degree, so

$$r\ cos\theta = -\ ae$$

this will be the projection is here, the projection of *r*, this is your *r* from this place to this place.

So, it is a projection is lying here from this point to this point and we are putting here minus sign because we are taking distance from the focus, this is your focus, so on the left hand side, this is -ae and it is appropriate to do that because this angle is greater than 90 degree. So, now if you put here l + e $\cos\theta$;

$$\frac{\mathrm{l}}{(\mathrm{l} + \mathrm{e}\cos\theta)}\ cos\theta\ = -\ ae$$

and then expand it, so l cos θ will insert for this also a $(1 - e^2)\cos\theta$, this equal to $-\ ae\ (1 + e\ cos\theta)$.

And a cos $\theta - (ae)^2\ \cos\theta$, so what we see that these 2 terms get cancel out and also a and a from here this cancels out and we get here in this place

$$\cos\theta = -e$$

So, if we insert this in this equation, so what the result we will get is

$$b^2 = \frac{\left(a(1-e^2)\right)^2}{(1-e^2)^2} - a^2e^2$$

you can see that $1 - e^2$ this cancels out and this leaves us with

$$.= a^2 - a^2e^2$$

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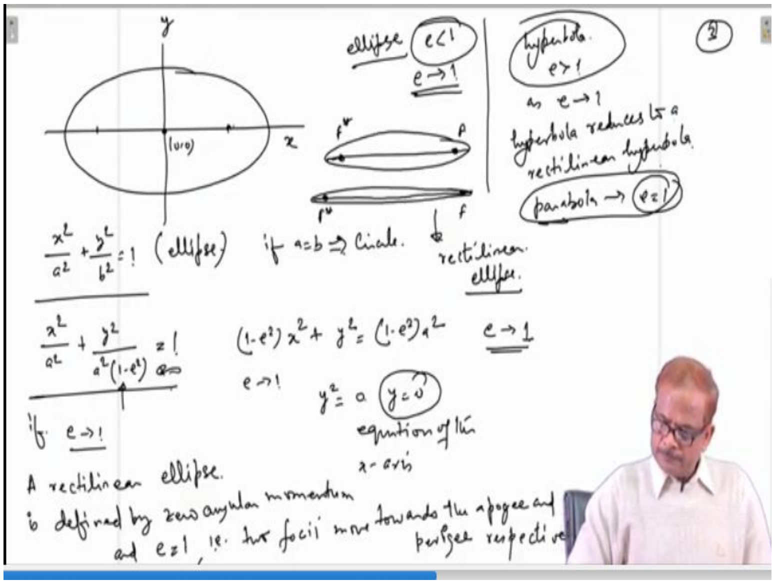

And this implies

$$b = a(1 - e^2)$$

this is what we are taking this, so this is magnitude wise. A simple format of an ellipse with origin at the centre of the an ellipse, here X- axis and Y-axis taken along this direction, so this we know that we can write this is as from our geometry, so this is for an ellipse is a equal to b, this implies a circle, this gets reduced to a circle.

Now, if we have inserted for b here in this place, so if e tends to 1, so what happens in that case? If *e* tends to 1, so this will be tending to 0, so $1 - e^2$, so we cannot divide it like this, so we have to little rewrite it, so we can write it as

$$(1 - e^2)x^2 + y^2 = (1 - e^2)a^2$$

this is after rewriting, So, as e tends to 1, so this gets reduced to y^2 equal to 0 or y equal to 0.

So, what happens in this case; this is the equation of the x-axis, so that means you have an ellipse, the focus is here, so as the eccentricity becomes 1, okay, right now the eccentricity is having some fractional value for the ellipse, $e < 1$, so as *e* approaches 1, so in that case so this gets degenerated into *a*, so it will be looking something like this, you are moving the, it is becoming more and more eccentric.

So, it will move toward the end of the ellipse, okay the focus is moving, this is *F* and they are this is *F**, here this is *F*, this is *F**, so this gets generated into a rectilinear ellipse similarly, for an hyperbola, the same thing is applicable for the hyperbola, e is greater than 1 and has e approaches 1, the hyperbola reduces to a rectilinear hyperbola. Similarly, the parabola will also get converted into; for the parabola already e equal to 1, so we will skip this case.

We will consider only the ellipse which is $e < 1$ and for the hyperbola where $e > 1$ and as e approaches 1, this gets reduced into the either rectilinear ellipse or the rectilinear hyperbola, this is how we say. Remember that e is approaching 1, e not equal to 1, let us say e approaching 1, so rectilinear ellipse that is two focii moved towards apogee and perigee.

