Space Flight Mechanics

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Module No # 10
Lecture No # 49
General Perturbation Theory (Contd.)

Welcome to lecture 49 so we have been discussing about general perturbation theory so in that context we are going to discuss about the parameter variation. So if you remember last time we to multiple body problem.

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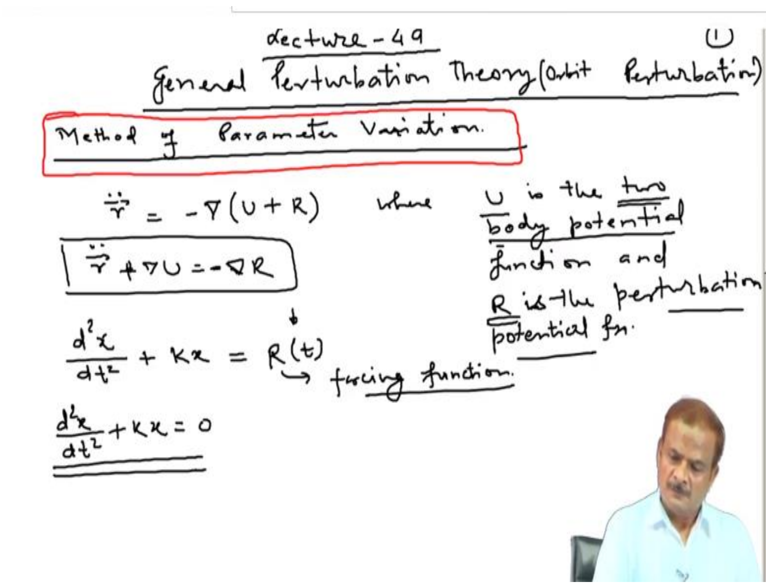

So and in that we derived the equation

$$\ddot{\vec{r}}_i + \nabla U = -\nabla R$$

where U is the 2 body potential function and R is the perturbation potential function. So if we are only dealing with the gravitational force so in that case you can get the orbit in the case say we have the aero dynamics force also presents so it cannot be represented in terms of the potential function because it is a dissipative force it is not conservative gravitational force is conservative.

And therefore for that also we need to treat it little separately but for the time being what we assume that this is our equation given where U is the 2 body potential this important to remember and R is the perturbation potential. Now if we need to solve this problem so we know of course that we can write it like this equal to $-\nabla R$. So this appears in a usual way like if we have the equation of the spring mass system and say we write it like this.

So in where this R and this R they are not the same this simply the forcing function so if we have this kind of system so what will be the solution of this? So of course we are aware of that the solution to $d^2x/dt^2 + k \times x = 0$ this will be normally a sinusoidal solution will get for this as we will do little later. But what will be the solution for this? But there are various methods for solving this problem but there is one method it is called the method of parameter variation as written here.

And especially in the case we are the parameters are varying slowly so that can case we are going to treat.

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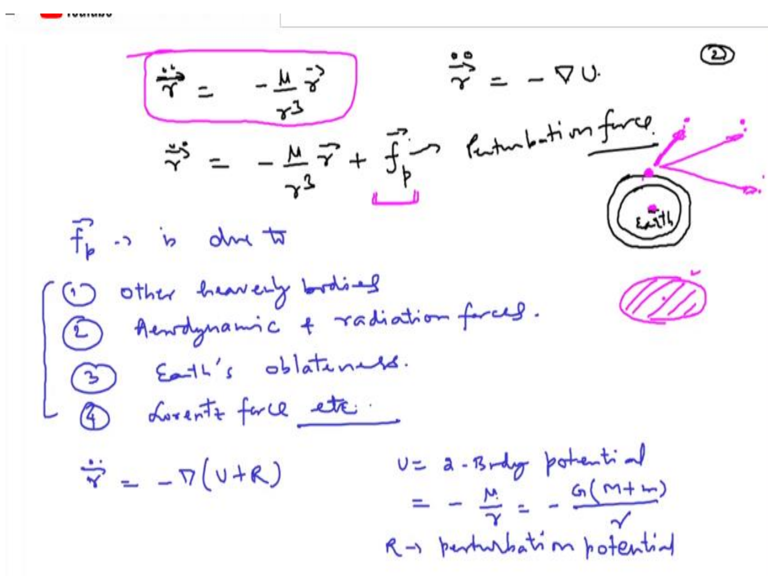

So already now we have the basic function

$$\ddot{\vec{r}} = -\frac{\mu}{r^3} * \vec{r}$$

which we have written in terms of potential. So once we write in terms of potential this gets reduce to $-\nabla U$ and if there is one more term so in that case r^3 r and plus say some function we write here as f where f is a function and if we write p so that means it is a perturbation force. And we are looking for solution to this problem so we have satellite here moving an orbit around the earth this is your earth and the satellite is moving around the earth.

