Space Flight Mechanics
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Lecture No - 62
General Perturbation Theory (Contd.)

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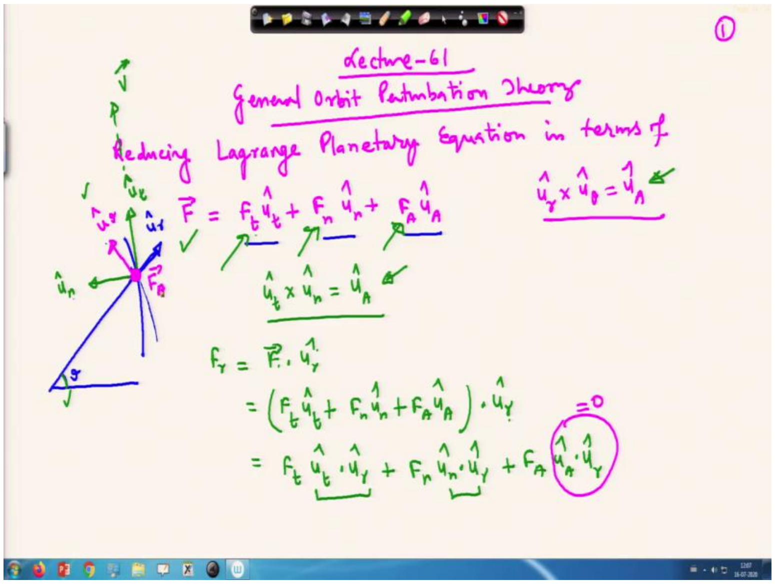

Welcome to lecture 61. So we have been working with the reduction of the Lagrange planetary equation in terms of F_r, F_θ and F_A. Now we reduce this in terms of F_t, F_n and F_A as it says written here. So for this the model we are using. In the earlier case u_r was in this direction and I will change the orbit little bit and u_θ was here in this place and then $\hat{u}_A$ was like this. So we have written, here in this case, but we are going to do instead of u_r now we will be using $\hat{u}_t$ which is the tangent vector in the velocity vector directions.

$$\vec{F} = F_r\hat{u}_r + F_\theta\hat{u}_\theta + F_A\hat{u}_A$$

$$\hat{u}_t \times \hat{u}_n = \hat{u}_A$$

Along the direction of velocity of the satellite this is$\hat{u}_t$ and u V I will remove from this figure because we were in the; instead of we are in the same direction as the vector I am going to show for this is $\hat{u}_n$. So instead of u theta now, we have $\hat{u}_n$and then perpendicular to this so $\hat{u}_t \times \hat{u}_n$ this we are going to write as $\hat{u}_A$. So in both the cases in the previous case also u_A appeared and here also we are using this symbol.

So already we have discussed how this is you u theta because the angle theta is shown here so things I am not going to discuss in this place. So, with the equation written for the F, F_t equal to u_t F and in terms of A say. Now in the previous equation lecture if you see here we have got this equation in terms of F_r perhaps we have done some mistake again here in this place again u_θ, this is $1 - e^2$ in F_r this part is θ.

$$F_r = \vec{F}.\hat{u}_r$$
$$= F_t\,\hat{u}_t.\hat{u}_r + F_n\hat{u}_n.\hat{u}_r + F_A\hat{u}_A.\hat{u}_r$$

So this is the correction that we need to do. This is where theta ok, so what we require that this quantity should be available. In all these places will be replaced in terms of F_t, F_n and F_A so our langrage planetary position which reduced to the in terms of F_θ. F_r and F_A. Now this will get reduced in terms of F_t, F_n and F_A and this is pretty simple ok not much work is required in this. So, F_r we can write as $\vec{F}.\hat{u}_r$; they are not perpendicular to each other ok.

But A and r the $F_A\hat{u}_A.\hat{u}_r$ this quantity will be 0 because they are perpendicular to each other. A is a vector which is coming out of the paper which I have not shown here it is a coming out of the paper F_A. Generally it was showed by the head of the arrow. F_A, can be shown like this. We will write $\hat{u}_A$, this is $\hat{u}_A$ which is shown by this pink dot which is coming out of the paper.

