Space Flight Mechanics
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Module No # 11
Lecture No # 51
General Perturbation Theory (Contd.)

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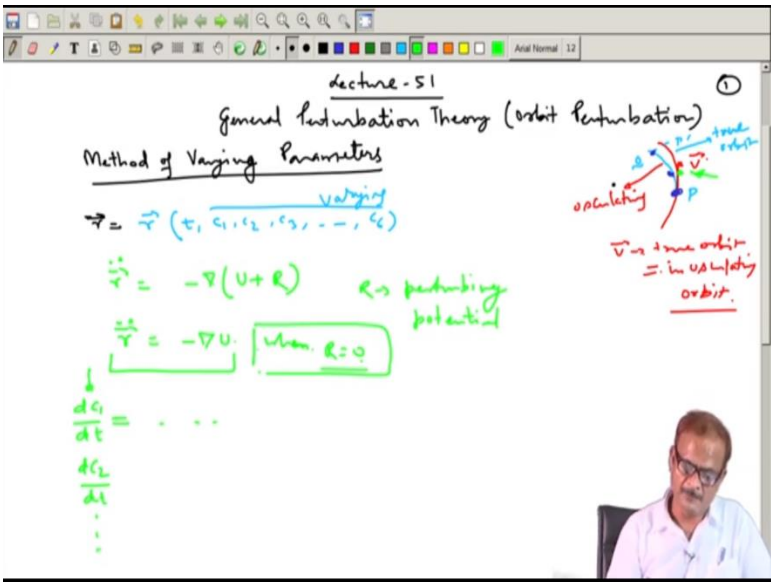

Welcome to lecture number 51. So we are discussing about the general perturbation theory and later to that especially the method of parameters variation or method of varying parameter. So in that context we discuss one problem and how does it work . So the same thing we are going to apply to the orbit problem where the orbit is perturbed by the third body .

There after we will take into a count other forces obviously we will have the planetary force and we will have the aerodynamic drag, solar radiation drag can also be there solar radiation pressure. So but we will get confined to up to the aerodynamic one. So how much I can cover within the given number of lectures so it is a still it is not defined because many of the things may require explanation and that takes a lot of time.

So what I will do whatever I can over and the rest others I will give as soft copy after detailed materials explain in details by details so let us start with the problem we have been discussing. So we remember that the radius vector of the orbit from the center of attraction or may be with respect to the center with respect to which you are defining the so let us say that the radius vector is represented as r = r t and c_1, c_2, c_3 up to c_6.

So these are 6 orbital parameters but now here in this case this parameter are not constants and they are varying and this is obvious from the figure I have drawn here in this place. So say this is the initial point so at this point we have the velocity vector v which is the same v is the same in true orbit velocity vector in true orbit. This equal to in osculating orbit so in both the places so basically at this point velocity vector in osculating orbit or either in the true orbit it is the same.

This is the osculating orbit and already we have defined the osculating orbit and the true orbit. These are the osculating and the true orbit. So satellite is usually stay at the point p this is located at the point p here and if there is no perturbing force then it will follow and it will go to q. It will follow the paths of the osculating orbit it will go to q and of course if the perturbing force is not there so osculating orbit is equivalent to the Keplerian orbit.

But if the perturbing force is here then because of the change in the velocity at the next instance say in a short time this moves from if the forces is not present so they this will move from this place to this place while if the force is present so perturbing force is present. So because of the perturbation it will not move to this place rather it will move to this place as shown by the green. So the perturbation is causing the orbit to deviate from the, what it should follow and that we have written as $\ddot{\vec{r}} = -\nabla (U + R)$ where R is the perturbing potential.

In the absence of perturbing potential we will just have $\ddot{\vec{r}} = -\nabla (U)$ when R = 0 for perturbing potential is not there. So at that time we just get the 2 body problem and it will of course follow the paths soon by the blue line or this sky blue line . So if perturbing potential is present so how the parameter will vary? So, whatever, intension is to get this quantities $d c_1/dt$ $d c_2/dt$ and so on.

So these are the things we are looking for so if we know at what rate it is varying so if we integrate the corresponding equation we know the corresponding equation we can integrate and get the parameter at a future point of time.

