Space Flight Mechanics
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Lecture – 31
Body Problem (Contd.)

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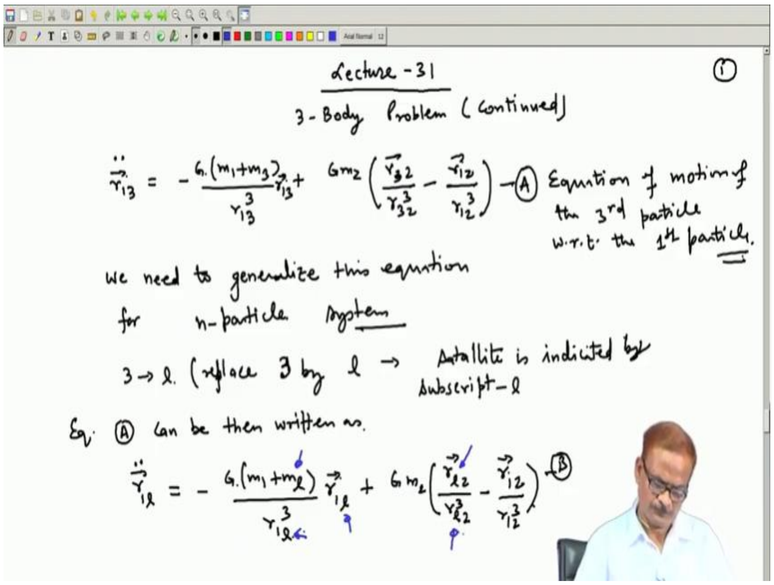

Okay welcome to lecture number 31 already we have derived district equation, this was the equation of motion or the position vector, how the position vector of the third particle it changes with respect to the first particle. So, equation of motion of the third particle with respect to the first particle and in one short we have not written.

This first we have written the equation of motion for the particle one then for particle three and then subtracted to get this equation and nowhere we have validated equation of motion which is described in inertial frame using Newton's second law and this we will always keep care of. Now we need to generalize it equation for n number of n particles or simply we can write this as n particle system.

For that what we that we write here replace 3 by l that is the satellite or even you can use the s notation to indicate that this is the satellite I have chosen just l. So, replace 3 by l so satellite is indicated by subscript l. Therefore the this equation we will write it as (A) equation (A) can be then written as $\ddot{r_{11}}$ = - G here we have missed out this term r_{13}, G $m_1 m_3$ divided by r_{13}^3. This we are replacing 3 we are replacing by land therefore we replace this by l here and this 3 also we need to replace by l.

And then again, we rewrite for okay so here in this case we have just replaced the satellite by the subscript l everywhere. So, 3 is replaced by l in all the places now in the next step we replace 2 by j.

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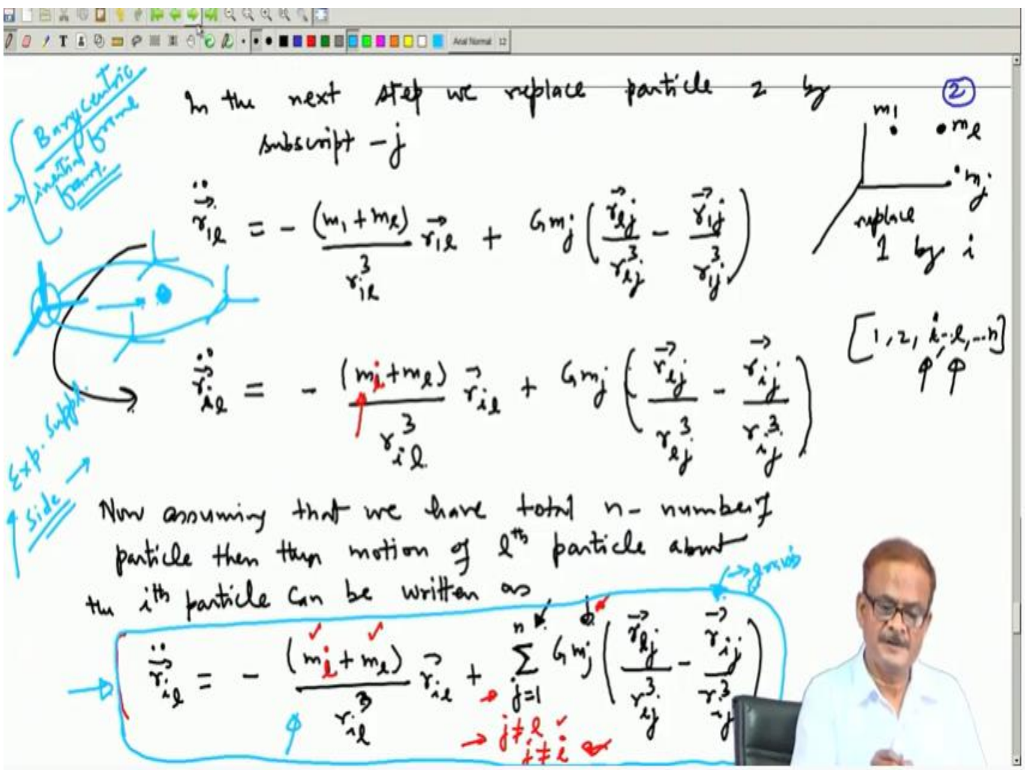