And also the green one the u_A is in the same direction. For u_t, u_n and u_A so we are left with estimating this two things u_t. u_r and u_n .u_r. Now if you look in this place this vector is the tangent vector and what this angle is that we have to write here. So, let us say this angle is $\hat{u}_t$ then you have $\hat{u}_t.\hat{u}_r$ this will be equal $\hat{u}_r$ magnitude $\hat{u}_t$ magnitude times cos α and what this quantity will be?

Now going back and looking into our earlier lectures, if this is r direction and this is the tangent direction, $\hat{u}_t$ along this direction. This is the θ direction, theta is like this. This angle we have written as α. So, V is along this direction, r is along this direction and $r\dot{\theta}$ will be along this direction. So what do we get? From this place; cos α equal to r dot will be here so $\frac{r\dot{\theta}}{V}$. So therefore this becomes equal to cos α, all there are unit vector.

And this quantity we are trying to evaluate here. So we have $\hat{u}_r.\hat{u}_t$ equal to cos α into $\dot{r}$ divided by cos α becomes $\frac{r\dot{\theta}}{V}$. Similarly other things we can work out.

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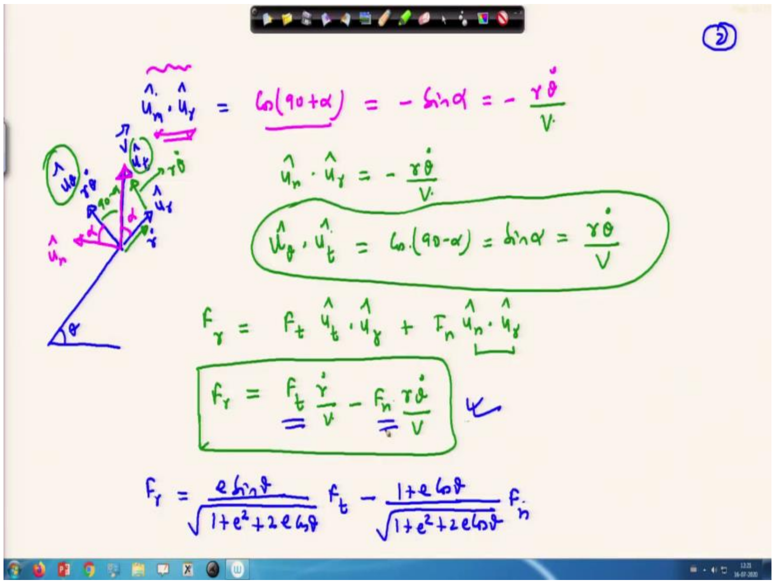

Now the next term is $\hat{u}_\mathrm{n}.\hat{u}_r$. The same way this is the angle between the u_n vector, this is θ and r direction we have taken here; $\hat{u}_r$ then V we have taken here in this direction and then r × $\dot{\theta}$ is along this direction r dot is along this direction perpendicular to the u_n vector, $\hat{u}_\mathrm{n}$ is along; $\hat{u}_t$ is along this directions of perpendicular to this will be; I will take; use another colour.

$$\hat{u}_\mathrm{n}.\hat{u}_r = \cos(90+\alpha) = -sin\alpha = -\frac{r\dot{\theta}}{V}$$

This is $\hat{u}_\mathrm{n}$, this angle we have written as α read that, this rotates α, for this angle also will also be equal to α. So therefore these quantities then we are looking for the angle between u_n vector and u_r vector. So how much this angle will be there shall be 90 + α the angle between them becomes cos; this quantity will be than equal to cos 90 + α equal to – sin α. And sin α is the quantity $r\dot{\theta}$ the quantity which is here this quantity is $r\dot{\theta}$.

$$\hat{u}_\theta.\hat{u}_t = \cos(90-\alpha) = sin\alpha = \frac{r\dot{\theta}}{V}$$

And this quantity r dot is along this direction, so then sin θ becomes sin α becomes $\frac{r\dot{\theta}}{V}$. So therefore $\hat{u}_\mathrm{n}.\hat{u}_r$ this is -$\frac{r\dot{\theta}}{V}$. Similarly we have $\hat{u}_\theta.\hat{u}_t$; $\hat{u}_\theta$ is the this vector here and $\hat{u}_t$ is this vector here. So this angle is 90 -α and therefore this becomes cos 90 - α equal to sin α, which we can simply write as $\frac{r\dot{\theta}}{V}$.