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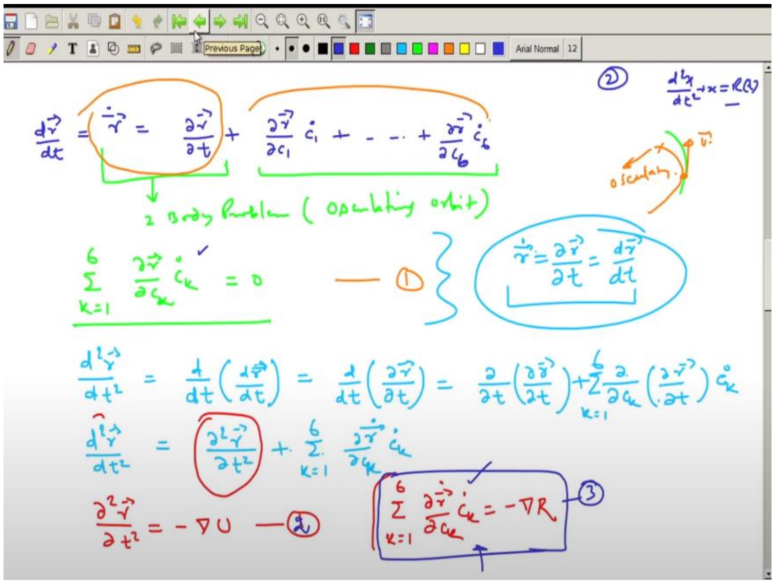

So we have started with writing r dot so we have worked with a simple problem which was $\ddot{x} + x = R(t)$. Here in this case we are going to work with the actual problem so in the actual problem we have dr/dt this can be written as ∂r/∂t + ∂r/∂c_1 × $\dot{c_1}$and so on c_6 into $\dot{c_6}$. So here this particular term this corresponds to your 2 body problem. And here in this case also we are writing this as the, our osculating orbit.

And therefore as early discussed this

$$\sum_{k=1}^{6} \partial r/\,\partial c_k \;*\; \dot{c_k} \;=0$$

This, parts must 0 because the velocity in the true orbit is the same as in the osculating orbit at any at the point of trajectory where they are touching each other . So at this point the velocity in the osculating orbit and this is the osculating orbit shown here and the true orbit this velocity remains same.

And therefore only this term will stay and the other term this term will be 0. So this gives us one equation another equation we can get in terms of the acceleration so while we write the equation for the acceleration. So this dr/dt here in this case this needs to be replaced by ∂r/dt because from the constraint from this constraint we have $\dot{r}$ equal to we are writing like this ∂r/∂t = dr/dt.

Using this we can write it in this way and this can be further written as ∂/ ∂t ∂r/∂t + ∂/ ∂c_k × k = 1 + 6. So this we are writing as ∂^2 r/∂t ^{2} + summation ∂/ ∂c_k I am using the summation notation here for making it little compact and this is your $\dot{r} * \partial r/\partial t \quad * \dot{r}$ we are writing like this way. So I will use the notation $\dot{r}$ and × $\dot{c_k}$ so $\dot{c_k}$ also will be present here in this place.

So immediately what we can see that this part actually if we go back and here lo into this one what we have written here this is

$$\ddot{\vec{r}} = -\nabla (U)$$

and which is nothing but $(-Mu/r^3) \times r$ and comes of vector. And this quantity is $\partial^2 r/\partial t^2$ this quantity is $\partial^2 r/\partial t^2$ square because in this case parameter is not varying. If the parameter varies then this is not valid.

So therefore from here immediately we can see that this quantity $\partial^2 r/\partial t^2$ this is $-\nabla u$ this is 1 and the other equation we name as the other part then becomes

$$\sum_{k=1}^{6} \partial r/ \partial c_k \, * \, \dot{c_k} = - \nabla R.$$

So what we have got here this is one equation let us name this is 1 we have already named, so we will name this as 2 and this we will name as 3. So you can see that this 2 equations they are written in terms of $\dot{c_k}$ here also it is written in terms of $\dot{c_k}$. So this can be solved for $\dot{c_k}$ where k varies from 1 to 6.

But the problem earlier we have taken there we did matrix in version and other things and using that we solved it we will not go by that method. Let the same thing but we have to do the exactly the same thing what we have done in the last lecture but we do it in a little different way so that it is convenient to work with.