So, next step we replace particle 2 by subscript j and if we do that, we will have $\ddot{r_{1l}}$this will be equal to

$$\ddot{\overrightarrow{r_{1l}}} = -\frac{(m_i + m_l)\overrightarrow{r_{il}}}{r_{il}^3} + \sum_{\substack{j=1 \\ j\neq l \\ j\neq i}}^{n} G m_j \left(\frac{\overrightarrow{r_{lj}}}{r_{lj}^3} - \frac{\overrightarrow{r_{ij}}}{r_{12}^3}\right)$$

Now suppose I have to replace suppose I replace this 1 by i if we do that. So, this will be il double dot I am taking time in doing this because I 'am going step by step r_{il}^3 G$m_j r_{lj}$. So, we have converted this all the equation previously we have written in a generalized notation. Now assuming that we have total n number of particles then the motion of l_{th} particle about the i_{th} particle can be written as $\ddot{r_{il}}$ now the perturbation term will be more.

Perturbation term we have to increase r_{il}^3; r_{il} + here the summation follows. So, Gm_j r_{lj}^3 minus and summation from where to where that we have to take care of. So, here in this case the summation is extending over this j. Therefore, summation will move from j to n and we will write here j = 1 to n but out of this out of 1, 2 say l and before this the i comes and then lastly the n comes. So, so out of this i and l will not be counted here in this place in this summation which is indicated here into this summation showing.

So this implies that we have to write here j is not equal to l and also j is not equal to i. So, these are the terms we need to put here j not equal to i because the its particle already its involved here now here also we have to change we have not changed here in this place though this is your in mi we have changed here but here this was missing. So, we need to change here 1 we have done this, this is also in mi.

So already the rts particle is involved here and lts particle this is involved here and here whatever is coming this is because of the jts particle. So jts particles I need to remove this j = l and j = y and sum it over and therefore this j is from 1to n but out of that we have to remove the case j = l and j = i and this is your equation which gives you a generalized form of equation of motion of the lts particle with respect to the its particle.

That means it maybe your satellite it can be moon with respect to earth or in general you to go about any planet say about the Saturn or you have the Jupiter. Jupiter have its own satellite so in that case accordingly you can describe this equation remains valid throughout provided we choose the frame; frame we need to choose okay properly. So in this case for the solar system we our frame remains the Bary centric frame or Bary centric inertial frame if you are going to this will serve you as the inertial frame beside this you can have your inertial frame located at the Jupiter.

So, if you are looking for how the with respect to the Jupiter if the satellite is moving so in that case you have to locate the initial fame at the Jupiter. Jupiter is moving around the sun so obviously this is not an inertial frame because it is accelerating towards the Jupiter is accelerating toward the sun and therefore it cannot be an inertial frame. But this frame does not rotate along with the Jupiter just adjust translating, translating means already I have described its orientation will always remain the same.

