Space Flight Mechanics
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Lecture No - 69
Orbit Determination

Welcome to lecture 69 we started with the reference frame which is required for carrying out the orbit determination. So in that context we were looking at various inertial reference frame which are available currently and out of that we have and those we have termed as the International Celestial Reference Frame then Fk5 and of course our other one the dynamical reference frame.

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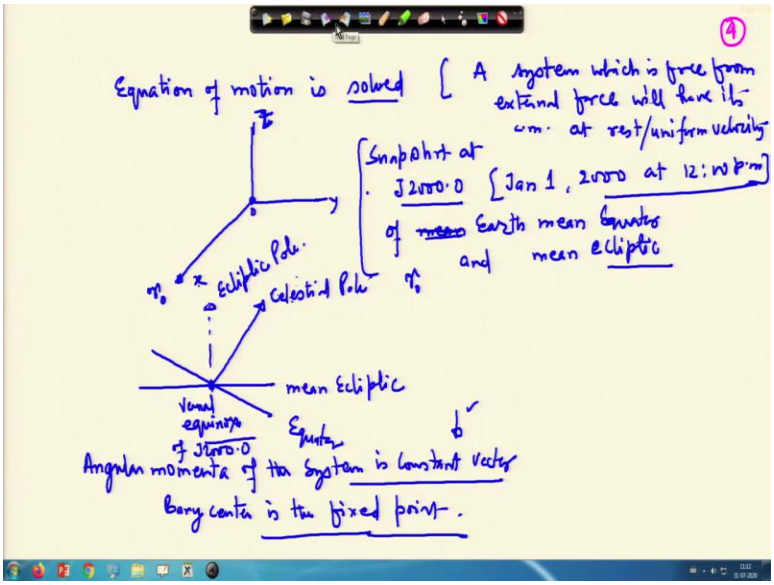

The dynamical reference frame same time as we look into the previous lecture.

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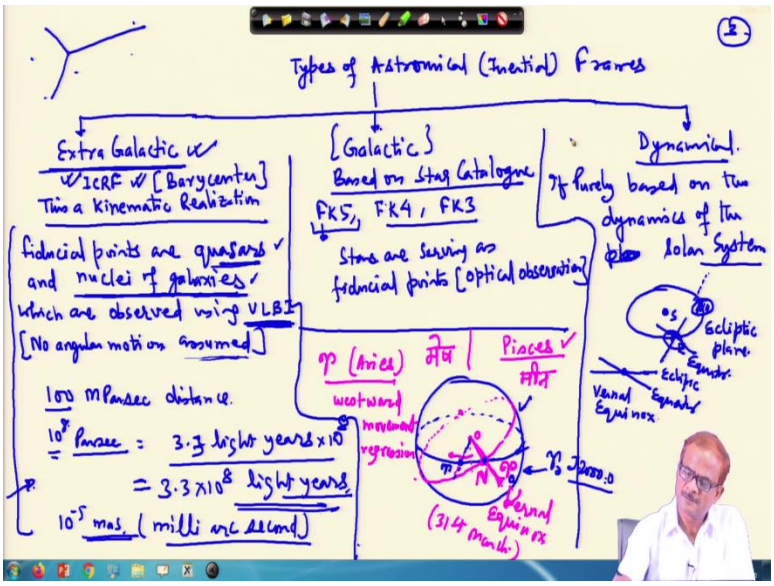

So here this you can the dynamic; there are if you look in the literature or in the books. In the books only the Fk5 and ICRF it will be mentioned. Dynamical reference frame is also there and this is purely based on the dynamics. So, we can call this as the earth mean equator of 2000. This serves as it one of the reference frame and Fk5 is another Fk5 also in this project 2000 is defined means at the January 2000 1st January 2000 at 12 pm.

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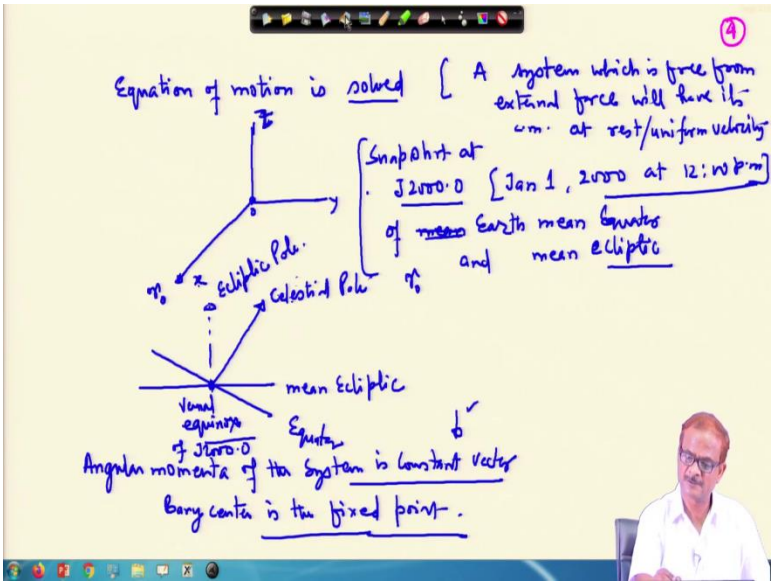

