Space Flight Mechanics
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Lecture-54
General Orbit Perturbation Theory (Contd.)

Welcome to lecture 54. We were working with evaluation of Lagrange bracket.

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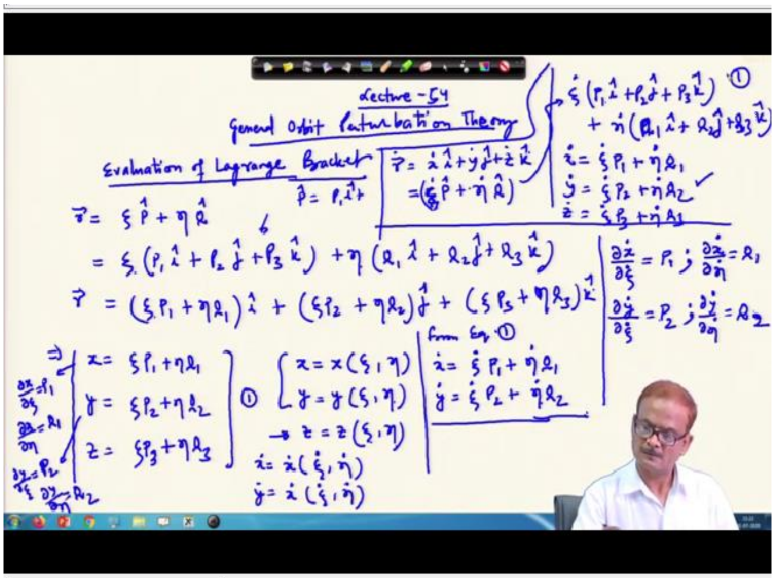

So, we continue with that, so in that context we wrote

$$r = \xi \hat{P} + \eta \hat{Q}$$

This is Q_1 $\hat{\imath}$. See using this short notation of representing $\hat{p}$ in terms of P_1 $\hat{\imath}$ and so on as we have written here. So, we are avoiding longer equations write $P_3 \times \eta \times Q_3$. So, let us name this as a equation 1 here. So, this implies that x is a function of ξ, η, y is a function of ξ, η and z is also a function of ξ η.

But while dealing with orbit, we are not interested right now in this part, only we are interested in this part from equation 1, we will get here $\dot{x}$ equal to. Now if we differentiate this part we can see that we get this as $\dot{\xi}$ P_1 + $\dot{\eta}$ Q_1. And from where this is appearing this is appearing directly from this equation

$$\dot{r} = \dot{x}\,\hat{\imath} + \dot{y}\,\hat{\jmath} + \dot{z}\,\hat{k}.$$

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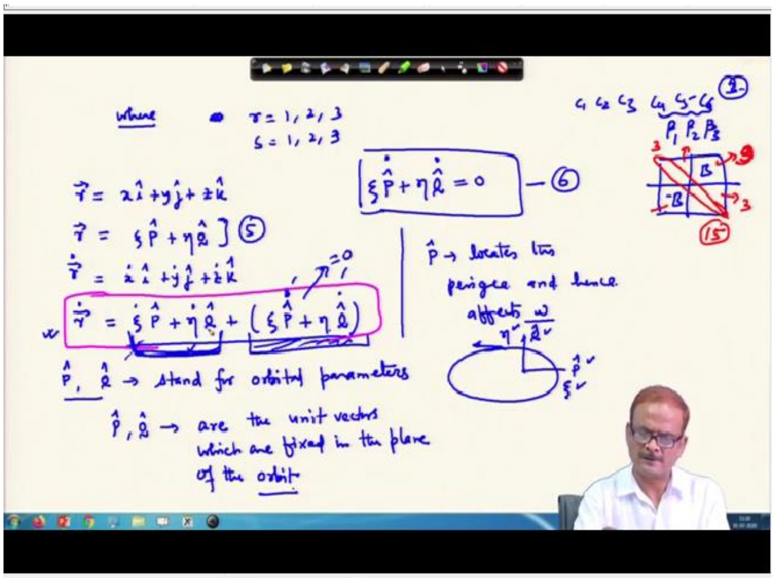

And so in that equation if you look back this particular part it is only $\xi\,\hat{P} + \eta\,\hat{Q}$, so what we were writing, this is visual from this place because this part is 0 ay. Therefore we are left with only $\dot{\xi}\,\hat{p} + \dot{\eta}\,\hat{Q}$.

$$\dot{r} = \dot{\xi}\hat{P} + \dot{\eta}\,\hat{Q}$$

and $\hat{p}$ already we know this is $P_1\hat{\imath} + P_2\,\hat{\jmath} + P_3\hat{k}$.

