Space Flight Mechanics
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Lecture-55
General Orbit Perturbation Theory (Contd.)

Welcome to lecture 55. So, we have been trying to evaluate the Lagrange bracket. So, in that context we are working with certain mathematics.

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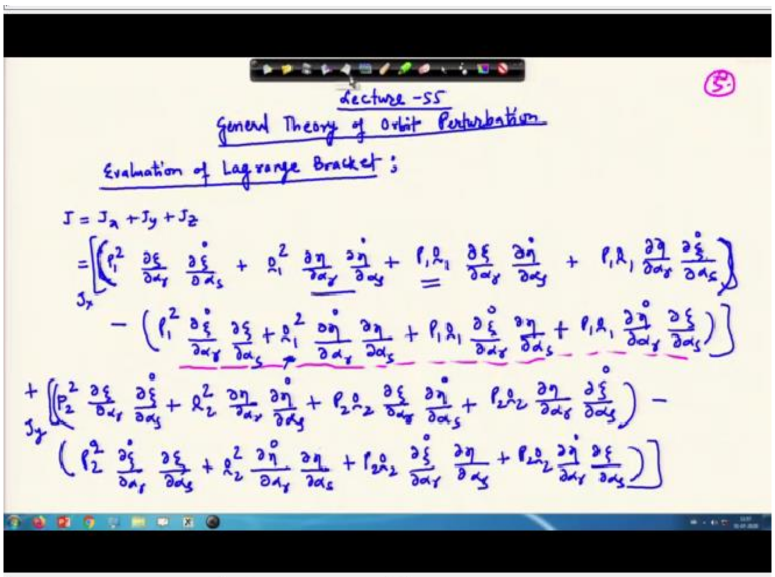

So, if you remember, in the last class we wrote this J equal to J x + J y + Jz.

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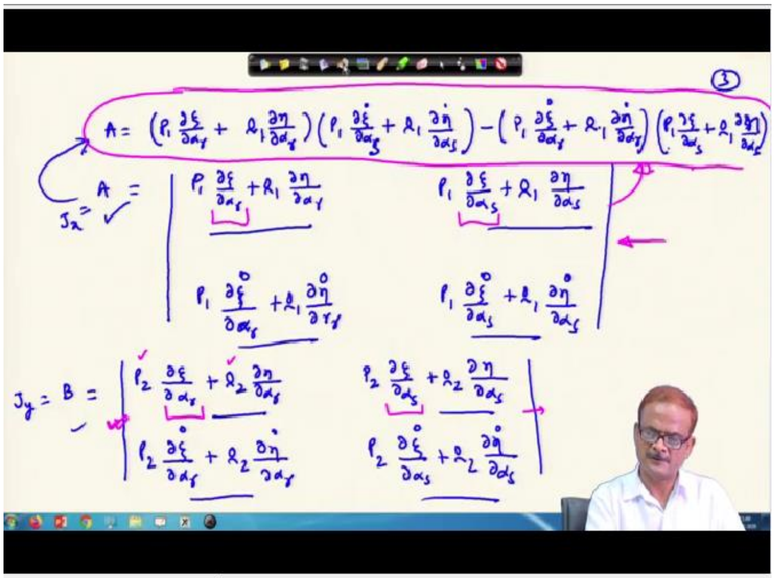