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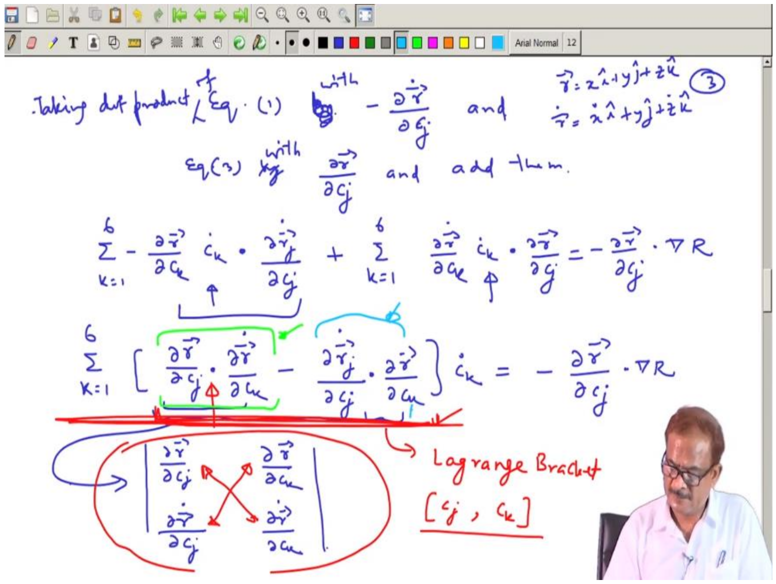

So we multiply equation 1 - $\partial \dot{r}$ / ∂c_j and equation 3 by $\partial r/\partial c_j$ and add them. So if we do that immediately what we can see the first equation was k = 1 to 6 $\partial r/\partial c_k \times \dot{c_k}$ and then multiplied by so multiplying in all so here we take the actually what we do this multiplication here means because it say vector is involved here so we will write in terms of dot product.

So multiplying taking dot product the right word is taking dot product of equation 1 with - $\partial \dot{r}$ / c_j and equation 3 with $\partial r/\partial cj$ and add them. So here dot product once we take and minus sign so minus sign will be here and plus the other one is $\partial \dot{r}$ / $\partial c_k \times \dot{c_k}$ both are in both the places $\dot{ck}$ is present and this we have to take the dot product this is dot product here with $\partial r/\partial c_j$.

And therefore, on the right hand side we can see that the quantity will be $\partial r/\partial c_j$ the for this quantity the equation 1 on the right hand side is 0. So therefore, whatever we multiply that side remain 0 only thing this equation will count so ∇ R it will come as product of ∇ R and which a minus sign. And we can immediately see that $\dot{ck}$ this is common here. So we can take $\dot{c_k}$ as a common k = 1 to 6 and we can write this as

$$\sum_{k=1}^{6} [\partial\vec{r}/\partial c_j \times \partial\dot{\vec{r}}/\partial c_k - \partial\dot{\vec{r}}_j/\partial c_j \times \partial\vec{r}/\partial c_k]\dot{c_k} = - \partial\vec{r}/ \partial c_j \,.\, \nabla R.$$

Try in effect what we are doing we are combining these 2 equations. We are trying to solve these 2 equations and here we have dot products. So we put the dot product in it. Now R is defined as $x\,\hat{\imath}$, y $\hat{\jmath}$ and z $\hat{k}$ and therefore $\dot{r}$ can be defined as $\dot{x}\ \hat{\imath}$, $\dot{y}\ \hat{\jmath}$ and $\dot{z}\ \hat{k}$. So if we use that notation immediately we can see that this can be by separated into 3 parts each of the terms can by separated into 3 parts.

And therefore we will get equation in a format which will involve x, y and z $\dot{x}$ $\dot{y}$ and $\dot{z}$. And other thing I would like to point out though this is not the correct thing to tell but immediately what we can see that if we ignore this dot product and assume that r is a scalar and at that time you can see that this is $\partial r/\partial c_j$ and $\partial r/\partial c_k \, \partial \dot{r} / \partial c_j$ and $\partial r/\partial \dot{r} / \partial c_j$.

So this particular part it appears as determinant only thing that here you we have dot product while we consider the determinant so in that case the dot product will not appear . But if we see way of remembering let see easy to remember . And once you multiply them the in between you just place the dot product and your job is done. In fact later on we will see that r can be replaced by x, y and z and z and $\dot{r}$ can be replaced by $\dot{x}$ $\dot{y}$ and $\dot{z}$. So in effect what this equation is going to give us we see it on the next page.