$$F_t\, \hat{u}_{\text{t}}.\hat{u}_r \;+\; F_r \hat{u}_{\text{n}}.\hat{u}_r \;+\; F_A \hat{u}_A.\hat{u}_r$$

$$F_r \;=\; F_t \frac{\dot{r}}{v} - \; F_n \frac{r\dot{\theta}}{V}$$

So, we have got all these terms here and therefore F_r can be written as $F_t\, \hat{u}_{\text{t}}.\hat{u}_r$ + F_n from the previous derivation in this place we use it $\frac{r\dot{\theta}}{V}$. and then $F_r \hat{u}_{\text{n}}.\hat{u}_r$ – rθ – $F_n \times \frac{r\dot{\theta}}{V}$. So, immediately you can see that our equation has got reduced into this format. So just we need to replace in the equations we have derived here the F_r, F_θ, F_r we have got here; in the same way we can get the other terms.

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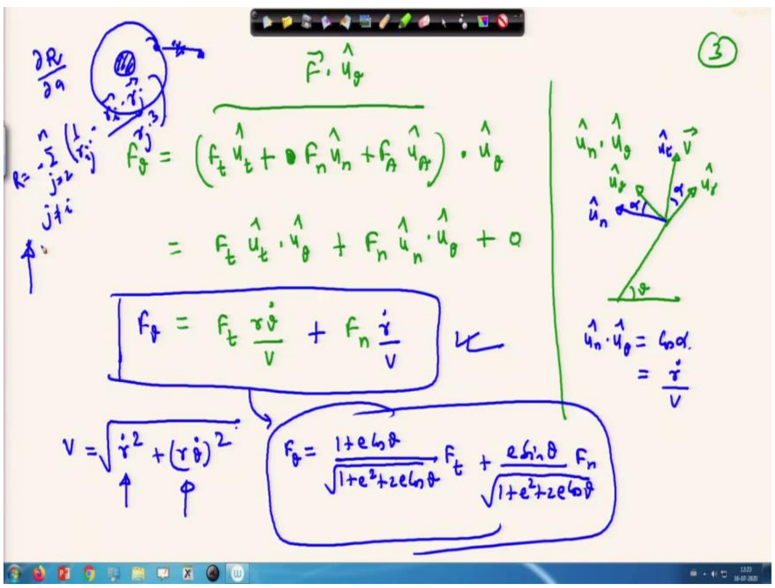

F_θ is $\hat{u}_t$+ F_n $\hat{u}_{\text{n}}$ and $F_A \hat{u}_A.\hat{u}_\theta$ this is A vector the component along the u_θ direction and from this place then we have u_t all these are whichever the coplanar vectors and not perpendicular to each other. they will not be 0 rest others will be 0 and this term will be 0 because they are perpendicular to each other $u_t.\hat{u}_\theta$ this place r $\dot{\theta}$ u_n and u_θ we have not worked out so we need to do that.

$$F_\theta \;=\; (F_t \hat{u}_t +\; F_n \hat{u}_n + F_A \hat{u}_A\,).\hat{u}_\theta$$

$$F_\theta \;=\; \left(F_t \frac{r\dot{\theta}}{V} + \; F_n \frac{\dot{r}}{V}\right)$$

$\hat{u}_n$ dot com so for this we have drawn a figure here. This angle here is α, this angle is α $\hat{u}_n . \hat{u}_\theta$ this will be cos α and cos α is nothing but $\dot{r}$divided by V, this is our F_θ. Now one more step is to be taken that V needs to be replaced in terms of the common terms that we are using. So here if we insert these values ok we will get the V and similarly $\dot{r}$ we have to write.

