Space Flight Mechanics
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Module No # 11
Lecture No # 52
General Orbit Perturbation Theory

Welcome to lecture number 52 so in the last lecture we derived the Lagrange bracket.

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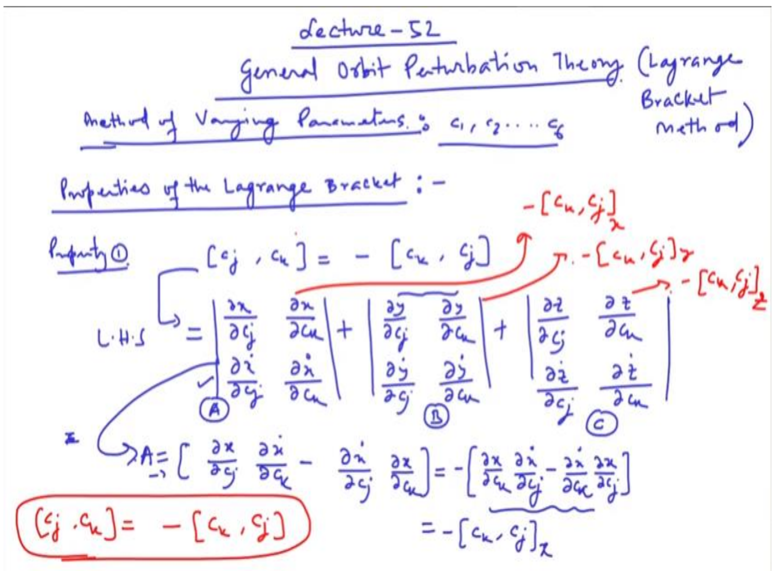

And therefore the problem we have been working with we call this as the general orbit. Actually I wrote general perturbation theory but we are because working with the orbit so we call this as the general orbit perturbation theory. And using Lagrange bracket method because of Lagrange bracket is appearing here and the Lagrange it was the person who did this for on his name this is called the Lagrange bracket.

And especially why we are calling this as the varying parameter because the parameter as c_1, c_2 this are the osculating parameters they vary with time. So in the last lecture wherever we finish we need to I was trying to go into the properties of the Lagrange bracket. This I wanted to tell in the last lecture but it is better it will take it may take little more time. So it is better to do here now the first property I list as property number 1 c_j, c_k this is immediately evident from that determinant which we have written $\partial x/\partial c_j$ $\partial x/\partial c_k$ and $\partial \dot{x}/\partial c_j$ $\partial \dot{x}/\partial c_k$.

So this quantity on the left hand side this LHS this is equal to plus similarly for the y term $\partial y/\partial cj$ $\partial y/\partial c_k$ $\partial \dot{y}/ \partial c_j$ and $\partial \dot{y}/\partial c_k$. And one more term for the z have to write so this is your left hand side. So what we can see that the quantity let us take only the first one this can be written as $\partial x/\partial c_j \times \partial \dot{x}/ \partial c_k$ as we have written earlier also from this part we came to this one but this is easy to remember as compare to this one.

And this can be written as let us say this is part A this is B and this is C so A equal to then we can take the minus sign outside and write this as $\partial x/\partial c_k \times \partial \dot{x}/ \partial c_j - \partial \dot{x}/ \partial c_k$ and $\partial x/\partial c_j$. Immediately you can see that if I do it for all of them so what this quantity is this is the quantity c_k, c_j and only the x part. Similarly we will have so therefore in this place we can replace this with $-$ c_k c_j and only x part this can be replaced by $-$ c_k c_j only y part and this can be replaced with $-$ c_k,c_j only z part.

So what does this imply then? This implies that c_k c_j after summing all of them the x part y part then the z part all of them are coming which minus sign. So this becomes equal to c_k c_j.

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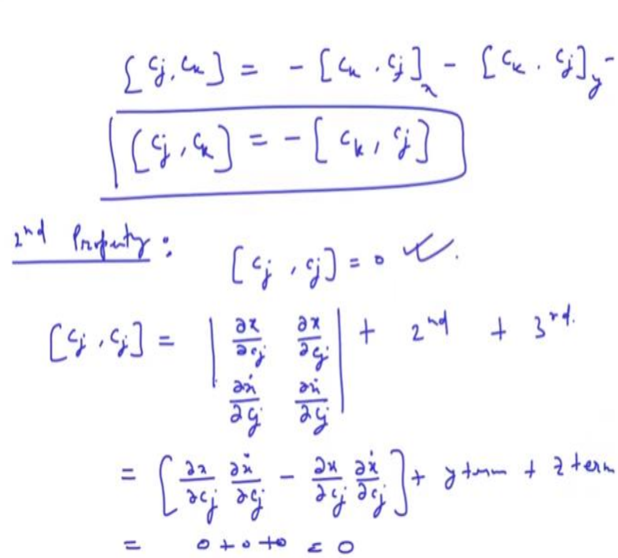