As I am showing here however correction can be given and these are advanced topics which you can find in book by Seidelmann. So, it is called the explanatory supplements by Seidelmann. So, you can refer to that book if you have access to that book okay? So every time we need to take into account one inertial frame and depending on the situation, we have to locate it if required proper correction has to be given and all those things what are required for modeling these things these are given in not explanatory supplement by Seidelmann.

Seidelmann exactly I cannot remember this all the alphabets in his name so I will leave it, but it is a vice explanatory supplement by Seidelmann. So, you can look into that it starts with side something like that Seidelmann in some of the libraries that this book may be available. Now we have got this so this is our main term, and this is our perturbative term I have stated earlier and here in this case now one more point I would like to say here see this perturbative term here we are taking only due to gravity. So this is only due to gravity.

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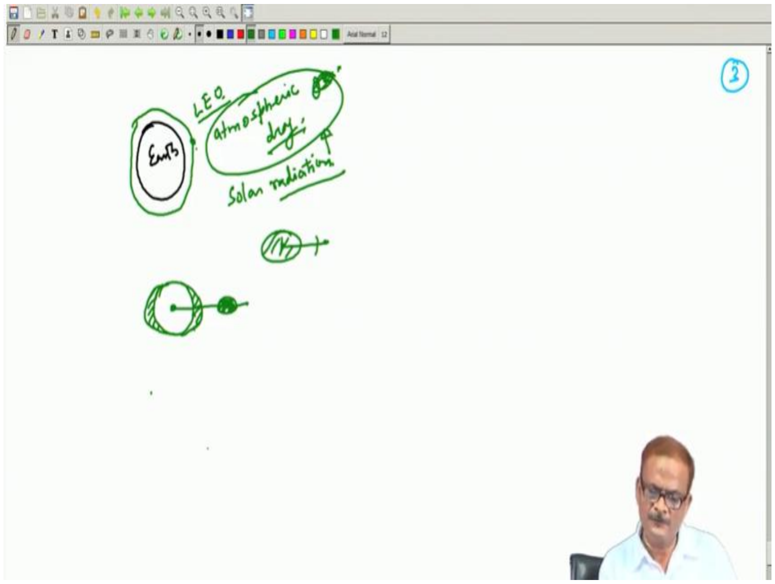

But in the case where a non-gravitational forces are present just in the case of say earth is here, this is your earth and around this satellite is moving in the orbit, so it is going here in the orbit and if it is in the low earth orbit LEO low earth orbit. So at that time the atmospheric drag will also act on this atmospheric drag this will also act on this and this one we cannot represent using the potential function while this term we can indicate it was in potential function and to therefore if we write in terms of potential function it becomes a little easy to handle.

Then we will have the solar radiation as I have mentioned earlier solar radiation then earth is not a perfect sphere okay? If it is a perfect sphere, then there is no problem but if it is not a perfect sphere and satellite is also of not you cannot treat the satellite as a point if you have a little larger satellite. So, then the situation becomes different earth is oval at that means extra mass present here it is a nonuniform.

So because of that our equation here what we are writing here in this place this gets we have to write it in a proper way this is just for a particle mass system it is written but if you have not particle mass system then that situation is different if earth is a particle satellite is a particle you are treating like this this is okay. But earth is a finite mass and that too often it the situation becomes different so all those things they can appear as the perturbative term okay and we need to model them properly so we will take this term as much as possible as we advance. Okay so for now we will look into the Bary centric formula.