And there is another planet an earth is moving around the sun from planet or the sun or the moon it is there and that is going to affect directly the motion of the satellite. Directly indirectly whatever so already we have written the equation for that because of the earth the equation will be governed by this but this is a very simple one. In fact our earth is oblate and because of that we have also the perturbation or term appearing it will appear here in this place once if we take the shape of the earth to the oblate shape.

So in that case this extra term will also appear so this we will take care of as we progress first we will learn what is the parameter variation method and thereafter will come to that. So this perturbation due to oblate of the earth perturbation due to other planets all this things will be present in the system ay, f_p is due to other heavenly bodies and then due to aerodynamic forces can be aerodynamic and radiation forces earth oblateness.

And then can be the Lorentz force so whatever the forces it is a existence of the possibility of a existence of a whatever the forces are there so we can add here in this place in this list. So with this we are given that $\ddot{\vec{r}} = -\nabla (U + R)$. ay so here we have

$$U = 2 \text{ body potential} = -\frac{\mu}{r} = -G \times (M+m)/r$$

Where M is the main body and m is the smaller body .

And r is the perturbation potential as we have written earlier ay so this states the scene for what we need to do ay we have to solve this problem to in order to get the trajectory of the satellite.

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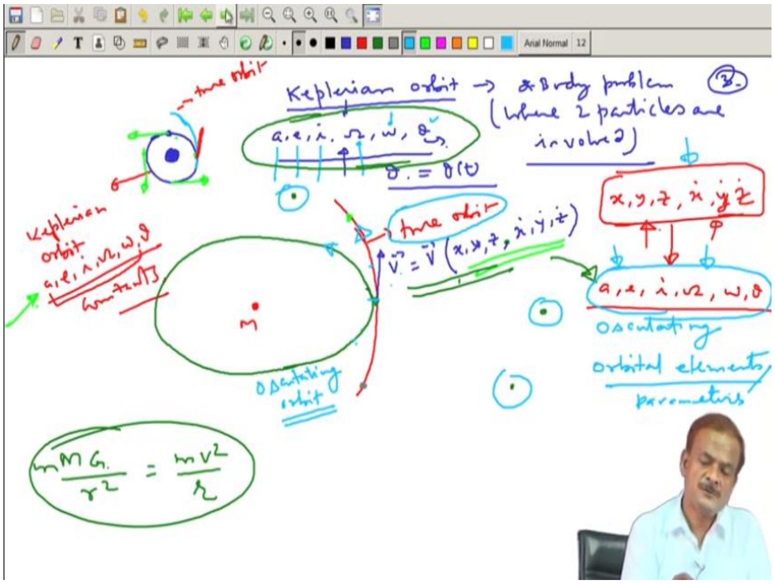

So actually what is happening that if you remember that if we have a perfectly a spherical and uniform mass body and around that one satellite is going in the circular orbit or electrical orbit. So this we have called as the Keplerian orbit this is the 2 body problem where 2 particles are involved. What if your main body it is not the spherical or it is a non-uniformed density then the orbit will not represent in cannot be represented in terms of the Keplerian orbit means you have a, e, i,Ω ,ω and Θ this were the 6 parameters of the system.

So in that case in the Kaplerian orbit all of them were constant except theta this was a function of time theta was a function of time misses through normally. But, where the perturbation has in the case represented by this particular equation, so in that case this trajectory will not lo like this. But rather that trajectory will evolve so here it will lo like this ay instead of this the orbit is changing ay it is not going in a close one it is an open orbit and it will vary over a period of time like this.

And then e will change i will change Ω which is the; what we call as the nodal angle then argument of perigee all this will change theta of course it is changing already. So the solution to this equation this will give us the 2 orbit in the true orbit the velocity at this point this is the true orbit shown by the true orbit and this is your Keplerian orbit where a, e, i ,Ω ,ω and Θ these are present and out of this are constants this 5 are constants.

Now in this orbit if we assume that say if I assume that actual orbit is the true orbit is going like this is the true orbit. And here the center of attraction mass m is there and some small mass moving here in this orbit which is show this mass is moving in this orbit. And the presence of other planets heavenly bodies it will perturb the orbit and because of this it is going like this and what we are calling as the true orbit.

So in the true orbit given at any point we have the position it is x, y, z and $\dot{x}$, $\dot{y}$, $\dot{z}$ if it is available. Say in the true orbit these are the position and velocity may be with respect to the main body as we have done earlier. So if we get this x, y, and z. So immediately using this we can calculate a, e, i, Ω ,ω and Θ as we have done for the keplerian orbit. But here in this case we call this as the osculating orbital elements or orbital parameters elements slash parameters.