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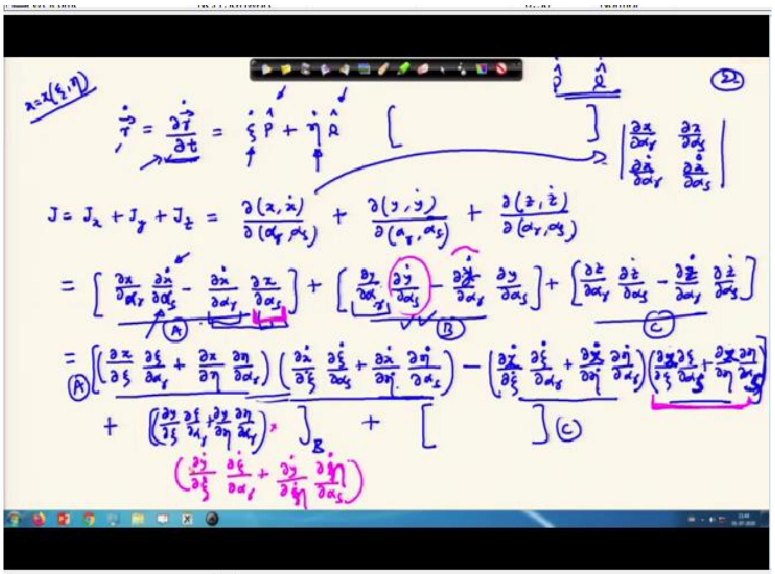

And thereafter we expanded them like this. So we need to evaluate each Jx Jy and Jz.

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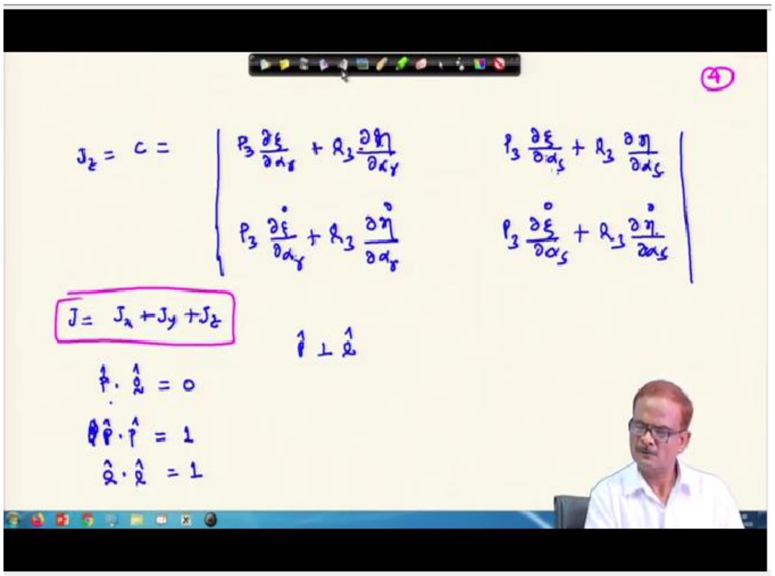

And add them. So, we need to expand them and add them. So, if we do it the exercise from the previous lecture. So, what we get here, just P_{12} $\partial\xi/\partial\alpha_r$. This you can check yourself if only thing that you need to take paper and pen and multiply them and write here, I am directly writing here in this place. This part, first we are writing Jx, this minus P_{12}. You can see immediately that how the pattern is changing.

It is the same thing as we have written here in this place. Only thing the dots are changing. This is your Jx, this part is Jx. Similarly, we need to write the other part which is Jy. So, instead of here this P_1, we will have P_2 here in this place. So, P_2 $\partial\xi/\partial\alpha_r$. Rest other things we have to copy from above, you can check, guess yourself again/multiplying Q_2 square. Instead of P_1 Q_1 here, this becomes P_2 Q_2, rest other things remain same.

And this - P_{22}. Then we pick up this part here, this part and replace P_1/P_2 and Q_1/Q_2. This is αs, from this part we are writing, so this dot will be here. Next we have to just replace in this Q_1/Q_2 and rest other things will appear as it is. P_2 Q_2 $\partial\dot{\xi}/\partial\alpha_r$. Again here in this place we have done the mistake, this part is dot is not there. So, this is your Jy part. And one more term we have to write, which is the Jz.

