Space Flight Mechanics
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Lecture - 44
Restricted 3-Body Problem (Contd.,)

Welcome to lecture number 44. So, we have been discussing about the restricted 3 body problem in that context, we looked into the expression for the Lagrange range points and thereafter, we also worked out what will the location of those Lagrange points in terms of the earth moon system and earth sun system.

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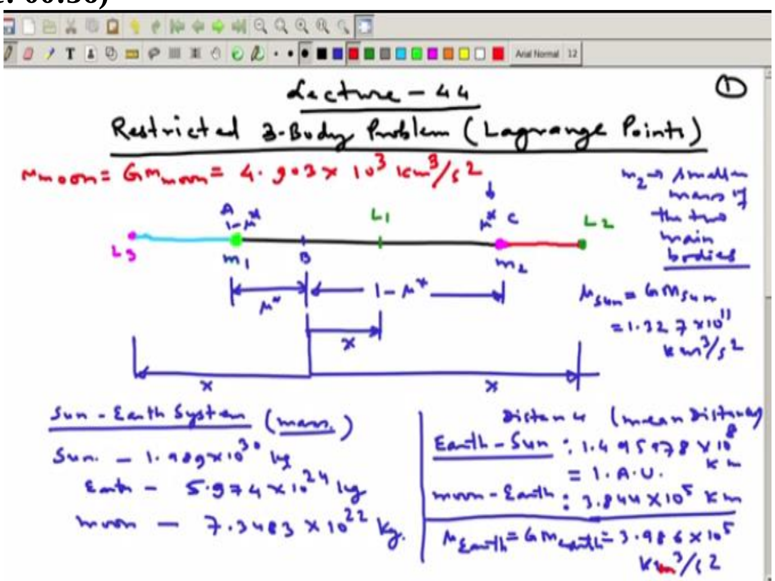

So, we proceed further with that. So, already we have looked at the Lagrange points we have denoted by x. So, you need distance from here to here to this Lagrange point and this has been denoted by X. So, from here to here, to L_2 this is again X the same way to L_3 this is also has been indicated by X, so for the sun earth system and sun moon system. So we will list some values of this mass of the sun, earth and all those things.

So that μ^* can be estimated and all the cases the μ into we are taking as the smaller mass. Mass of the 2 main bodies, so for sun the mass is 1.989×10^{30} kg, for the earth 5.974×10^{24} kg and for moon this is the mass of the sun, earth and moon. Now the respective distance is earth-sun distance 1.45978×10^{8} . And this is written as one astronomical unit, this is the mean distance. For the moon-earth system this is 3.844×10^{5} km.

So, these are the mean distances also, we can list μ sun which is nothing but G times m sun 1.327 $\times 10^{11}$ km cubic per second square, generally μ earth 3.986 $\times 10^{5}$ km cubic per second square. So, these are the values given to us and using this we have to okay for moon also we can note down because for moon also we required some wherever we can note down μ moon 4.903 10^{3} km cubic per second square.

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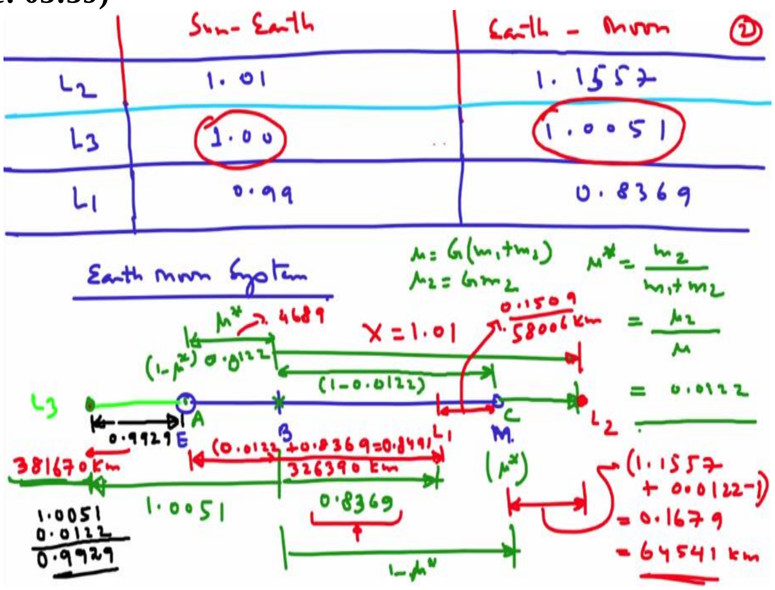