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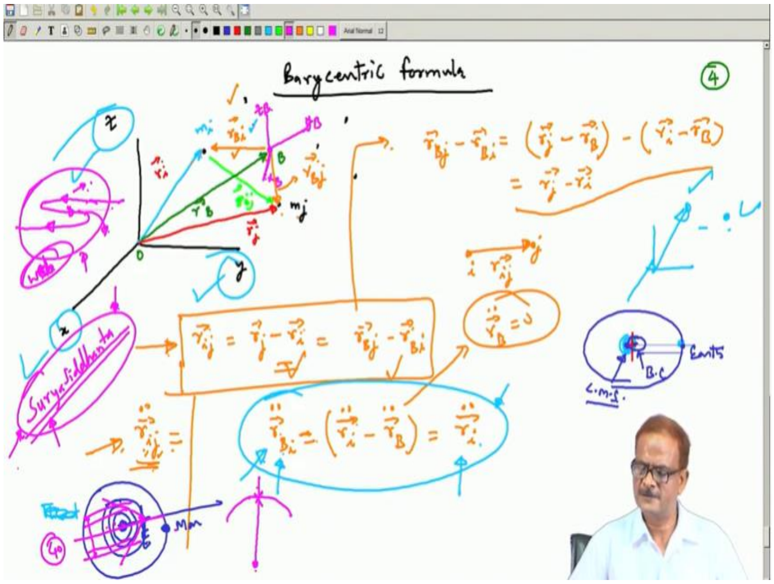

So, here I am not going to derive the whole equation I will show you that either you write in an inertial frame as I have done earlier or with respect to the Bary centric frame both are same this is x, y, z and we have number of particles here okay and let us say the Bary center is located here in this point.

This is our Bary center mi and m_j we show here on this point for clarity this is r_j and this is r_i, r_{ij} and this is Bary center is lying here. So, Bary center to the Bary center we are this letter say at this point is B to if you can integrate this as rr_{oB} this is the point o and about the Bary center you have the difference frame located these are three axis of the Bary center X_B, $Y_B$and Z_B and we can connect this point r_{Bi} on this side we can show this as r_{Bj}.

So what we can notice from this place that

$$\overrightarrow{r_{ij}} = \vec{r_j} - \vec{r_i}$$

and already we know that the center of mass of this mutually this is a very visible from the basic principle that system of particles which is moving under its mutual gravitational act a system of particles or the particles which are moving under mutual gravitational acceleration and a free from external force like the drag or any other thing solar radiation or any other magnetic force from the external sides.

So those things are absent only the gravitational force is present the mutual gravitational attraction. So the system of particle which is free from the external force moves it is the central mass moving with constant velocity and I will show it definitely I will do that for two particles already we have done this and along the same line it can be done for the three particle system okay?

So, Bary center is a point which is the center of mass of the system and it moves with constant velocity and depending on the initial condition it may be either at rest or moving with constant velocity these are the two situations. So here r_j minus r_i is equal to r_{ij} we have written and this can also be written as $r_{Bj} - r_{Bi}$ which is obvious from the this figure and you can check also this can be checked here like

$$\overrightarrow{r_{Bj}} - \overrightarrow{r_{Bi}} = \vec{r_j} - \overrightarrow{r_B}$$

from here this O will be removed from this place this is simply r_B and then - r_i this vector how this vector will be expressed this will be expressed by

$$\vec{r_i} - \overrightarrow{r_B} = \vec{r_j} - \vec{r_i}$$

which is true on the left hand side we have written.

So this is the basic fact that once we are looking for motion of one particle with respect to other so it does not matter which inertial frame we are using the motion equation of motion would be described in inertial frame but it is a independent of the reference frame chosen and this will be visible from this point okay the if you are interested in writing like this the equation of motion for the jts particle above the its particle so vector going from this is the jts particle this is the its particle and this is your r_{ij}.

So this will depend just on the either you write it here in terms of this or either here in terms of this both are equivalent as shown here in this place? And therefore the way we tend to describe the motion of one particle with respect to other it is in dependent of the reference frame and we will complete this. So for this point is very much visual and we do not need to elaborate any further on this okay?

So, now let us look into this issue here r_{Bi} which is the point here so motion of the its particle we are looking for $\ddot{r}_{Bi}$. So, what this quantity will be $r_i - \ddot{r}_B$ and as I will be showing you that $\ddot{r}_B$ this quantity is going to be 0. So, therefore this blitzer dues to $\ddot{r}_i$ so this is the fact here that the motion of the its particle with respect to the Bary center which is B okay it is shown here okay with respect to the Bary center it will be the same as the motion of the its particle with respect to the x, y, z differences and this fact can be used to simplify your problem wherever it is required.

So, with this we can conclude that the distills okay for here before writing I would like to state few more things if you look into the books okay many of the books there describe the equation of motion in terms of Bary center but it is not necessarily you write in any frame okay and thereafter you will do the subtraction to get the motion of one particle with respect to other you get the same result it is not different.