And the other part $Q_1\,\hat{\imath} + Q_2\,\hat{\jmath} + Q_3\,\hat{k}$. So, from here immediately we can see that $\dot{x}$ n becomes $\xi \times P_1 + \dot{\xi} \times P_1 \times \dot{\eta} \times Q_1$. Similarly $\dot{y}$ equal to $\dot{\xi} \times P_2$ and $\dot{\eta} \times Q_2$ so on. So, this is what we have written here in this place and $\dot{z}$ also can be written in the same way, $\dot{z}\,P_3 + \dot{\eta}\,P_3$ we are not interested in $\dot{z}$ right now ay.

Therefore, we see that $\dot{x}$ this is also a function of $\dot{\xi}\ \dot{\eta}$ and $\dot{y}$ also it is a function of $\dot{\xi}$ and $\dot{\eta}$. Moreover, from this place we will need some of this derivatives to be evaluated later on x $\partial\dot{x}/\partial\dot{\xi}$, this becomes P_1. And the $\partial\dot{x}/\partial\dot{\eta}$, this is Q_1 along the same line $\partial\dot{y}/\partial\dot{\xi}$ is Q_1. And $\partial\dot{y}/\partial\dot{\eta}$ is Q_2. Moreover, we can observe from this place. This gives us $\partial x/\partial\xi$ equal to P_1.

And $\partial x/\partial\eta$ equal to Q_1. So, these values are the same. This also $\partial y/\partial\xi$. This gives us ∂y, this is Q_2 P_2 $\partial\dot{y}/\partial\dot{\xi}$ is P_1 Q_1 , in this part we have written wrongly here. From this place we lo into this $\partial\dot{y}$ y

$\partial \dot{\xi}$ is P_2 and similarly $\partial \dot{y}$ y $\partial \dot{\eta}$ that becomes Q_2. So, here this will be P_2 and $\partial y/\partial \eta$ this becomes Q_2. So, these are some of the information we required to solve this problem.

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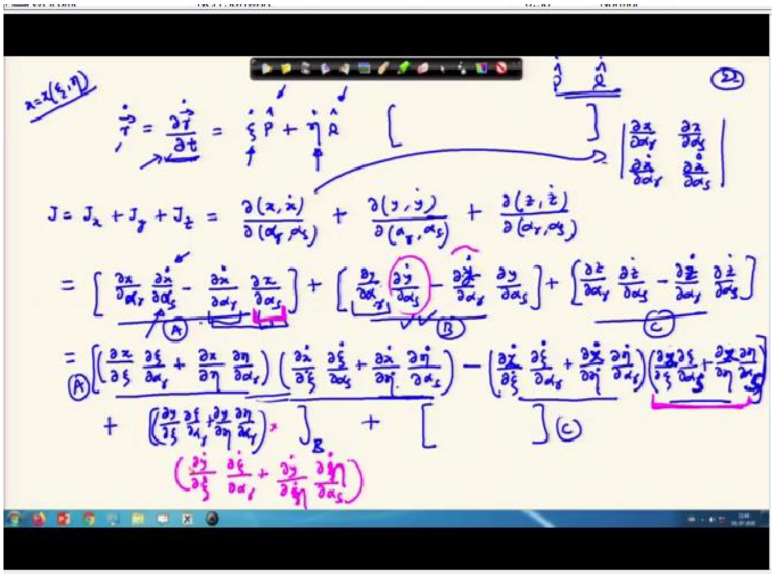

So, $\dot{r}$ is nothing but $\partial \dot{r}/\partial t = \dot{\xi}\hat{P} + \partial \dot{\eta}\ \hat{Q}$. The principle we have used here the other part as I told you earlier, $\dot{P}$ and $\dot{Q}$. This part is not appearing, but the same principle we are using the osculating orbit and the true orbit. We can see that $\dot{r}$ equal to $\partial \dot{r}$ / ∂t. So, this is for the velocity in the osculating orbit and the true orbit, they are the same.