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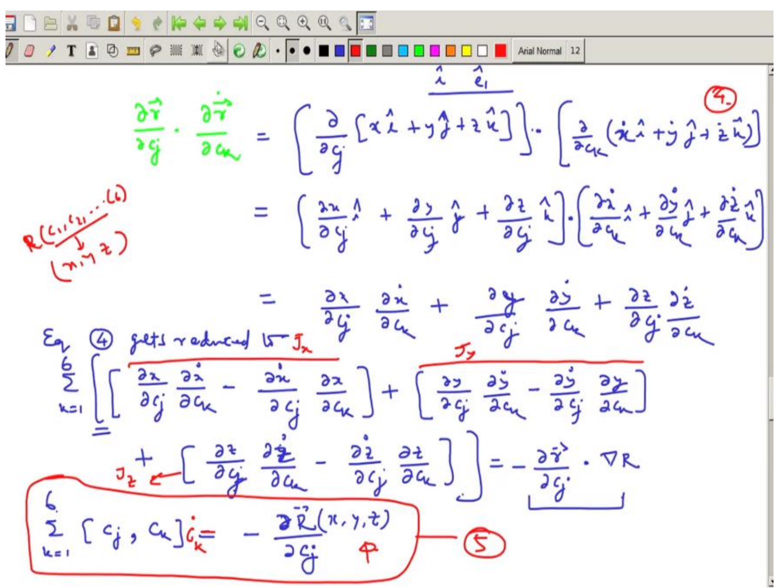

So the quantity which is inside the bracket this quantity this whole thing from here to here it is a called a Lagrange bracket and this is represented by c_j, c_k and therefore this is also called as Lagrange bracket method. So the if we take the let us take the this first term we take just this term and work with this $\partial r/\partial c_j \, \partial \dot{r} / \partial c_k$ this is what we have here . So immediately what we can observe that this quantity is we can write in terms of $\hat{\imath}$ or either $\hat{e}_1$ whichever is convenient.

Let us continue with $\hat{\imath}$and we have to take the dot product of this with respect to with the other one ∂c_k $\dot{x}$ $\hat{\imath}$, $\dot{y}$ $\hat{\jmath}$ and $\dot{z}$ $\hat{k}$. Therefore this gets reduced to $\hat{k}$ and then and dot product. So this gives us $\partial x/\partial c_j$ $\partial\dot{x}$ / ∂c_k + $\partial y/\partial c_j$ ∂ $\dot{y}$ /∂c_k dot product results in a scalar quantities and ∂z dot ∂ck.

So early the other part you can write other part means the second term these term can also be written in the same way only thing here this is dot the other one is with c_k with subscript k r is appearing. So there is an exchange of the terms like where with the subscript k rj appear in. So there is an exchange of the terms like where y dot is the where the y will become it will become y where the x is their that becomes x.

Therefore the Langrage bracket the equation we can write as this quantity we are writing as cj k in quantity in the bracket we are writing as the Langrage bracket. So therefore equation this let us write this as equation 4. Equation 4 gets reduce to k = 1 to 6 and then in the bracket the terms will separate out $\partial x/\partial c_j \times \partial x/\partial c_k$ minus now this becomes $\partial\dot{x}$ from where this is coming from the second term from this term.

You do this exercise by yourself I am leaving up it to you. Therefore we will have a total of such 3 terms comes $\partial y/\partial c_j$ $\partial\dot{y}$ / ∂c_k $\partial\dot{z}$ ∂c_k and on the right hand side we have $\partial r/\partial cj$. ∇R which is - 1. This also we can expand and you can check what this quantity exactly. So on the left-hand side this is k = 1 to 6 the quantity which is inside this bracket that we write as c_j, c_k this is Lagrange bracket. Summation over and what this quantity is we will do it on the next page and then come back to this point.

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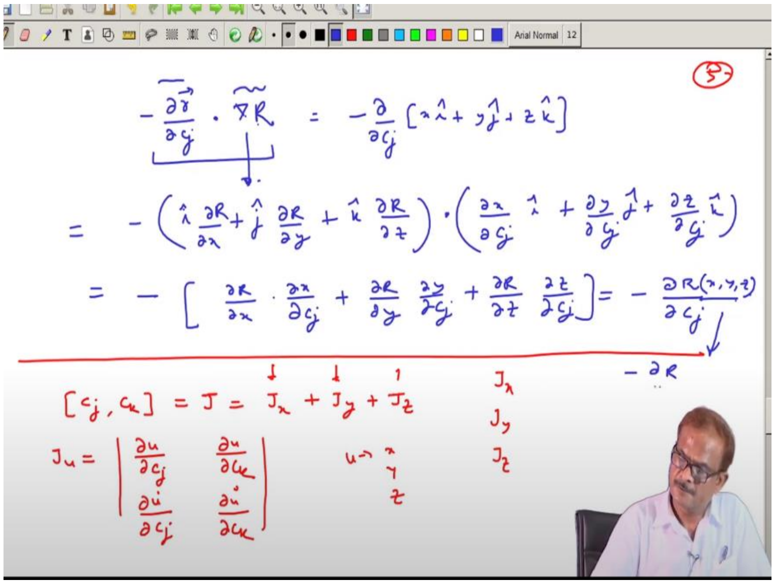

$$- \partial\vec{r}/\partial c_j \,.\, \nabla R = - \partial/\partial c_j[\, x\,\hat{\imath} + y\,\hat{\jmath} + z\ \hat{k}]$$