Now, I will show why there are different ok which the only some journal paper they mention about that in but in the book I have not seen this to be mentioned.

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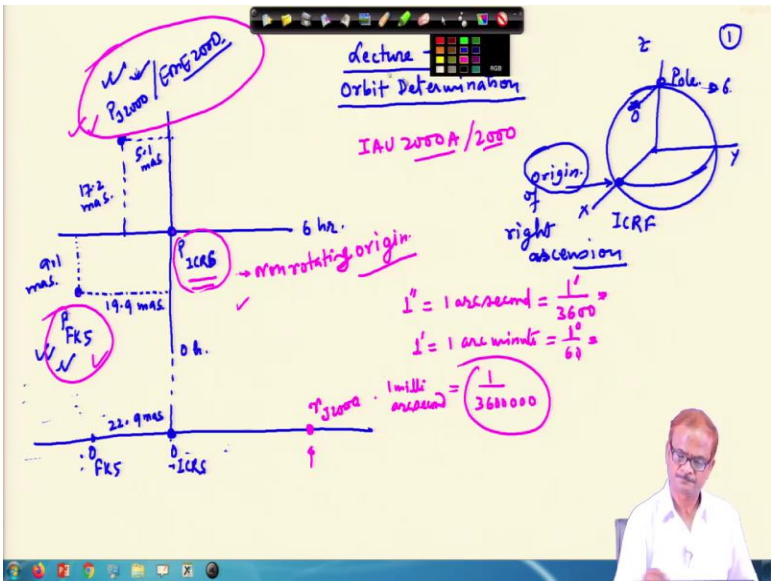

So, if I say that this is my International Celestial Reference Frame and this will be defined as the pole, pole of this X Y and Z. So if I write here the pole of ICRF or the pole of the ICRF whatever you want to write because in that we are trying to realize it then the pole of the Fk5 is just shifted like this pole of Fk5. This distance is 19.9 milli arc second. Angular distance 9.1 milli arcsecond. This is 6 hours this is 0 hour. Looking from the top in this direction parallel to this we can sum this as the 0 hour.

So, in this direction this will be 6 hour and so on. So total 24 hours that will make. And the J 2000 or the EME 2000 what we call is pole of J 2000 it is located here. This distance is 17.2 milli arcsecond and this one is 5.1 milli arcsecond. So you can see that the pole of Fk5 and pole of J 2000 and the pole of ICRF they are not the same that means they are not the same reference frame but the ICRF has been constructed using the quasars and the Galactic nuclei in such a way that it lies as close as possible to the P J 2000 and PFk5 the pole of Fk5 and pole of J 2000 as close as possible.

Because there should be continuity arbitrarily we cannot do. So, whatever the measurements where are you are done earlier this was defined so on that the Fk5 was defined. In this thousands of stars were added. Based on stars so from that then this is fixed and P J 2000 this is of course dynamical one and we can also write in terms of EME 2000 earth mean equator of 2000. Now if we look the same thing. So just below this, if we write the origin of the ICRS.

Origin of the ICRF International Celestial Reference system .The origin of ICRS that means this point on the sphere we will call this as the origin. This is called the origin of right extension and in short, we just call it origin of right extension and in short I will be using this as the origin. So here are the origin of the ICRF is located then at a distance of 22.9 Mili arcsecond. Your origin of Fk5 is located,

So that means that one is another reference frame. So the inclination; the pole is different and also the origin is different. They are not the same so you can see the changes and similarly we will have the EME or gamma J 2000 which belongs to P J 2000 we will not write P because P stands for the pole. So this is only written as the J 2000. So this is your reference to the vernal Equinox. So all these frames are quite close to each other, this 99.9 milli arc second.

So 1 milli; 1 arc second this is equal to 1 by 3600. 1 arc second is nothing but I am let us write this. 1 arc minute is one degree divided by 60. So 1 arc second is written like this. This is this much of degree and 1 arc minute if you; 1 arc minute and here will replace this with. And 1 arc second its symbol is given like this. 1 arc minutes is given symbol like this. So one degree divided by 60 that gives you 1 minute.