I will write this part here c_k c_j what we are doing that we are writing this as c_k c_j the x part c_k c_j the y part and $-$ c_k c_j the z part, so this is the summation the minus sign can be taken as the common this will result in c_k c_j. So this is the first property of the Lagrange bracket the second property c_j, c_j this equal to 0 and that we can check immediately with this if you replace k with j so immediately you can see that all this quantities will vanish.

So in this case c_j, c_j this will be equal to say one part I am just writing here $\partial x/\partial c_j$ $\partial x/\ \partial c_j$ and $\partial \dot{x}/\ \partial c_j$ $\partial \dot{x}/\ \partial c_j$. And similarly the second term and the third term so what this quantity is? This is $\partial x/\partial c_j \times \partial \dot{x}/\ \partial c_j$- $\partial x/\partial c_j \times \partial \dot{x}/\ \partial c_j$. Similarly for the y term and for the z term therefore all of then it is 0 this gives a 0 and thus we get the second property.

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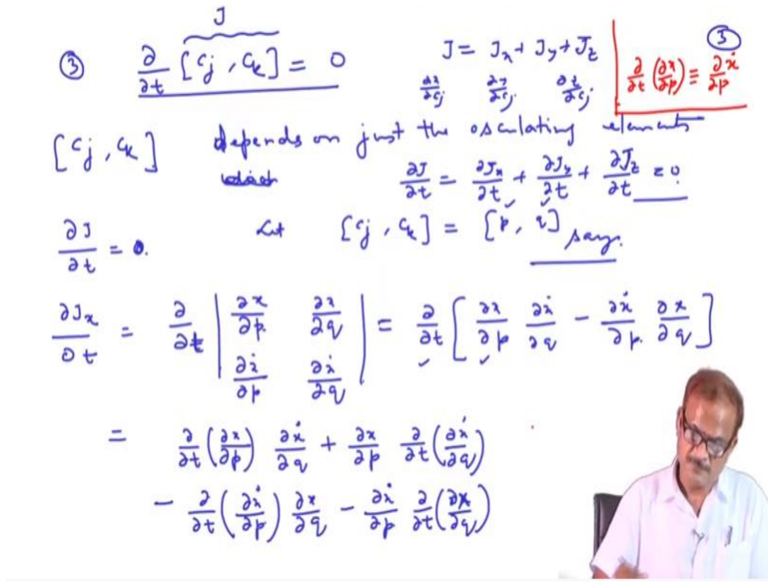

The third property is of great importance in solving the problem which is states that $\partial/\partial t$ $\partial c_j/\partial c_k$ this equal to 0 c_j c_k depends on just the osculating elements ay. So Lagrange bracket this is not a function of time this is what it says and it is a very important. Now one more thing we need to say like before we take to this property I can strict here if itself. This c_j c_k it can be given j can varies from 1 to 6 and k also varies from 1 to 6 so that means it is a 6 by 6 matrix.

So let us say that each of the bracket we are writing as in short notation c_{11}, c_{12} to c_{16} and similarly from here c_{61} to c_{66}. So each of them it is representing Lagrange bracket so in view of this second property your diagonal elements are all 0 because c_{11}, c_{22} from this one you can see that all of them will be equal to 0. All this quantity will be 0 c_{33}, $c_{44} = 0$ $c_{55} = 0$ and c_{66} this equal to also 0 so your diagonal elements they vanish.

So total there are 36 element here in this place so out of 36 once diagonal elements are 0 so 36 – 6 only 30 needs to be estimated. The 30 Lagrange brackets need to be estimated because the 6 are 0 in view of the first property this we have written here in this place we can see that the Lagrange bracket c for suppose we write this as c_{13} and on this side this will be c – c_{31} . So in view of the first property this says that the terms on this side the elements of this matrix on this side the elements it will be negative of elements on this side.