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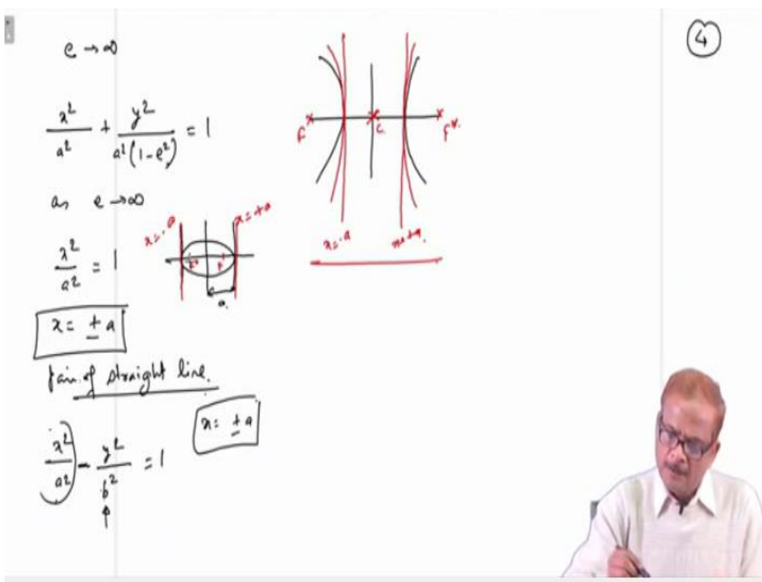

Now, what happens if e tends to infinity, so in the case of an ellipse, so as e tends to infinity, this is

$$\frac{x^2}{a^2} = 1$$

this gets reduced to this equation,

$$x = \pm a$$

So, this is a pair of straight lines that means your ellipse gets degenerated into a pair of the straight lines, so here your ellipse will get degenerated, this was your ellipse, this is the distance from here to here a.

So, this will get degenerated into a pair of a straight lines which will be this is $= -a$, on this side this is $x = +a$, this is also applicable to the hyperbola. In the case of hyperbola, you have

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

in the case on the hyperbola you write this quantity just ellipse part will change and instead of writing here plus, we are writing this with a minus sign.

And the same conclusion you will get because as e tends to ∞, b will tend to a very large value, magnitude wise and therefore only this part is left, okay so this gets reduced to x equal to plus minus a, so there if you have hyperbola like this, so this will degenerate as straight lines as e tends to a higher and higher value to a straight lines, this is $x = -a$, $x = +a$, this is the centre.

So, about the hyperbola, we can keep discussing something more and there is no end to all these geometric configuration but I want to end this part here and start discussing about the central force motion.

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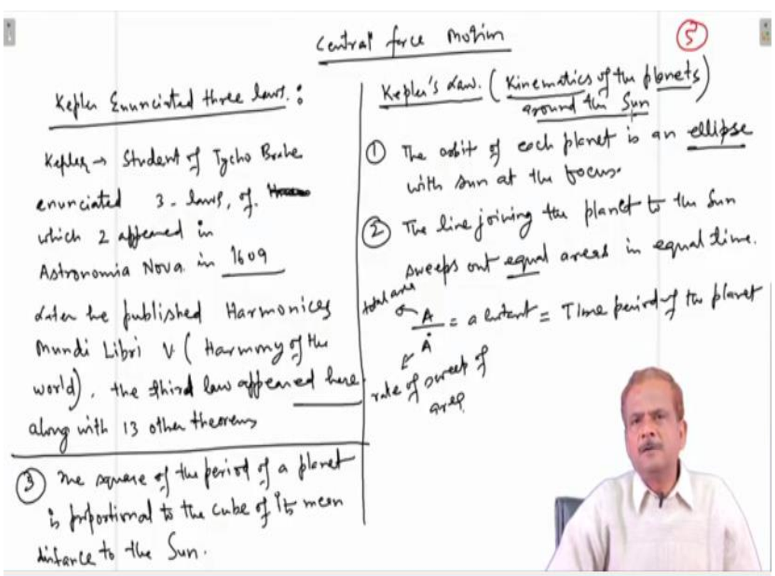

So, we have start with the central force motion, so when we think of the central force motion, so the most natural movement is of the planetary system about the sun or the motion of the satellite around the earth, so and this study can be credited to especially to our Kepler who enunciated, so Kepler enunciated 3 laws and this came from he was the student of Tycho Brahe and he assisted him; Kepler assisted him the Tycho Brahe in all the measurements and the data which was collected, later it was analysed by Kepler and he came to certain conclusions, okay.

And those conclusions were later published and he has written a number of books, okay so some of the books he has mentioned theses 3 laws, so Kepler was a student of Tycho Brahe enunciated three laws of which two appeared in Astronomia Nova in 1609, later he published Harmonices, Mundi Libri V that is harmony of the world, the third law appeared here along with 13 other theorems.

So, what are those 3 laws of Kepler, so they are basically known as Kepler's laws and they described the kinematics of the planetary system, planets; kinematics of the planets around the sun. So, the first law is which planet move around the sun in an elliptical orbit, so this can be written as the orbit of each planet is an ellipse with sun at the focus. The second law this is the line joining the planet to the sun sweeps out equal areas in equal time.

Another way this can be written as a divided by a dot equal areas, sweeps out equal areas in equal time that means the rate of sweep of area is constant, okay so this is a constant and if this is the total area of the ellipse and this is the total area and this is a rate of sweep of area, okay then this is written as the time period here, this is the time period of the planet and the third law we can write here in this place.

The square of the period of a planet is proportional to the cube of its mean distance from the or the mean distance to the sun, these are the 3 laws regarding the planetary motion, he enunciated and these are very important but it was unfortunately, it just explain the kinematics, why this is happening; we cannot explain from here but later once the Newton's law were used to describe the motion of the planets as a particle around the sun. This problem was solved.

And all the 3 laws can be derived from the Newton's law, so we will stop here this lecture and we will continue in the next lecture okay, where we have start with the Newton's law and look into the central force motion, thank you very much.