Space Flight Mechanics

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Lecture No - 70
Orbit Determination (Contd.)

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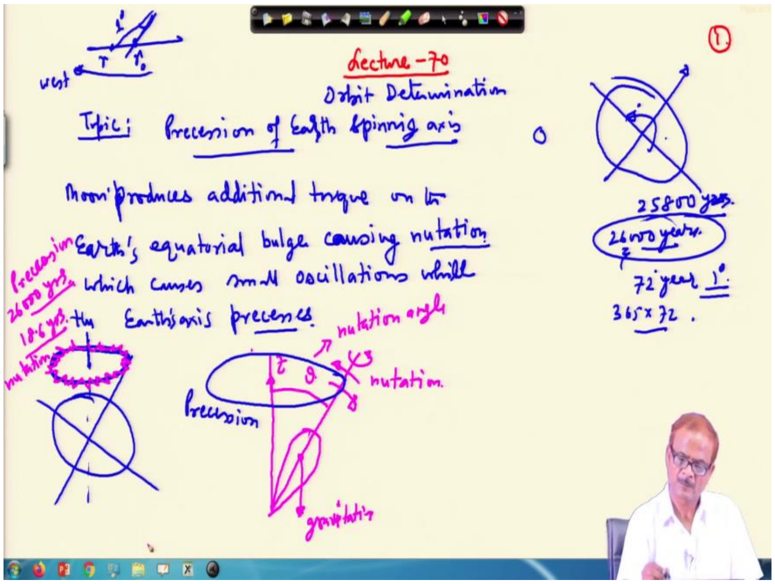

Welcome to lecture 70 we are discussing about the unique solar precision and in addition the ecliptic also that gets; it can also get perturbed because of the other planets and besides moon also affects the motion of the earth. So this was the last point we have discussed in the last lecture. The moon produces on the earth’s equatorial bulge causing nutation. There is a problem in the book by Mechanics Engineering by Bear Johnston.

Where it is a given the earth inertia is given along different axis and then the sun is given here. The mass of the earth is given in it is required to find out the torque acting and thereby finding the period of precession of the earth. So that comes out if I remember it turned out to be around 25800 years and total play station here for the earth's axis because the moon also it is contributing that turns out to be around 26000 year.

So that means every 72 years 72 years, 1 degree regression of the nodal line takes place that means on the ecliptic. If this is the gamma 0 say; so next gamma in after 72 years this will change here in the backward direction this is moving west by 1 degree in 72 years. So if you multiply this by 365 into 72 it may turn around the same value around 26000. So, moon produces additional torque on the earth equatorial bulge causing nutation while the earth's spinning axis precesses that means the earth spinning axis is precessing.

This is the Luni solar precession already we have done this. Here on this there we will be riding the nutation. Actually, this is of; strictly speaking how do we define the precessing motion. If this is the axis of say z direction and you have a top here, this is a top at in this place. So this angle theta we call this as the nutation angle. That means any motion here in this direction arises in this direction is called the nutation motion nutation.

While motions of this axis like this; this is called precession. So over the precession and the nutation is riding. Ok and therefore and this period is around 18.6 years nutation period and while the precession period is 26000 years around. This is the precession period and this is the nutation period. So, your top is spinning on its axis and where the torque is acting; torque is acting due to the gravitation and that makes it precess.

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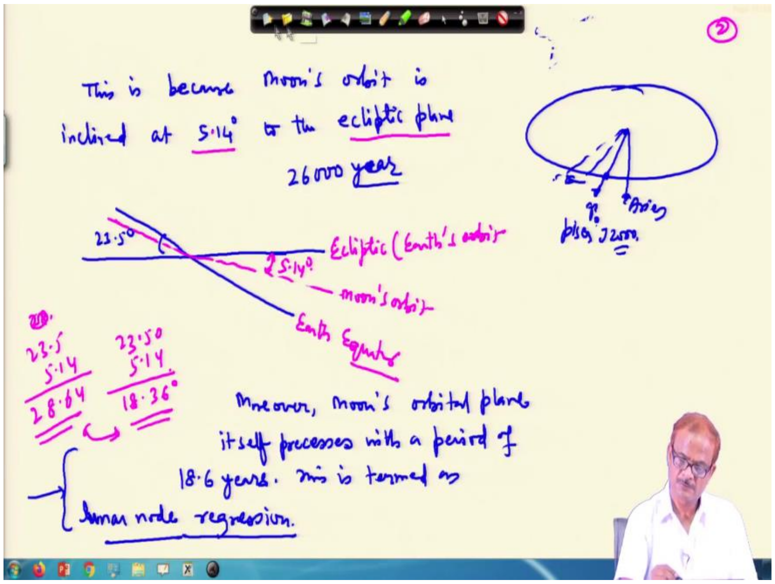

So earth's axis causes the nutation this is because moon's orbit is inclined at 5.14 degree to the ecliptic plane. If this is the ecliptic plane, this is your equator. This angle is 23.5 degree approximately. Hence moon's orbit you can show it like this. This is 5.14 degree. The Moon's orbit.