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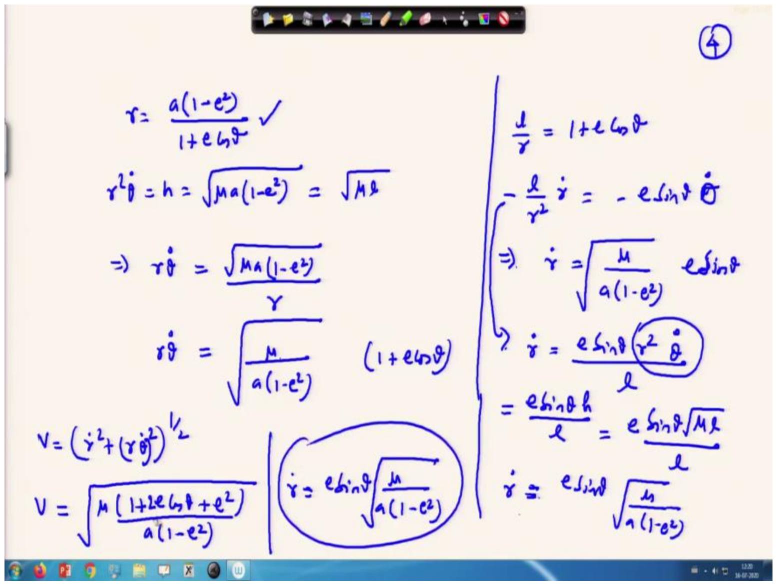

In this since we have done in quite details while working with the two body problem when the central force motion, this is $\sqrt{\mu \times e}$ root instead of r from this place you can check it this part. So for your ready difference I am because we have done it quite in the early in the lectures for therefore I am driving it again here in this place. This can be simplified to $\dot{r}$ equal to e sinθ.

$$r = \frac{a(1-e^2)}{1+e\ cos\theta}$$

$$r^2\dot{\theta} = h = \sqrt{\mu a\ (1-e^2)} = \sqrt{\mu l}$$

$$r\dot{\theta} = \sqrt{\frac{\mu}{a(1-e^2)}}\ (1 + ecos\theta)$$

$$\dot{r} = \sqrt{\frac{\mu}{a(1-e^2)}}\ esin\theta$$

So we need to replace a l and r in this expression $r^2\theta$ into this quantity is it is sin θ it divided h divided by l and h is $\sqrt{\mu \times e}$ so e sin θ, finally you can get to the format $\dot{r}$equal to 3 sin θ times mu by $a(1-e^2)$ and you can see from this l is equal to $a(1-e^2)$so result is immediately visible.

Now therefore V becomes $\left(\dot{r}^2 + \left(r\dot{\theta}\right)^2\right)^{1/2}$

$$\dot{r} = \frac{esin\theta r^2\dot{\theta}}{l}$$

$$\dot{r} = esin\theta\sqrt{\frac{\mu}{a(1-e^2)}}$$

$$V = \left(\dot{r}^2 + \left(r\dot{\theta}\right)^2\right)^{1/2}$$

$$V = \sqrt{\frac{\mu(1+2ecos\theta+e^2)}{a(1-e^2)}}$$

So, inserting these values you can insert it and check it this value will turn out to be mu times 1 + 2e cos θ+ e^2 $a(1-e^2)$ under root $\dot{r}$ already we have got here. We have $\dot{r}$ equal to $esin\theta\sqrt{\frac{\mu}{a(1-e^2)}}$. So, we utilize these 2 expressions in the equation we have written here F_θ and F_r. And once you insert this and then F_r gets reduced to $\frac{1+ecso\theta}{\sqrt{1+e^2+2ecos\theta}}F_n$.

$$F_r = \frac{esin\theta}{\sqrt{1+e^2+2ecos\theta}}F_t - \frac{1+ecso\theta}{\sqrt{1+e^2+2ecos\theta}}F_n$$

We are replacing in terms of t n and not this F n this part and this part. Generally the F theta gets reduced to 1 + e cos theta, so this completes the derivation for this particular section. N ow what remains that if r is given so this $\frac{\partial R}{\partial a}$, if it is not in the terms of force, then we have to estimate it says in the three, we are one part main what is here and then the satellite is moving around this and then it perturbed by third body.

If it is perturbed by the third body so for that the expression already we have written like that expression we have written as

$$R = -\sum_{\substack{j=2 \\ j\neq i}}^{n}\left(\frac{1}{r_{ij}} - \frac{\vec{r}_i - \vec{r}_j}{{r_j}^3}\right)$$

.

This expression we have written now we need to express it in a particular way to formulate our problem but that is going to take much longer time. So therefore I am not going to cover this part in the class.

I will provide supplementary material for this. So, we will chapter here and then we start with a new topic in the next class will close this here itself.