Just like in the needle of watch it is moving in so this is 60 degree is divided into 60 degrees so 1 minute you get 60 seconds the same way it is defined. So, 1 arc second 1 minute divided by 3600 second. So, 1 milli arc second will be 1 divided by 3600 and milli means? So this is 1 milli arc second. This is very small quantity. you can see for our practical purpose. You can assume them to be coinciding.

$$1' = \ arc\ minute = \frac{1^o}{60}$$

$$1'' = \ \ arc\ second = \frac{1^o}{3600}$$

$$1\ milli\ arc\ second = \frac{1^o}{3600000}$$

But ICRS is a different convention and for converting the coordinates from the inertial to non inertial frame. They have their own convention. So, ICRS convention it is different little and also based on the non-rotating origin the latest one, non-rotating if I get time I will go into this.

Otherwise, I will supply you with the materials. So here the precision and nutrition involved the latest theory is being written as a IAU 2000A and 2000.

So these are the 2 models available where the precision nutation had been combined together. And what we will do here; in this Framework or either in this frame we will assume them to be the same. So in that we will assume precision and nutation to be 2 separate phenomenon, and we will model them and finally we will transfer from the celestial reference frame to the Terrestrial reference frame. And what the terrestrial reference frame is will that also we will come to know across time.

Terrestrial frame is a frame fixed to the earth, but there are certain definitions for that and it is a rotating along with the earth. Ok, but the inertial frame that are non-rotating non accelerating. Inertial implies that it is a non-rotating non accelerating. So, this way we see that this frames are different but for practical purposes, they appear to be the same. But for precise modelling, very, very precise modelling you require to differentiate between all these three.

And in your conversion you should be consistent you cannot do that you are taken something in that P J 2000 then in the and gone to Terrestrial on from Terrestrial again converting so there you are taken Fk5 for ICRFS so that way things will be wrong then.

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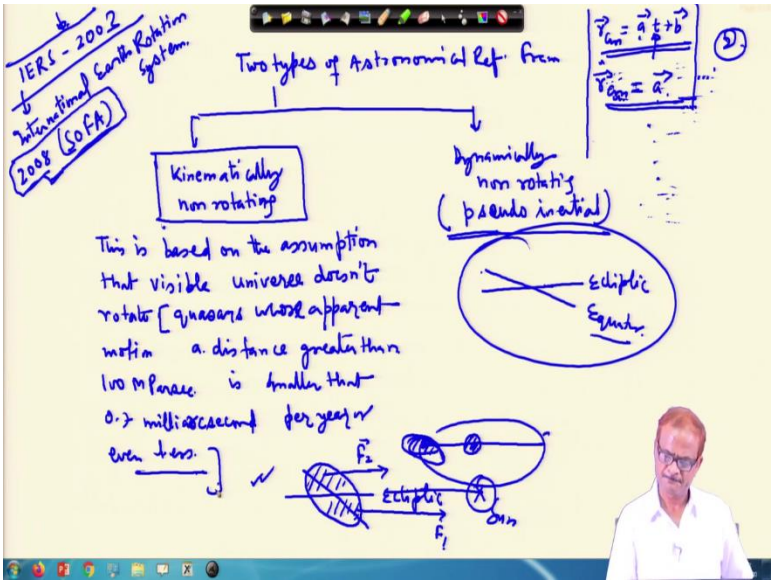

So in the reference frame already I have listed that one is dynamically defined another one non dynamically defined. So we have 2 types this is what we have discussed earlier in the last lecture. Here the types of astronomical reference frame how they are fixed. And now based on the type we can write 2 type of astronomical reference frame one is defined kinematically which already I have told and another one is define dynamically.

So, dynamically non-rotating this is pseudo inertial because you are taken a snapshot of that and then working with this. And kinematically non rotating, this comes from the fact that you have chosen the fudicial points such that they are not showing on the in the celestial background. They are not showing any motion. So, this is based on; this is based on the assumption visible universe does not rotate and visible universe what we are doing here.

We are using the quasars whose apparent motion at a distance greater than 100 Mega Parsec as earlier I have told you just smaller than 0.7 milli arc second per year or even less. While in the dynamically one as I have told that this is based on the motion of the solar system and you have to write the equation of motion of the solar system and in that we find the fudicial point where the centre of mass of the system this does not move the; this we have written as the centre of mass. So, r times r centre of mass; m times r centre of mass.