So with this value available to us we can explore what are the locations of those Lagrange points so already we have solved and in the case of the L_2 point or the L_2 point and then the sun-earth system and the earth-moon system. Already we have written last time 1.1557, similarly for L_3 we have for the earth sun this is 1.00 and the earth-moon we have written 1.0051, this we walked out last time.

So, remember the L_1 is the intermediate point; L_2 is the rightmost point we are integrating and L_3 is the leftmost point in our configuration as shown here in this place. So, therefore for the earth moon system we can show all these points like this here the earth is located, this is your earth and this point moon is located, this point is your barycenter say accordingly as the values are given here. So, we can see on this side, L_2 is line here on this side and from X is being measured from this place.

So, L_2 is located here and X is measured from this point, so this is 1.01 on the normalized scale, similarly on this side, L_3 is line here. So L_3 is located here, and it is a distance from this point then, according to this 1.0051 and L_1 is given to the 0.8369. So somewhere from the barycenter this is located at certain point this is L_1 and distance from here to hear this is 0.8369 on the normalized scale. Here in this case the mass of moon we have we are integrating by μ^*.

So, the earth mass is being integrating by 1 - μ^*, this distance from distance from this place to this place, this we have indicated by μ^* and distance form this place to the moon this is 1 - μ^*. So, μ^* this distance or either from mass because it is a normalized scale is a non-dimensional therefore,

$$\mu^* = \frac{m_2}{m_1 + m_2}$$

or μ_2 divided by μ where, μ is nothing but G times (m_1 + m_2) and μ_2 is nothing but Gm_2.

So, on that scale, the μ^* turns out to be 0.0122 this distances is 0.0122. So, what will be the distance up to the moon, it can be computed this will be 1 - 0.0122 from the barycenter, the location of the moon, you can check immediately where the barycenter we will line with respect to the center of the earth. Therefore a rough calculation we can do here in this place.

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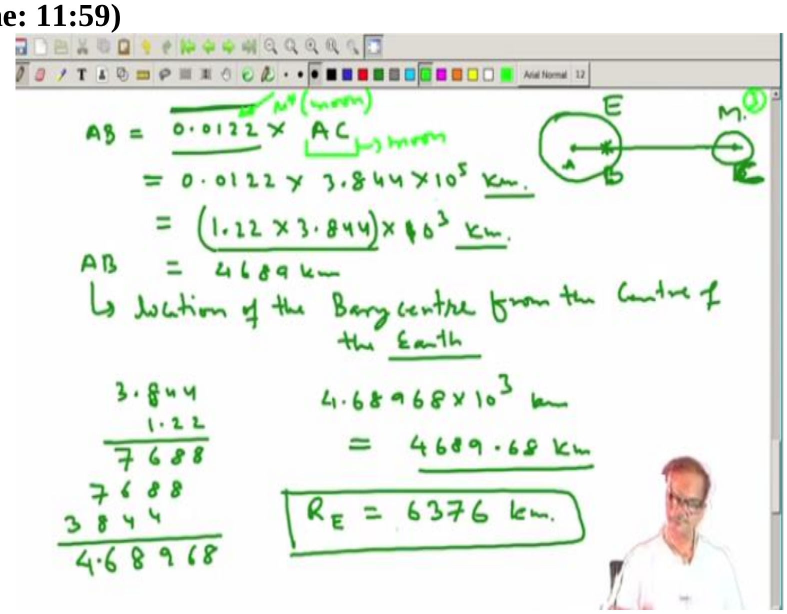