So if you see if I write it like the say for 3 by 3 matrix if I write it appears like this suppose this is -5 so this will be +5. If this is 4 so this will be -4 if this is -3 so this will be 3 so this forms a skew symmetric matrix. So out of 30 then because of on the elements on one side the elements on this side are just negative of elements on the other side which elements on this side just negative of this and therefore immediately we can see that we just need to estimate only evaluate only 15 bracket needs to be evaluated.

So out of 36 we just need to evaluate 15 brackets to solve this problem and this eases the process a lot. And the third property we have to list which will help us in solving the problem so I will do this part just here. Say if this quantity if I write this as j this implies $\partial J/\partial t = 0$ I have to prove so let c_j c_k we write this as p, q. And J we know of course J = Jx + Jy + Jz because we have to take derivatives like the $\partial x/\partial c_j$ $\partial y/\partial c_j$ and $\partial z/\partial c_j$ and similarly with respect to k also and in addition the $\dot{x}$, $\dot{y}$, $\dot{z}$ also involved.

So $\partial Jx/\partial t$ this quantity will be $\partial/\partial t$ and j we are writing in terms of p and q so therefore it is just for convenience ay nothing to do with I can equally write in terms of j and k. But writing in terms of p and q at least there are 4 symbols here involved here only 2 symbols are involved so that is why we are writing like that. So this gets reduced like this $\partial x/\partial q$ instead of p, c_j and c_k we are writing just p and q.

And of course $\partial J/\partial t$ this is $\partial Jx/\partial t + \partial Jy/\partial t + \partial Jz/\partial t$ this equal to 0 this is what we are going to prove. Differentiating see once we differentiate the quantities ay let me go step by step rather than jumping we have taken just the partial derivative we need to go on the next page. can we now one thing I would like to mention here you must aware of those things $\partial/\partial t$ $\partial x/\partial p$ this is equivalent to writing the exchange of the operator we can write this as $\partial/\partial p$ $\dot{x}$.

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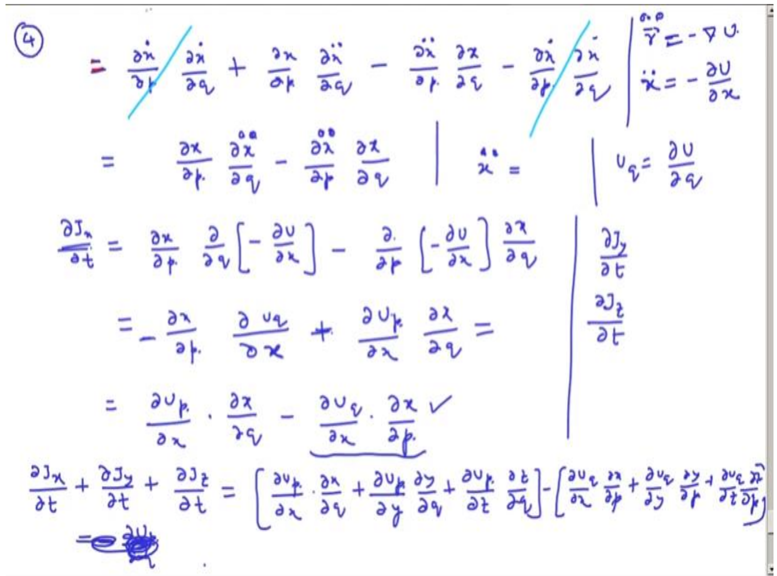

So if we follow this notation so immediately we can see that the quantity here gets reduced to $\partial\dot{x}/\partial p$. And then the next point to $\partial\dot{x} / \partial q + \partial x/ \partial p$ and ∂x similarly here we can write here in this place this t instead of writing it $\dot{x}$ we will write it in terms of $\ddot{x}$. So

$$\partial Jx/\partial t= \partial x/\partial p\ \partial\ddot{x}/ \partial q - \partial\ddot{x}/ \partial p\ \partial x/\partial q\ \partial\ddot{x}$$

So here this is $\partial\ddot{x}/ \partial p \times \partial x/\partial q - \partial\ddot{x}/ \partial p \times \partial x/\partial q - \partial\dot{x}/ \partial p$ – du $\dot{x}/ \partial p$ and this term then gets $\partial x\ \partial\dot{x}/\partial q$. And immediately we can recognize that this term and this term they are the same with opposite sign and therefore can cancel out. Thus we have $\partial jx/\partial t$ now $\dot{x}$is the quantity we have written as only if we write in terms of say $\ddot{r}$ we are writing for the perturbation free orbit or the osculating orbit as $- \ddot{r} = -\nabla U$.