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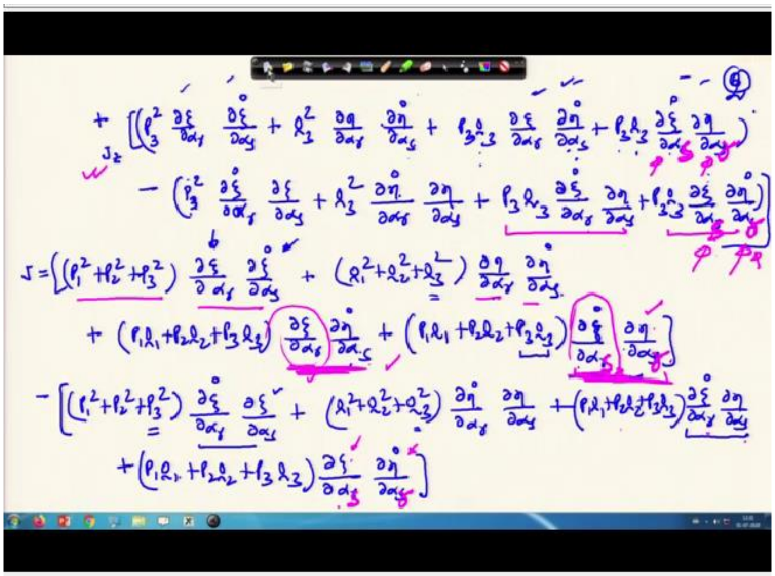

And + Jz we have to write here. So, Jz is here. The same thing we need to copy from the previous one, only thing P_2 will be replaced by P_3. So, this because $P_{32}\partial\xi/\partial\alpha_r$ $\partial\xi/\partial\alpha_s$ Q_{32} P_3 Q_3, this – P_3 Q_3, so we should follow the right term ∂dot and this is ay, this part is ay and P_3 Q_3 $\partial\eta/\partial\alpha_s$ and $\partial\xi/\partial\alpha_r$. So, here only ξ is appearing, η, η is appearing as η and $\dot{\eta}$

While here this is getting mixed up. So, these things we have to check that we are writing it properly and the same way, the P_{32} ay, we follow the same pattern. Only thing this time we have $\partial\dot{\xi}/\ \partial\alpha_r$ as we have here in this place. Once we are subtracting it, so this is appearing with $\partial\dot{\xi}/\ \partial\alpha_r$. So, it will follow the same pattern. Q_3 square $\partial\eta/\partial\alpha_r$.

So, these are the 3 terms the determinants, which has been expanded. Now, we can combine the terms which are common. Like, we can write here, J equal to P_{12}+ P_{22} + P_{32}. So, already we have written this quantity here. So, this consists of, this implies this is P_{12}+ P_{22} + $P_{32}$is equal to $_1$. Similarly this implies Q_{12}+ Q_{22}+ Q_{32} equal to $_1$.

And this implies P_1 Q_1 + P_2 Q_2 + P_3 Q_3 equal to 0. So, this information we are going to use, and there/ the equation will get simplified. So, we combined these terms and then it los like this. So, the other term will come with the whole thing in the bracket - P_{12} P_{22} P_{32}. You can see that here this dot was not there, so here dot is appearing, here dot was there.

So, dot is getting over from that place. So it says from nearly a reputation with exchange of the dot, so Q_{12} Q_{22} $Q_{32} \times \partial\dot{\eta}/\partial\alpha_r$ and then plus $P_1 Q_1 + P_2 Q_2 + P_3 Q_3 \times \partial\xi/\partial\alpha_r$ and $\partial\eta/\partial\alpha_s$, now just on recap this particular one so $\partial\xi/\partial\alpha_r$ and $\eta/\partial\alpha$, $\partial\dot{\eta}/\partial\alpha_s$. P_1 Q_1 . We can check this term, whether this is correct or not directly from this place.

Because we are writing the third term. So, accordingly we can correct if there is any error. So, just let us look into this. We are writing P_1 Q_1 P_2 Q_2 and P_3 Q_3. So, we have combined the term so P_3 Q_3 we are using here in this place. So, P_3 Q_3 . We just need to check these terms and just wondering some sign has been triggered P_1 Q_1 and this part is . And then P_1 square it was there, this also .

this mathematics is a little long but it is nothing there is very great in this, we just have to combine the various terms. This is the whole thing, whole cross, what we have to work with. So, instead of working at this level, if we just verify the term that we have written correctly or not. So, our job will be done P_3. We will expand this particular term related to this place. So, from here we get $P_{32} \times \partial\xi/\partial\alpha_r \, \partial\dot{\xi}/\partial\alpha_s + Q_{32} \times \partial\eta/\partial\alpha_r$ and $\partial\dot{\eta}/\partial\alpha_s$.