And velocity in the osculating orbit, it will depend only on ξ and η, not on the unit vectors. So, therefore the other part is 0. with this information we are ready to work with the Lagrange bracket. This is what we were looking for and then we wrote these quantities as α_r α_s, the first one . It will be better all the time while you are working. So, write it in the determinant format and only then work with this.

Otherwise, there may be chances of mistake. But if you are conversant with the notation, you can directly write this here in this way also . So, the determinant as we can see, this will result in what I am writing, similarly the other part $\partial \dot{y}$. Now we insert the corresponding values, $\partial x/\partial \alpha$, how much this quantity is. This quantity we can write as. because x is a function of ξ and η.

Therefore, we can write this as $\partial x/\partial\xi \times \partial\xi/\partial\alpha_r + \partial x/\partial\eta$ and $\partial\eta/\partial\alpha_r$ and then $\dot{x}$. So, this is $\partial\dot{x}/\partial\xi \times \partial\xi/\partial\alpha_s + \partial\alpha_s$ minus this quantity. So, for this also we need to write here. So, you can see that if we try to write for all of them, how long this will be. So, it becomes pretty difficult to write all these quantities.

But because of the some of the repetition, we can write in a quicker way. So, this becomes $\partial y/\partial\xi$, see I could have escaped all this mathematics and directly I could have written the final equation, what do you miss all of them. It is a in order to how the things are developing. So, if you know that you understand the subject better and that the main objective here. So, $\partial y/\partial\eta \times \partial\eta/\partial\alpha_r$.

This is a bracket only for the first one, say this is suppose A, this part is B and this part is C. So, this is just representing A. Thereafter we have to also write the same kind of bracket for B. And same kind of bracket for C, only thing they will be in terms of, here sorry this part while working we are dividing this part is x, because still we are working with the A, the first bracket. So, here this is x. All these are x.

And then this part becomes the first part here $\dot{x}$ and then this becomes x . Similarly, we will have for y and $\dot{y}$ z and $\dot{z}$. Now we need to simplify will first use this only work with A .

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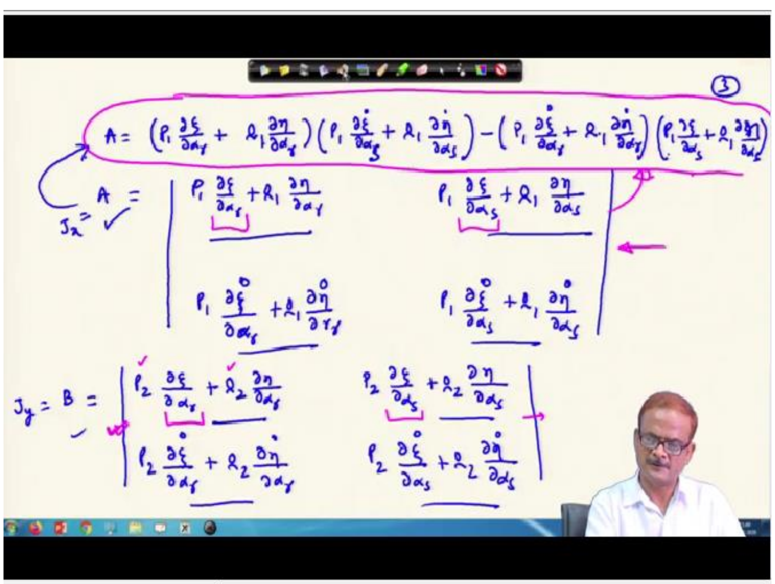

So, let us work only with the A, so A can be written as the first part $\partial x/\partial \xi$. This is nothing but P_1 , as we have written earlier $\partial x/\partial \xi$, this is P_1, this quantity here. So, the first one is P_1 and then multiplied $\partial \xi/\partial \alpha_r$ multiplied $\partial \xi /\partial \alpha_r$ + the second quantity ∂x then $\partial x/\partial \xi$ $\partial x/\partial \eta$. So, $\partial x/\partial \eta$ is Q_1 . So, therefore, we write here Q_1 and this become $\partial \eta/\partial \alpha r$.

So, this way we can compute this part A $\partial \dot{x}$ / $\partial \xi$ already we have seen that, it is the same thing the P_1 $\partial \dot{\xi}$. Here one more correction is there while we are writing we are missing that point. These are $\dot{\xi}$. Wherever $\dot{x}$ is there. So, if we lo back here. So, it is a function of $\dot{\xi}$and $\dot{\eta}$. Similarly $\dot{y}$ is a function of $\dot{\xi}$and $\dot{\eta}$.