So in terms of $\ddot{x}$ this will become $\ddot{x}= - \partial U/ \partial x$ similarly for y and z it can be written. So therefore this gets reduced to $\partial x/\partial p \times \partial/\partial q$ exchange of variables this is minus that gets plus and this curve is not there because we have u q is where u q is nothing $\partial U/\partial q$. So what does this imply we can write it properly

$$\partial Jx/\partial t =\partial Up/\partial x \times \partial x/\partial q - \partial Uq/\partial x \times\partial x/\partial p$$

So and if we do this operation for all of them so therefore if we add for all the terms like $\partial Jy/\partial t$ similarly we can get the terms for the we need to be evaluate $\partial j/\partial t$ and ∂ j z/∂t and along the same line it can be done. So this will held us $\partial Up/\partial x \times \partial x/\partial q + \partial Up/\partial y \times \partial y/\partial q$ along the same line we have to write for the z ∂q and the terms corresponding to this then we have to also use so this will come minus sign – ∂Uq/∂x here I am not able to write this I will continue here on this side with minus sign ∂Uq/∂x × ∂x/∂p + ∂Uq/∂y × ∂y/∂p/∂z × ∂z/∂p.

And this we need to work out so what this quantity gets reduced this quantity gets reduced to ∂U p/∂we go on the next page.

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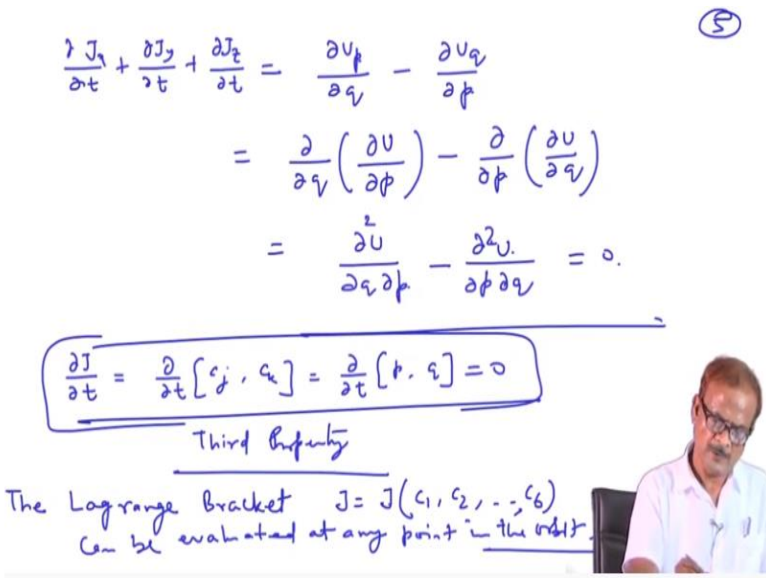

So therefore ∂Jx/∂t + ∂Jy/∂t plus this can be written as from here the first term is nothing but from the partial differential concept this turns out to be ∂U p/∂q. And the second terms this term this turns to be ∂Uq/∂p from – ∂Uq/∂p this terms is nothing but ∂/∂q and already we have written ∂Up = ∂u/∂p this equal to 0. So therefore ∂j/∂t which we have written as ∂/∂t cj, ck which we expressed in terms of p and q this quantity = 0 which is the third property.

$$\partial J/\partial t=0$$

And this 3 properties will help us solving this problem Lagrange bracket can be evaluated at any point in the orbit ay. Now we need to discuss some more thing.

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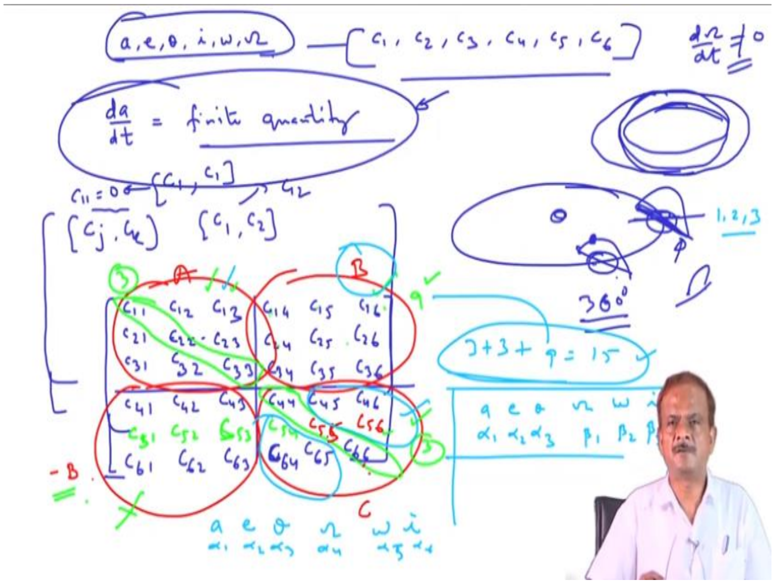