Osculating means kissing so what does it mean it implies that suppose that these velocity here. So V this will be function of x, y, z, and $\dot{x}$, $\dot{y}$, $\dot{z}$ it is changing with position and the components is indicated here. So with position it is V will change here also if we lo here in this orbit so here also the depending on whether it is elliptical or circular if it is circular V remain the same but if it is elliptical it is keep changing.

So this change is a different issue once it comes back to the same it again the satellite will come back the to the same position, but here in this case as you can see that satellite is not going to come back to the same position because of the perturbation it is changing all the parameters. See in the true orbit that is a velocity and using this velocity at any point I can draw a circle or ellipse about that point not circle say it will be an ellipse.

Because if it is a circle then the velocity as to satisfy like $MG / r^2 \times m$ in this must be satisfied mv^2 / r if it is circular orbit. So without a stating whether it is a circular or elliptical or whatever it may be if you have the particle at this point and this is the corresponding velocity so using this velocity you can get this elements. Though the satellite is not going to follow this route but rather it is going to take to this part ay because of the presence of other heavenly bodies.

And this we are calling as the true orbit while this we call as the osculating orbit. So osculating orbit will exist at one point another point you go another osculating orbit will be there but this is only at a particular instant of time it is a existent says only for a particular instance of times at other time it will be not be there in that position. While the Keplerian orbit if given the same velocity it will exist for all the times this is the difference between the keplerian orbit and the osculating orbit. Both are got we get a, e, i etc., in this same way.

But in the case of the Keplerian orbit you get the heavenly body repeat the same orbit again and again. But in the case of osculating orbit we are just getting an orbit but the particle is not following that path you should get this point that using this we can get this but particle is not going to go in this orbit. And this we are calling as the osculating orbit and why this is happening with this is because of the this perturbation term. So this is the concept of osculating orbit.

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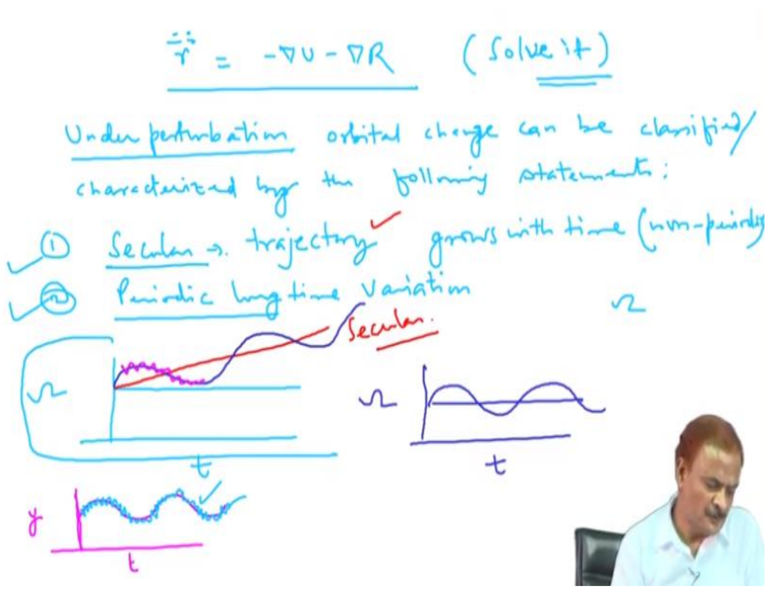

Now our jobs is to solve the problem which we have written as $\ddot{\vec{r}} = -\nabla (U + R)$. So under the perturbation or vital change can be characterize by the following characterize or classified slash characterized by the following statements. It can be secular orbit secular changes this implies trajectory grows with time this is non-periodic. And the other one it is a periodic term periodic long time variation so these are the 2 things present and beside this we will have the short time periodic variation also present.

So secular implies say if we plot some Ω and versus t so in that case we know that if that is no variations in Ω. So Ω should be constant so it should come like a straight line ay for Keplerian orbit. If there is no perturbation so in that case Ω plot with respect to time it appears like this but in the case this is a secular perturbation. So it will appear like this secular it may be non-linear it may be linear whatever it is possible.

In the case of the periodic variation so periodic variation which may write over the secular one it may appear like this or periodic variation if we list say the periodic variation can appear like this also. Above this we write the short periodic terms. So short periodic term will appear like this so in a signal you might have noticed that if we have a signal and say we plot here some y or some voltage signal in electrical engineering voltage signal.