So, accordingly we have got these quantities here, which are we are going to use. So, here also we were missing this be supplied this is $\dot{\eta}$, wherever the dot part is there. So, correspondingly on ξ and η we also the dot is present. So, here in this part also the dot is present. Similarly in this part also the dot is present and this part is . So, with this, then we have $\partial \dot{x}$ /$\partial \dot{\xi}$.

This is same thing as P_1. So, this we have already written. And this is $\partial \xi$ /$\partial \alpha$. This is $\partial \alpha$ yes ay, other part is here with respect to α is. So, this is α s, not α r. So, this is α s, similarly the other part. Then the next part, this part of the writing $\partial \xi/\partial \dot{\eta}$ $\partial \xi/\partial \dot{\eta}$ we will look here ∂. We are completing this part the first part we have already written.

This part is already we have written, this part we are writing. So, $\partial x/\partial \dot{\xi}$. We are loing for $\partial \dot{x}$ y $\partial \dot{\xi}$, which is nothing but P_1. So, the P_1 we are supplying here, and thereafter we have $\partial \dot{\xi}/\partial \alpha_s$. This we have done and then the other part $\partial \dot{x}$ / $\partial \dot{\eta}$ $\partial \dot{x}$ / $\partial \dot{\eta}$. This is Q_1 , so next we write here Q_1 and the $\partial \dot{\eta}/\partial \alpha_s$.

So, this is $\partial \dot{\eta}/\partial \alpha_s$. So, this way we can complete. And I could have written in terms of the determinant itself of and writing in terms of determinant it is a little it will not a spread out and it will be easy to work out. rather than working in terms of the expression, the way we are writing here. So, anyway that also already we have written in terms of this, so we will follow this part.

Thereafter, we will have the other part, which is this part we have done. Now this part we have to do. So, $\partial\dot{\xi}\partial\dot{x}$ / $\partial\dot{\xi}$ this equal to P_1 and then $\partial\dot{\xi}$/ $\partial\alpha_r$ and the same way, then $\partial\dot{x}$ / $\partial\dot{\eta}$ will be Q_1. So, this is Q_1 and then we get $\partial\dot{\eta}$/ $\partial\alpha$r $\partial\dot{\eta}$/ $\partial\alpha_r$ and then in the next term we take ∂x/$\partial\xi$, this is P_1.

This is P_1 and then $\partial\xi/\partial\alpha_r$ over quantity, this is α_r, this quantity see the first quantity we have written this part we have written, the other quantities with respect to s. So, this would be α_s. So as the equation gets complicated. So, it is quite likely that we omit a few symbols here. So, this part we are writing , this part is here. So, this part is with respect to α_s.

Therefore, we have to write it with respect to α_s. So, $\partial\xi/\partial\alpha_s$. And similarly the other part ∂x/$\partial\eta$. So, ∂x/$\partial\eta$ is nothing but Q_1. So, Q_1 and thereafter $\partial\eta/\partial\alpha_s$. So, this is for A and the same thing if we use the determinant form so simply this becomes P_1 $\partial\alpha_r$ + Q_1, writing in terms of determinants it is a little convenient because we do not do mistake in that case.

So, this is how your data los like. And once we multiply, so we get it in this form, ultimately, we have to use this form. we need to reduce the whole equation. So, we are bound to use this equation. So, following the same line, then the B can be written as from determinant part I am writing this, this easier to do P_2 $\partial\xi/\partial\alpha_s$ + Q_2. Because this is related to y and therefore, P_2 and Q_2 are appearing.

Rest other things will be mention. This is your part B, which is corresponding to this one. This is much easier to write. So, what we need to do, we pick up this ∂y/∂r $\partial\alpha_r$, we are writing in terms of α, here also we see $\partial\alpha_r$ × $\partial\dot{y}/\partial\alpha_s$. So, ∂y/$\partial\alpha_r$ the quantity we can check ∂y/$\partial\alpha_r$ ∂y/we have done mistake here.

So this part then we have not written here ∂y/$\partial\alpha_r$ this would have, we can expand it and if we expand. So, this we have to write again in terms of η ξ and η. So, ∂y/$\partial\xi$ × $\partial\xi/\partial\alpha_r$ + ∂y/$\partial\eta$ × $\partial\eta/\partial\alpha$r This is the; this part . Thereafter, we need to write for this part. So, this part, similarly we can put here in the bracket multiplied here.