And then + P_3 Q_3 P_3 Q_3 in this particular part we are multiplying. So, $\partial\eta/\partial\alpha r$ and $\partial\dot{\eta} \times \partial\alpha s$. Similarly, we will have here Q_3 $P_3 \times Q_2 \, \partial\dot{\xi}/\partial\alpha_s$ timed $\partial\eta/\partial\alpha_r$. So, there is exchange for this term and then minus, taking this one, so – P_3 square $\partial\xi/\partial\alpha_s + \times \partial\dot{\xi}/\partial\alpha_r + \partial r$ and then $\partial\alpha_s$.

On paper pencil with patience it is ok . But without writing it frequently it becomes a little difficult to work with $\partial\alpha_s$. So we will write here $\partial\alpha s$ terms $\partial\eta \times \partial\dot{\eta}/\partial\alpha_r$, P_3 $q_{32}\partial\dot{\eta}/\partial\eta \, \alpha_r + P_2 Q_3 \, \partial\alpha_r$. This is the way it is written. So, now we can check from the corresponding from whether we have written correctly or not.

Except for writing working on a paper it is very easy, but writing here on the board where we do not get the equation in front of me is occurring from one page to another page. It is quite tough. So, here we have $P_{32}P_3$ Q_3. And here P_3 and Q_3 is also there, also will check for this, whether this is correct or not $P_{32}\partial\xi/\partial\alpha_r \, \partial\xi \, \partial$dot$/\partial\alpha_s$ P_3 square $\partial\xi/\partial\alpha_r \, \partial\dot{\xi}/\partial\alpha_s$.

So, this is appearing P_{32} and then Q_{32}, only the plus sign $Q_{32}\partial\eta/\partial\alpha_r\ \partial\dot{\eta}/\partial\alpha_s$. So, Q_2 square $\partial\xi\ \partial\eta/\partial\alpha_r\ \partial\dot{\eta}/\partial\alpha_s$. So, this part is also fine. so this is done. This part is done. And then, this minus sign. The same thing is repeating only thing that her P_{32} and $\partial/\ \partial\eta/\partial\alpha_s$. Let me check this part again $P_{32}\partial\xi/\partial\alpha_s$.

This is α_s and $\partial\dot{\xi}/\partial\alpha_r$, so these are some of the mistakes we do while writing the subscript gets misplaced. And then the other thing becomes $Q_{32}\times\ \partial\eta/\partial\alpha_s$ and $\partial\dot{\eta}/\partial\alpha_r$. So, this thing we have to take care of – $P_3\ \partial\xi/\partial\alpha_s$, P_{32}– P_{32}. So, in the minus sign we have $P_{32}\partial\xi/\partial\alpha_s\ \partial\dot{\xi}/\partial\alpha_r$, this part.

This is ay $\partial\xi/\partial\alpha_s\ \partial\dot{\xi}/\partial\alpha_r$. Similarly, Q_3 dot $\partial\dot{\eta}/\partial\alpha$ this also we verified, Q_{32}, we have already verified the rest, $\partial\dot{\eta}/\partial\alpha_r$ and $\partial\eta/\partial\alpha_s$, this also . So, Q_{32} with minus sign here $\partial\dot{\eta}/\partial\alpha_r\ \partial\eta/\partial\alpha_s$, this is also that part . Now rest we just need to verify this part, this part, this part and this part.

So, P_3 Q_2 P_3 Q_3, it is appearing as a common term. So, $\partial\xi/\partial\alpha_r$ and $\partial\dot{\eta}/\partial\alpha_s\ \partial\xi/P_3\ Q_3$ which is appearing here, P_3 Q_3 $\partial\dot{\xi}/\partial\alpha_r$. So, this is the problem I cannot get back and forth, P_3 Q_3 $\partial\xi/\partial\alpha_r\ \partial\eta/\partial\alpha_s$. So, these things are while we have expanded. So, after expanding we are combining the terms together.