Say we have we are aware that we have parameters a, e, i, Ω ,ω and ϴ these are the 6 parameters. And this we are representing c_1, c_2, c_3, c_4, c_5 and c_6. So here in this case in that true orbit case what is happening that da is not a constant but rather da/dt this is a finite quantity. This simply implies that it is a varying and now in whether in the or this while the planetary perturbation is present.

So at that time whether this is changing or not changing it is matter of time we will explore that which quantities are more important dominate and which quantities are small. So all this things can be explored like for 1 example I will give you the case of the sun synchronize satellite where the satellite is always facing the sun it is orbit is always facing the sun the plane of the orbit. So as the sun is here; and the earth is rotating in orbit.

So orbit is say like this and you always want your orbit to face here in this direction so now it goes here in this place or goes here in this place if it remains parallel means it will not be pointing towards the sun ay. So to point towards the sun it must rotate like this and this nodal line notation it results from the non-spherical shape of the earth rather than being a sphere let us say oblate it is a oblate like this.

And because of this the nodal line can rotate ay so this is one part orbital perturbation where d ω /dt this is a finite quantity this is not equal to 0. And it as to rotate at a particular rate so that in 1 year it goes by turns right 360 degree and this corresponds to a particular angle of inclination for particular altitude you need have a particular inclination. So all this things we will to explore it is too early to discuss all this is issues .

But here these are the 6 parameter which we are representing like this and we are looking for how these parameters are changing with time if the perturbation is present. So our the c_j c_k the terms the brackets which we are taking we will have the terms like the first one is c_{11} which is equal to 0 the short notation is we are writing like this. This is c_1 we are writing as c_{11} the next one c_1, c_2 this will write as c_{12} as I stated earlier.

So this way we will have this matrix and in this matrix we will let us say we write it like this c_{11}, c_{12} to c_{16}. Similarly c_{31} in between we have c_{21} all those things will go there c_{22} and here c_{32}, c_{33} we divide it into this 4 quadrant like this. Similarly this part we write as c_{14}, c_{15}, c_{16}, c_{35}, c_{36}, c_{44}, c_{45} and c_{46} and the last one the fifth one I am skipping c_{61}, c_{62}, c_{63}, c_{64}, c_{65} and c_{66}. So this let us say that I represent this as A and this as B .

So this part then because of the Lagrange bracket property it can be written by –B and this we will write as C. Out of this quantities are 0 as I discussed earlier this are 0 so I need to evaluate in this quadrant only how many terms it is a skew symmetric and therefore 1, 2, 3 only 1, 2, 3 quantities have to be estimated here. Here in this part in this also only 3 so 3 needs to be determined here and how many here in this part here B total 9,

1, 2, 3, 4, 5, 6, 7, 8, 9 so 9 here and as we get this immediately with minus sign this is available so this part we do not have to work , only this and this here also because it is a skew symmetry so c51, c52, c53, c54, c55 and c56. So c44, c55, c66 these are 0 only work diagonal terms are represent and because of the skew symmetry. So these terms will be negative of this terms here so only 3 needs to be determine here so 3 + 3 and +9 from this place so that makes it 15.

So as I states earlier only 15 brackets we need to be evaluate and rest we forget so and one more part let us say that a ,e ,i ,Ω ,ω and Θ we write as α_1, α_2, α_3, α_4, α_5, α_6 or either c_1, c_2, c_3, c_4, c_5, c_6 whether this way. So instead of doing this what we will do that we will write a e Θ as α α_1, α_2, α_3 and Ω ,ω i as β_1, β_2, β_3.

So that our indexing will be only over 1 to 3 that I will show you in the next lecture what does this imply and so this will ease our process that you can see here in this bracket only 1, 2, 3 terms are there 1, 2, 3 rows 3 columns and 3 rows are there in this B . So it is better to replace in terms of 1, 2, 3 so we do not have to carry in the terms like the 4, 5, 6 and the next time I will show you so we stop here thank you very much.