Because the Bary center is also an inertial frame and you are choosing another difference inertial frame individually you are writing the equation of all the particles so newton's law is perfectly applied in that case there is no change in no error done in that and thereafter once we subtract the equation for different particles by dividing by the corresponding mass.

So we get the relative motion equation it is so easy to work with and therefore do not get this confused if somewhere it says written with respect to the Bary center or I am using here this x, y and z frame and in most of the places I have used this kind of notation that this is my frame these are the particles I have written equation of motion for this particle and then for this particle and then subtract it to get the equation of motion of one with respect to other.

So, there is no problem in that again and again I have repeated that this is valid okay or either you do with respect to the Bary center that is also valid this one is much more easier to comprehend much more easier to write okay. Bary center it may be a little complicated from places to places but it may be required the production time that you need to express your equation of motion with respect to the Bary center.

So, in the case of sun-earth system what exactly the thing is happening? Your center of mass is located inside the sun itself somewhere this is your Bary center here in this place okay and then the sun will be moving about this Bary center though this is the Bary center and center of mass of the sun is located as soon by this blue dot here this is Bary center and this is center of mass of sun and here this is your earth the center of mass of the sun will be moving above the Bary center like this and about this Bary center which will also move like this.

So, this is the mutual motion but if you are looking for in that case what you get you have the about the very center you can describe the motion of the sun and you can describe also the motion of the earth. But most of the time we are interested in finding with respect to the earth how my satellite will look like or with respect to the sun how the motion will appear. If you look into the as we look at the sun from earth so how it is the motion appears.

So again, it is a from that we call the geocentric point of view while sitting on the earth we are looking at the motion of the sun. So because it is diurnal motion means it is daily motion it is rotating on its axis because of that you see that sun is rising it is going in the sky and then it is a setting it is raising in the east and then setting in the west and depending on your this latitude it will appear either overhead at some times or it may be inclined many situations they appear.

So geocentric point of view how the sun will appear to move above the earth so this can be one view okay similarly the other planets what other planets what happens if I have the sun here and say the earth is here shown by e and somewhere the mars is here. So these are the orbits now looking at the from the earth how the mars will appear to move this can be another question then inside we have the mercury and Venus.

So how the mercury and Venus will appear to move so you will see that as the sun is moving appears to move about this earth as we see from the earth then the Venus it remains confined to this range mercury remains confined to this range. So around perhaps a 40° as seen from the earth this angle is confined to some 40° it is not a 40° on the each side I do not remember exactly.

But is always this will be visible other planets may not be visible at some time but the mercury and Venus they are always visible from the sun from the earth because they are moving above the sun and their deviation will not be more than 40° if I remember correctly okay. So in that case that also is described in the book by what we call in the book called the Surya Siddhanta, this is a book which deals with the planetary motion.

So, this describes the motion of the planet with respect to the earth it is a little complicated and only the geometry is used to solve the problem. Because earlier the computers were not used so used so using the geometry this motion were described and therefore you might have heard the retrograde motion of certain planets, so it is basically while you are looking from the earth. So how the other planet they appear to move it is you going like this?

Suppose some planet is going like this so this is called a they call it the direct motion and after some time from the earth it will appear to move in the backward direction and then again it will become in the forward direction. So, this is because just we are looking from the earth sitting on the earth, we are looking at different planet. So, because of this our position this kind of motion appears so this is called a direct motion and this motion this is called a retrograde motion/

Okay and you can look on the webpage if you search for this the retrograde motion of the planets as seen from the earth so we you will get the figure exactly because I cannot make that much good figure here in this place and using the graphics you can have a look of all those figures. So, it is interesting that the same thing what we are doing today this was done much earlier in time but in a different way.

And obviously at that time there is certain difference in the distances but not much but the precision in the angle which is a very good because it was based on geometry, so the angles were maintained trend distance was not of much importance in this case. People were interested in knowing the position of the various planets and the sun from time to time. So, we close this chapter here and then we will move to the next lecture. Okay thank you very much.