In different field different kind of signals can be present so we can have the actual signal which may be periodic it may be varying and above that noise rights. So noise it is a non-periodic it is a different kinds of noise can be represent so depending on the nature of the system it showing that over a periodic long periodic term the it may be in form of a short periodic or it may be totally white noise whatever it is possible.

This is who we do not take here our main concern is about this point which we are discussing that the orbit and change in a circular way or in a long time periodic variation way. And short time periodic variation may also represent.

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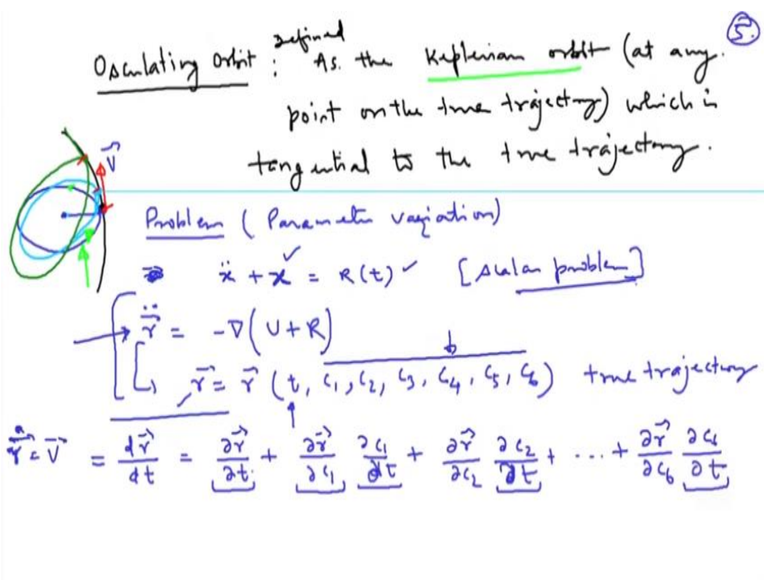

So the periodic osculating orbit it will define now with this introduction is defined as the Keplerian orbit at any point. So osculating orbit is defined as the keplerian orbit at any point on the true trajectory or the true orbit whatever you write on the true trajectory so this is the true trajectory on that any point we take defined as the Keplerian orbit which is tangential to the true trajectory.

So that means here if V is the velocity so V velocity is common to both the true trajectory and also the Keplerian orbit. So but the Keplerian orbit we will lo like this and if it is a Keplerian orbit say given the velocity V so I can get a Keplerian orbit at this point. So how it is different from the osculating orbit the Keplerian orbit osculating orbit will be present only at a particular instant of time as soon as we take the next instant of time.

So the true this osculating trajectory it will differ now the osculating trajectory at this point it will lo like this. Here I am taking this point ay I go to the next point I take next point here this is the center of attraction. So, at this point trajectory will lo like this at this point it will be tangential. But here it is a very distorted fewer the varying does not take so fast it is a slow variation and here it is of hearing that within very short distance so much of change as taken place this is not a true representation this is just for exemplifying this particular problem.

So we have here this point and this point so at 3 points 3 orbits are shown so this are the osculating orbit. But our Keplerian orbit is this one where here the particle will keep going or the body will keep going again and again over the same route again and again so that we call as the Keplerian orbit. So there is a different between the osculating orbit and the Keplerian orbit in the since that osculating orbit exist only at a particular instant of time while the Keplerian orbit will be forever because it is a 2 body problem.

So it will be always with respect to that particular body in the center of attraction it will always represent. And it will follow the same route because that is no perturbation ay so to understand what is the parameter; variation method? We consider one problem parameter variation. So here we take one simple problem say $\ddot{x}$ this is scalar problem

$$\ddot{x} + x = R(t)$$

If we have this problem this is a vector problem so it because which is applicable in 3 dimension while here only in x.

So the solution to this if the parameters are varying so it can be written as r= r and it will be function of then t and then the orbital constants which will be varying over a period of time c_4, c_5, c_6. These are the 6 orbital parameters so which with time they are varying so this is your giving the true trajectory. If we get the equation for the r in vector terms so we get the true trajectory how it is a varying in the interstellar space orbit respect to the main body because this equation we are writing with respect to all the time. It can be also with respect to the barycenter.

So here in this place dr/dt this can be written as $\partial r / \partial t$ and then $\partial r / \partial c_1 \times \partial c_1 / \partial t$. We are applying the partial differential and so on so this is representing velocity which we also write as $\dot{r}$. So $\dot{r} = v = dr / dt$ so the contribution is not only due to the variation with respect to time but also due to the variation of the parameters which are involved here.

And we are going to utilize this information for solving this and problem so we will start with the next lecture we will stop here thank you very much.