So, only thing that we need to put the corrections here. These are the corrections here you can note. These are the subscripts, we have to put properly. So this is Jx Jy and Jz. So, in the Jz terms we have got exchange. So, I have done all these corrections. Here also this correction is applicable and in this place also the correction is applicable. And we can verify from this particular one, this $_2$ we can verify.

$\partial\xi/\partial\alpha_r$. So, this is $\partial\dot{\xi}/\partial\alpha_s\ \partial\dot{\eta}/\partial\alpha_s$. So, this is $\partial\eta/\partial\alpha_r$. So not only here in this while we take the minus sign with the minus sign the $\partial\dot{\eta}/\partial\alpha_r$. here this was see in this place this is $\partial\eta/\partial\alpha_r$, here this is $\partial\dot{\eta}/\partial\alpha_r$ and $\partial\eta/\partial\alpha_s$. So, this is the change which has taken place.

In this place also you can notice that this is $\partial\dot{\eta}/\partial\alpha$ this one. So this become $\partial\eta/\partial\alpha_s$. And this is $\partial\eta/\partial\alpha_r$. So, this has become $\partial\dot{\eta}/\partial\alpha_r$. So, this is the proper sequence, so just a simple mathematics, simple multiplication nothing else, only thing that gets a little longer and we have to patiently write the things in proper way there. So, that the different subscriptions, do not get mixed up .

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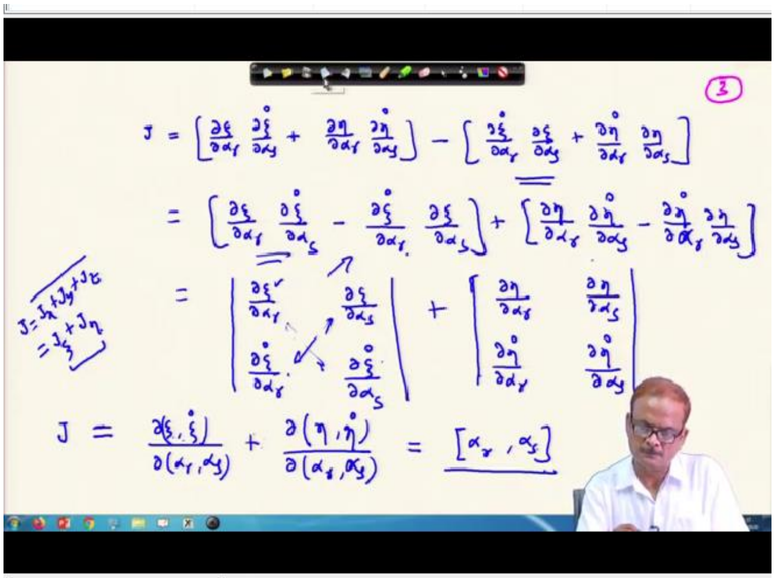

With this information now we are aware that this quantity is equal to 1. So, this is also equal to 1. This quantity is equal to 0, this quantity equal to 0. This is 1. This is 1. This is 0. And this is also 0. So our equation then gets simplified a lot. Therefore, the equation for the J this becomes $\partial\xi$/we pick from $\partial\xi/\partial\alpha_r\, \partial\dot{\xi}/\partial\alpha_s$. And then from this place $\partial\eta/\partial\alpha_r\, \partial\dot{\eta}/\partial\alpha_s$.

This part is 0, this part is 0. So, this gets eliminated. And then we get from this only the minus part. So, this minus $\partial\dot{\xi}/\partial\alpha_r\, \partial\xi/\partial\alpha_s$. And then this part $\partial\dot{\eta}/\partial\alpha_r$ and $\partial\eta/\partial\alpha_s$. Rest other comes up 0. So, we consider a long mathematics, it has got reduced to a very simple format. Now we combine the terms in this way. From here, then we bring the term , here again $_1$.