So, this becomes $\partial\dot{y}$ this quantity, now with respect to $\dot{\xi}$. Because it is appearing as a function of $\dot{\xi}\partial\dot{y}$/ $\partial\dot{\xi}\partial\dot{\xi}/\partial\alpha_s$. And $\dot{\xi}$this is $\dot{\eta}$, $\dot{\eta}$ $\dot{\xi}$ and $\dot{\eta}$ function, so we are using this.Then we need to take up this part. So, similarly write this part. So, instead of writing here in this format here which are directly written in the matrix determinant format which is much easier to work with .

So, you can check this step, because so much of mathematics, if we keep doing this in the lecture, so it would not be possible to cover this course.

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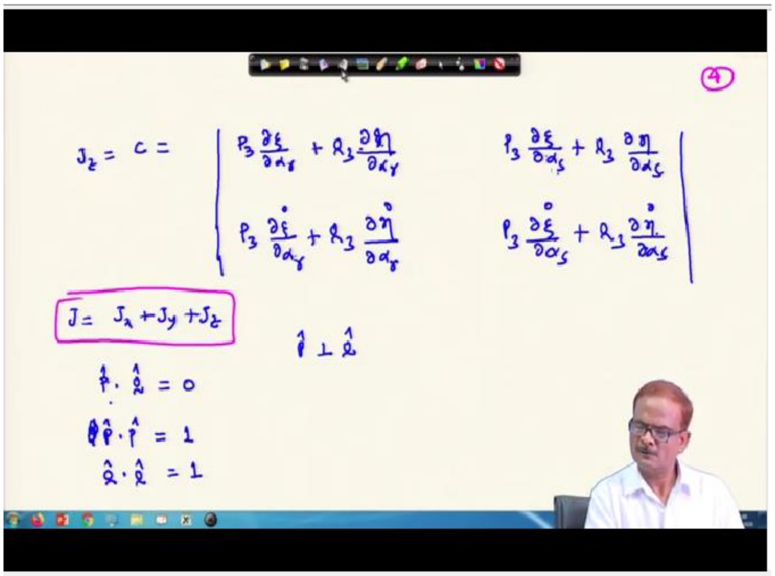

Similarly, the C part we can write as. Now, instead of this P_2 P_1 P_2 η starts with P_3. So, P_3 and rest of the things will appear in the same way those of you can see that from here. This part is the same as appearing here $\partial\xi/\partial\alpha$ $\partial\xi/\partial\alpha_s$. So, they are appearing in the same way except for the coefficient P_2 and Q_2. So, from here, it is very much clear that we just have to repeat it.

We do not have to do much exercise here. So, $P_3 \times \partial\xi/\partial\alpha_r + Q_3 \times \partial\eta/\partial\alpha_r$ similarly here $P_3 \times \partial\xi/\partial\eta$ r α_s. This is . First we are differentiating so $\partial\xi/\partial\alpha_s$ and $Q_3 \times \partial\eta/\partial\alpha_s$. So, here α_r is appearing, αs is appearing, α_s, α_r we can see α_s, α_r α_s, α_r. So, it is a repeating the same pattern.

And therefore, if we add all of them. So, this is basically what we have written as A, B and C, this is nothing but your Jx equal to A, this Jy and this part is Jz. So, J equal to J x + Jy + Jz. In addition, we are going to use the properties that $\hat{P}.\ \hat{Q}$, because both are perpendicular to each other. $\hat{P}$is perpendicular to $\hat{Q}$. therefore, this quantity is bound to be 0.

And moreover $\hat{P}.\hat{P}$ This will be because it is a unit vector. So, the product will be $_1$. Similarly, $\widehat{Q}$. $\widehat{Q}$. This will also be $_1$. So, this information we can utilize to solve this particular problem. Now, whatever we have written for the A. All of them, A, B and C all of them need to be represented in this form. Finally we have to use this form, but this form is write in the beginning easy to work with .

So, this need to be reduced in this format. So, we have already done this also needs to be reduced here in this format. And then all of them need to be added, because there is the plus sign and plus sign here. So, once add all of them we get J. So, for getting J we need to add all the 3 determinants. So, after adding them we can reduce them to get the proper equation of what we do the rest of the exercise we do it in the next lecture, we will stop here. Thank you very much.