Or maybe on the next page 0.0122 × let us say 6400 km. So this is the km, this will be the location of the moon from the barycenter means the barycenter is B and the rightmost to this point to be right at C. And this is your earth so this point we have named as A. So BC will be equal to, BC =

0 point sorry not the BC but AB this quantity is AB, the distance from here to here this is a $\mu^* =$ 0.12 so this multiplied by the actual distance we have to multiply it by the mean distance AB this equal to 0.122 multiplied by A times C.

So, this is 0.0122 times we go back here and this distance we have moon earth distance 3.844×10^4 km 10 5, 3.844×10^5 km. And if you work out this, so roughly this we can write as $1.22 \times 3.844 \times 10^3$ km and if you multiply this part so we can see that we are the barycenter is located from the center of the earth, so AB gives you location of the barycenter from the center of the earth and this turns out to be around 4689 km I will just check once here 3.844.

4.6 this is 4.68968×10^3 km. So, this is equal to 4689.68 km and what is the radius of the earth? Radius of earth is around 6400 km which we write as 6376 km. So, well we consider the barycenter, if this is the earth and here the moon is lying. So, the barycenter is will lying inside the earth itself. So, this is point A and this is point B. B is here, B is located here and this is point C. So, barycenter here in the case of the earth and the moon.

This is your earth and this is moon, so it is aligned inside the volume of the earth itself. The similar thing happens in the case of the sun earth system they are also the barycenter lies inside the sun and the calculation can be done in the same way only thing this you need to replace this quantity, because the μ^* in the case of sun will be different. This is your μ^* in the case of the sun here this has been taken for the moon and AC also in the case of the this is here moon or sun will be different accordingly if you put those values.

So you can calculate where the barycenter is located. Other distances also we can note it. Just if we look here in this place, this is 1.0051 and from there if we subtract 1. 0051 and from there we are subtracting 0.0122, so we get here. So here on the normalized scale, this distance from this place to this place. This is 0.9929 and on the actual scale this value will correspond to 381670 km.

So L_3 is located towards the left of the earth at a distance of 381,670 km. So, this way we can compute all the distances. Now, if we add this distance and this distance, so we get the location of the L_1 from the center of the earth and this distance will be 0.00122 + 0.8369. This distance I have written here, and this equals to 0.8491. And on the actual scale this distance corresponds to 326390 km.

So, this way all the distances can be computed. So, I am just writing some of the values here you can check yourself this value will be 1.1557 + 0.0122 – 1, 0.1679 and this will turn out to be 64541 km. Say considers the location of L_2 point from the moon on the right hand side is 64541 km. From L_2, from this distance from here to here, this will turn out to be on the normalized scale this is 0.1509 and on the actual scale this is 58006 km. So all these computations can be done. You check yourself, do it yourself, this point already we have written this distances is 4689 km as we have calculated on the next page.

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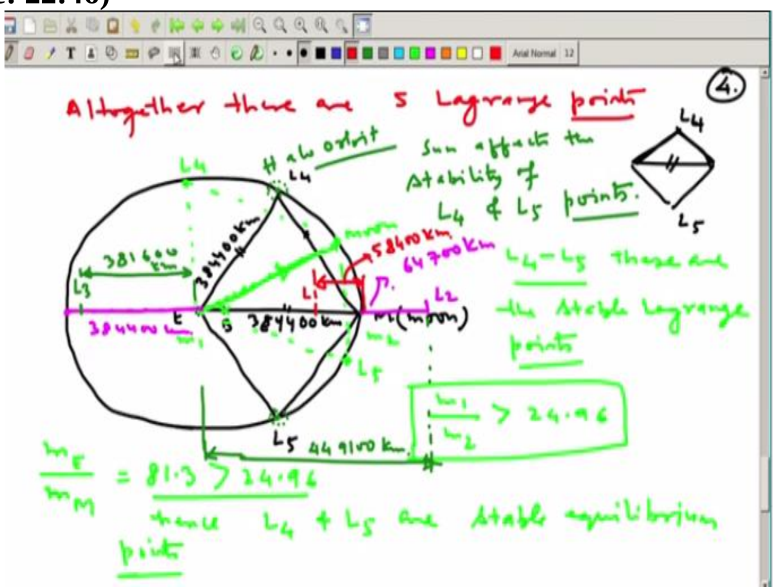