This part $\dot{\eta}\times\eta$, this is just η here, and in the determinants permit immediately you can immediately check this is nothing but whatever is written here $\partial\xi/\partial\alpha_r \times \partial\dot{\xi}/\partial\alpha_s$. This is the first term. And $\partial\dot{\xi}/\partial\alpha_r$ and $\partial\xi/\partial\alpha_s$, which is done just cross them. And the same way, this can be written. And in the shortcut notation we can then represent this as.

The first term we can represent as $\partial\xi$ $\dot{\xi}$ and because ξ and $\dot{\xi}$ these are involved, and in the denominator we have αr αs. So, J gets reduced into this format and this is nothing but your Lagrange bracket αr αs. So, this is in the orbit term. So, but we have been discussing that J equal to Jx + Jy + Jz. So, this has got reduced to the format in terms of ξ and vector, our variables remain the same α_r and α_s.

As they are the same, but the coordinate system we are using that had changed. So, this is a big simplification from 3 determinant to only using 2 determinants yet in this place.

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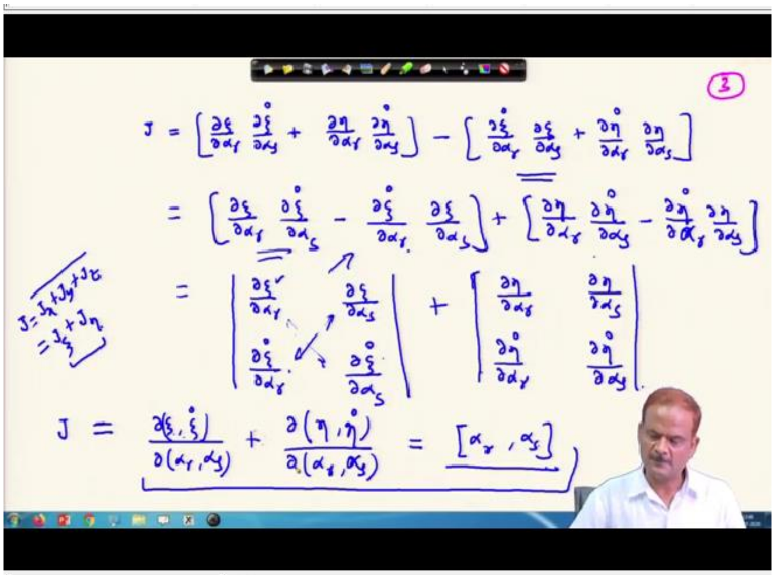

So, from the above derivation we can realize that ξ η can be used to address our problem and ξ η being co-planar. We can address the corresponding parameters a, e and n t with minus sign m equal to n × t I T. This also we have written T . So, time of perigee passes. So, this can be addressed using ξ and η in the plane of the orbit and using the parameters, α_r, α_s.

Where αr, αs what there are, these are referring to a, e etc. So, we have just looked into the bracket αthe Lagrange bracket α_r and α_s. So, you can compose using this the terms. So α_r, α_s from to which part it is belonging, it is a belonging to this part. In this part diagonal term is 0 and we have the total 9 terms are here present, out of this the diagonal 3 terms will be 0.

Rest other terms which are 6. So, 3 are on this side, three are on this side. So, we just need to evaluate those 3 and automatically because it is a skew symmetric. So, therefore, this part will be immediately determined, here we have to determine all the 9 and thereafter, this part 9, the other part 9 will be automatically determined, because of the minus sign, so 3 here, 9 here, and 3 here in this place.

So, total 15 we have to evaluate. So, this part is referring for your α_r, α_s, and this part is referring as earlier I have told, this is referring to α_r and β_s. While this part is referring to β_r and β_s. So, this way, we have to evaluate the Lagrange bracket. And once we do, but still we have not evaluated it, we have got it here in this simple format, using this we can proceed. So, we will continue in the next lecture and try to work out at least one next time. Thank you very much.