If you remember let me write it, if you recall earlier while integrating the equation of motion for the multi body system or whatever. We have got that V centre of mass. This can be written as

$$\vec{V}_{cm} = \frac{\sum V_i m_i}{M}$$

actually there we have used another notation we have written that

$$\vec{r}_{cm} = \vec{a}\, t + \vec{b}$$

this notation we have used where the centre of mass either moves with constant velocity if ‘a’ is non-zero we differentiate it. So we get this as ‘a’.

So, if a is non zero so it moves with a constant velocity if is zero so this is at rest centre of mass of the system remains at rest or either moves at uniform velocity therefore that serves as the centre

of the triad and the direction we are getting by the ecliptic and equator it is the intersection of that. So this way we have fixed this triad and besides this the perturbation from the planets it makes the ecliptic change it ecliptic is nothing but the orbit of the earth around the sun.

That gets perturbed because of the presence of other planets. Moreover equator of the earth also that get perturbed will discuss in on the next page. So these are the some of the important points we should keep in mind while working with all these things and one book I will refer you to or book or it is IVRS report of 2003 this is called the International Earth Rotation System this was published in 2003.

So at that time I have worked in this area. So I am aware of this 2003 report if you pick up so you will see and later on also reports have been published and one report came in 2007 or 8. Another report it comes by the name the SOFA this gives you all the just the programs. So it is programs for converting from one time to other time taken into account the crust motion and various other things are there.

So itself those programs can be directly taken and you can integrate in your main program with suitable modification. Ok so now we are going to because we are discussing about our frame. And we are already aware of it in the case of say the sun is there; earlier also I have mentioned and here the earth is moving around this and Earth is bulged like this. So, because of this and this is also inclined on the axis lets say inclined like this.

If this is ecliptic as mentioned earlier and the equator is here in this direction and its bulged the earth is bulged. And say the sun is here in this direction somewhere here the Sun is lying. So because of this the more force will act on this one and lesser force will act on this lobe. Separate out these two lobes. So more force will act on this say this is F 1 and this is F 2. So F 1 will be greater than F 2 because of this and this you will find a good discussion of this my lectures on the satellite attitude dynamics.

So because of this it causes the earth to precess about the; precess in a circle that I am going to show in on another page and that motion we called as the precession motion and besides this the

nutation is also involved. So there are the precision and the nutation motion involved which we need to discuss to understand this.

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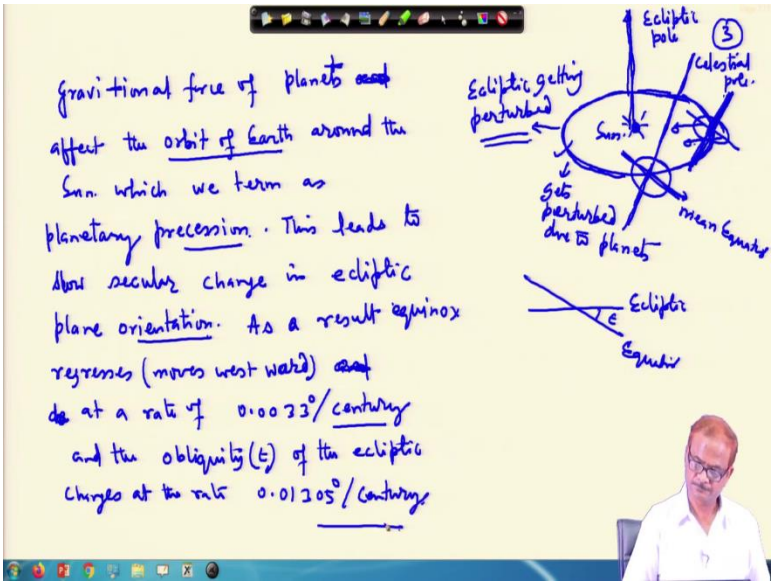

And will discuss it briefly not in very at a very large length. Sun is located here. Earth is here, it is inclined and this is the mean equator shown here and this is the celestial pole and this is the ecliptic pole means perpendicular to the, this ecliptic is the orbit of the earth around the sun. So this orbit of the sun this also orbit of the Earth this also gets perturbed due to planets. Sun -earth if this constitute 2 body system and the other bodies which are present Jupiter Saturn Mars so and in mid between you have two planets.

So, this you can take as the perturbing body so therefore the orbit of the earth also gets perturbed that means ecliptic plane also get perturbed besides because of the attraction of the sun ok if your well look at the earth here in this point where I was mentioning on this part, there will be more force and on this part there will be less force. Ok. So this houses the celestial pole to persists which we are; I am going to write little bit so that it is easy for you to refer to and later on I will supply the materials.