And we have to take dot product with ∇ R this is the vector and this is the vector so this is going to operate on this. So it is better to write it this way ∇ r = i × $\partial/\partial x\ \hat{\imath}$this is + this quantity we are writing here j × $\partial r/\partial y$ y $\hat{\jmath}$ × $\partial r/\partial z$. And take the dot product with $\partial x/\partial c_j$ $\hat{\imath}\partial y/\partial c_j$ y $\hat{\jmath}$ and $\partial z/\partial c_j$ $\hat{k}$. And this dot product we have to put it and there is a minus sign before this.

So this results in $\partial R/\partial x$ time $\partial x/\partial c_j$ + $\partial R/\partial y$ × $\partial y/\partial c_j$. And that results in you can see that this is nothing but $\partial R/\partial c_j$ where R is a function of x, y and z. Therefore we can utilize this information and we can complete this and write here this is - ∂R which is function of x, y and z/∂c_j. This is our equation 5 and we need to solve this equation we actually miss one part here this is the $\dot{c_k}$ so we have need to introduce this $\dot{c_k}$ here in this place and solve for $\dot{c_k}$.

So if we get the solution for the $\dot{c_k}$ so our objective is fulfilled. So here your R is the perturbation function this can also be written in terms of c_1, c_2 up to c_6 which is itself the function of x y z. And that is the reason I have written here this is R x y z. with this at hand let us write this c_j c_k = J = Jx + Jy + Jz and what this quantity are we can see from this place this quantity Jx this quantity Jy and this quantity in the bracket this is Jz.

So this quantity is being written there and we need to evaluate them. So Ju we can write as already I have written while in for the vector r but here I will write in terms of the scalar forgetting Jx, Jy, Jz what we need to do that replace this u with x, y and z and you get the corresponding Jx Jy and

Jz. So the basically your c_j c_k this is Lagrange bracket can be represented as a summation of 3 determinant which is being determinant as Jx, Jy and Jz this is not dot.

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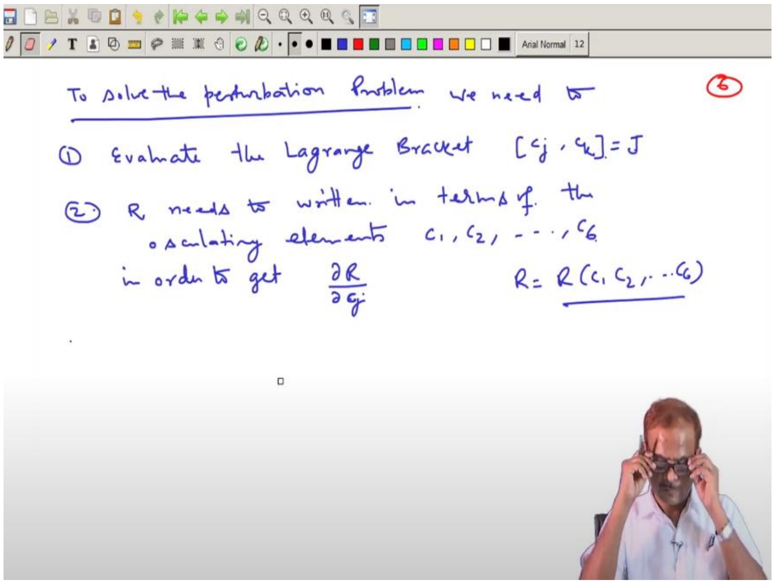

so with this at hand to solve the perturbation problem what we need to evaluate the Lagrange bracket c_j, c_k and also R needs to be written in terms of the osculating elements c_1, c_2 etc., in order to get $\partial R/\partial c_j$. So different value of j we can evaluate once its R is expressed in terms of c_1, c_2 to c_6 we can evaluate $\partial r/\partial c_j$. So this part here we write this as $\partial R/\partial cj$ where this needs to be expressed as c_1, c_2 to c_6.

Now I windup this part quickly and we will move to the next lecture so in this part just list few properties of Lagrange bracket or we maybe take it to the next lecture. So we will windup here so thank you very much we will move to the next lecture.