This is ecliptic which is Earth's orbit and this earth equator. So Inclination between the moon orbit and moon's orbit and Earth equator that varies 4.28 just add this 23.5 + 5.14. If we subtract from this 5.14, we get here this as 18.36 degree.

So moon's orbit inclination to earth equator it varies between these two values. This is because moon's orbit is inclined at 5.14 degree to the ecliptic plane. Moreover moon's orbital plane itself precesses within a period of 18.6 years and this is termed as lunar node regression. So, the precision motion and arising because of this sun and moon and the ecliptic perturbation if we had all of them, so the precision period turns out to be around 26000 years.

That means the vernal Equinox already I have told you this gamma 0 this refers to the vernal Equinox of the J 2000 and right now it is a pointing toward the constellation of Pisces. But earlier because it is regressing right now it is here that means earlier it was in the Aries and here right now, it is in the Pisces after sometime it will be in different. So the polar axis of the earth as it is precessing right now is pointing towards Polaris.

But in another for 5000 years this will be no longer pointing towards Polaris what we call is the Dhruv Tara. So the polestar will be no longer polestar for us after 5000 years and the reason is for that the Earth axis is precessing. So it precesses how it goes here in this direction. ok direction we have to show correctly.

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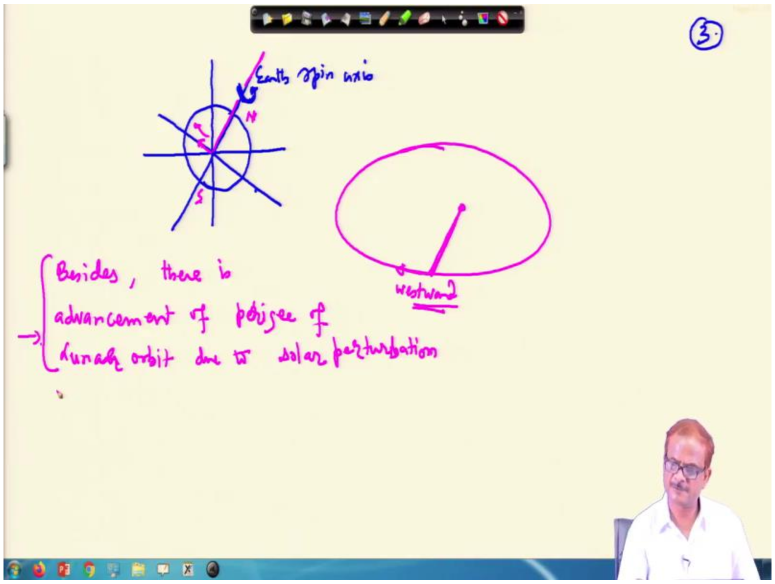

This is spinning like this ok anticlockwise let us see from the North Pole so it is spinning anticlockwise. And the nodal line, this is the sun and this is earth here. This is the nodal line. The nodal line it is regressing here in this direction westward. That means on the equator of the earth I will see this to move here in this direction. Besides there is advancement of perigee of lunar orbit due to solar perturbation very complex ok all of them are connected together.

So in the nutation model you will find there are hundreds of terms involved in that expansion. Already we have seen that we are expanding in terms of the Legendre polynomial or either the we have done in terms of Legendre polynomial and thereafter we have worked. So, due to solar perturbation; all this covers together the precision and nutation motions. Precision and nutation it is combined effect of the perturbation from the sun, from the planets and from the moon. Moon itself gets perturbed.

So let us draw one figure that will make it clear that what the precision is? And what the nutation is?