So, gravitational force of planets affect the orbit of Earth around the Sun which we term as planetary precession so that means the orbit of the Earth is getting perturbed means the ecliptic

getting perturbed. This leads to slow secular already in the general perturbation theory we have discussed all these issues how the; what does mean by secular itself which varies with time and it is not periodic. This leads to slow secular ecliptic plane orientation.

As a result Equinox regresses means moves westward at a rate of 0.0033 degree per century is a very small at a rate of 0.0033˚/ century and the obliquity which we write as epsilon. So, this is ecliptic and this is your epsilon and this is equator of the 05 per century.

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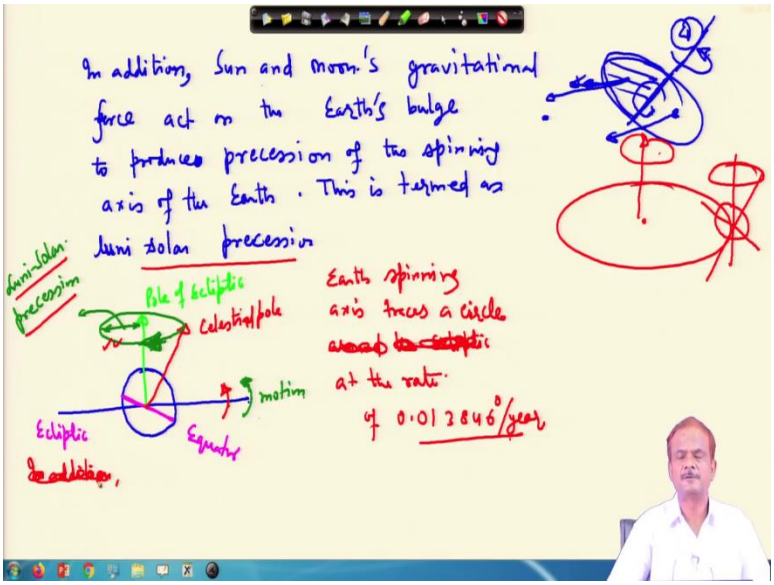

So, this is one motion involved so because of this you will see that this angle is changing and also on the equator this circular right now, this is gamma. So later on the gamma will come to this place. Later on this will come to this place. So this will move westward this is the west movement. In addition Sun and Moon is a gravitational force act on the earth bulge so it is acting on; these are the two things one we divided into two halves of the earth.

And this is one part force is acting here and if this is near to the sun you will have more force on this and on this and there will be lesser force. So as a result force is here more and here less in this direction, we will have a torque and because the earth is rotating on its axis first thing is spinning. The spinning body if torque is applied on this and it precesses, and it is very easy to mathematics is not very complicated if we will look into this satellite latitude and dynamic control force.

So there we will find all the details. So in addition sun and moon's gravitational force act on the earth's bulge to produce precession of; and this is termed as Luni solar precisions so that means if this is the ecliptic and the earth is so like this, this is the equator. In this direction we have pole of ecliptic and along this direction the celestial pole. So, this celestial pole this will go in a circle around this and this motion is called the precession motion.

It goes here in this direction it regresses and it is westward. It goes also here this way as shown by arrow. Luni solar precision. This is the major part besides this your ecliptic is also getting affected this also rotating this causing; some time back we have looked into this that in the general audit perturbation theory how the orbit of the satellite it gets subjected. So; similarly the earth is the satellite of sun its orbit also gets affected and that we call as the perturbation of the luni solar perturbation which I have shown you.

So this; so earth spinning axis traces a circle around the ecliptic pole see this is not the ecliptic pole but parallel to the elliptic pole. This is the ecliptic and its centre will be here and Earth is here. This is inclined like this, this is the pole. So, it will be precessing like this about this not about the ecliptic pole. Wherever I have written a ecliptic pole do not take it literally to the zero ecliptic pole.

Ok this is the rotation is taking place like this. I do not see written anywhere right now. But precession of spinning axis of the earth; this is termed as the luni solar precision. So earth axes traces a circle around the; it is better not to write it; traces a circle and at the rate 0.013846 degree per year. So when we have the luni solar precision and another perturbation is coming from the other planets which causes not only the ecliptic to move but also it will affect.

Mainly the orbit is getting affected because of the other planets. And sun and moon they are causing this to get affected. In addition moon also produces torque. So, in addition we will come to final conclusion so maybe it will continue in the next lecture. Thank you very much.