So, altogether we have 5 Lagrange points. Out of this, the 3 I have shown you, and the same way as you know the distance because it is on equilateral triangle L_4 and L_5. And this distance you are aware of so immediately you can calculate these distances. So it will be the same entry. So if I show earth here in this place and moon here in this place, and draw a circle and then draw it equilateral triangle like this.

So, this is the location of L_4, this is the location of L_5, this is your earth and this is moon barycenter is located here this is point B. Now, from here what we should notice that has the moon is rotating around the earth. So right now the moon is here, after some time the moon will come here in this place. So, what we need to do, we just need to join this point, draw an equilateral triangle here, this way.

So then this becomes your L_5 and this becomes your L_4, so L_4 and L_5 in the initial frame they are rotating, but if you sit down at the barycenter in the synodic frame which is rotating along with the earth moon joint life which is the line joining earth and moon which is rotating. So, if you are sitting on that and then also you are rotating. So, you will see that the L_4 and L_5 they are always located in the same place.

So, if you if that is your turn, so here my observation line will become this, the moon will be located here thereafter. So, again I will see that the moon this L_4 is located here, and L_5 is located here. So, in synodic frame are always located in the same place. Now, L_4 and L_5, it can be proved that these are the stable Lagrange points and for stability it is required that if I write this as m_1 and this as m_2, so m_1 divided by m_2, this should be greater than 24.96.

If this relationship is satisfied, then your Lagrange point will be stable. Now, here in this case, in the case of the earth and moon this ratio is 81.3 this is greater than 24.96 hence L_4 and L_5 are stable equilibrium point. So these are stable equilibrium points. On the other hand L_1, L_2, L_3 they are not the stable equilibrium points. So all the things for covering this whole the stability issue and other things it is not possible it will take at least next 15 lectures or more than that.

So, what I will do that whatever I am able to cover in this lecture, and the next lecture. So, thereafter I will give the handout working out all the details. So it will be circulated during the while the course runs, but the main things or the main results we will be discussing here in this place. So, here you have this distance is 384400 km the same way this also so all these 3 are equal to each other, from the moon on the right hand side.

Somewhere on this side your L_2 is located and these distances 64700 km and this circle already we have taken to be the full whatever the radius is there. So from this point to this point, this is your 384400 km. Now, the point L_3 is located, if you look here in this place for L_3 we have worked out 381670 km from this part we have written here 381670 km, so, we have, L_3 is located say here in this place, this is your L_3, about distance from this point to this point this will be 381600 km.

Now, because these are the stable equilibrium points and therefore, some Trojans are found around orbiting this point because these are stable, so if some particle is here, so those particles will be lying here in this place. However, because of the presence of the sun, the L_4 and L_5 points, it is equilibrium gets affected. But still we will see in the nature that the Trojans are located over this points, it is a orbiting there.

And this we call as the Halo orbit. And L_1 point is lying somewhere in between and that distance also we have written here say this is the point L_1 distance from moon to this place, this distance is 58400 km. Similarly similar case applies also to the sun only thing that the distances you have to look into properly it will go back and look here in this point L_3. So, sun-earth it is a 1.0 while for the earth-moon case this is 1.005.

So, there is a difference on the normalized scale. So, this we have to take care of while working with the system. So we will continue in the next lecture. Thank you for listening