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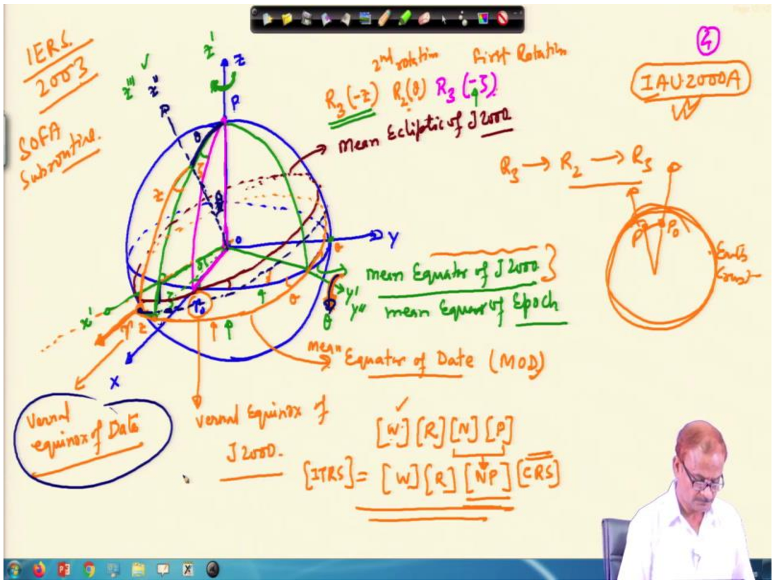

This will show as gamma 0 this as y and z. This is the celestial pole. This pointer is show as O. So the precession motion of the pole is modelled as first rotation is given about the R 3 by – z axis, $R_3(-z)$. R 3 means about the z axis by –z axis. Means we go here from this direction to this direction. This is 90 degrees this also remains 90 degree. This angle is zeta . So this is quite customary to show it here. This movement here has zeta. And this we write as mean equator of J 2000 or also we can term as the mean equator of Epoch.

$$R_3(-z)$$

Now once we have moved it. So this point comes here. And the gamma point we have chosen is the gamma 0 we have chosen your ecliptic plane it appears like this means the sun movement is in; if you look around the Sun if you project it on the earth surface, it will appear to move like this. So this is mean ecliptic of simply J 2000. This angle is your the inclination angle the obliquity that we speak about.

So our resulting axis now has changed. it has changed from this point to this point. This angle is zeta angle which we have shown here also in green. There after the next rotation we will give by theta. And theta rotation we give by, this is the first rotation. This is the second rotation. That means we are giving about the y axis. Ok next rotation we are giving about the y axis. So I will make it the whole thing green.

$$R_2(\theta)\ R_3(-z)$$

So, giving rotation about the y 2 we have to draw the line y also so once we have rotated the y-axis; this is the y-axis is here if we rotate about this we have given rotation like this is a negative rotation is not the right hand rule. So, it is the negative z direction it is directed. So, this is by zeta we can put a minus sign here. It is not required. zeta is enough because it is in the negative direction. Its anticlockwise direction.

Sorry it is the clockwise direction. If you look from the top, so it is a clockwise direction and therefore this is all put has a negative sign here. Then the y-direction also this will appear as you are in this place and the corresponding y plane. This plane we can show it here in this plane. So this is y, this is y prime, z remains in the same place z prime and x goes x comes here in this place. This is x plane.

Next rotation we give about this by theta, So if we give rotation by theta so what will happen? Giving a rotation like this it causes your; this plane which shown in blue to tilt y by theta. So that tilting can be shown as; I will show it by some other colour now? This plane you are rotating about this. So the whole plane it will rotate like this. So it is rotating like this. So this angle is theta this is positive rotation because it is you can see that this is looking from the y side this is anticlockwise it is coming over the green line. This is a positive rotation. And of course, we need to make it the circle complete like this.

$$R_3(-z)\,R_2(\theta)\,R_3(-z)$$

So this rotation has been given in the third rotation again we give about the z direction resulting z direction by angle z. Resulting z direction is right now. See here once we have rotated. So your; the whole thing will rotate and z direction it will rotate from this place to this place and it will come here. From here to here this is your angle z. Sorry, this is we have rotated by theta first. So this is rotated by theta about this from here to here. So this z-axis rotates to this place.

So this is your z double prime zz prime. And similarly the x-axis it will go in the same plane in which the green line is shown and it will go and get extended and cut here in this point. This will come and cut the mean equator here in this point. So that means that what I am going from this place to this place by dark blue colour, this is also theta angle. And this is not coinciding with the x axis line I will make it little more distinct.

This line is different and the green line if I extend here, so this comes and goes here, this is not cutting here in this plane. So it comes and cuts here in this point. So angle between them from here to this place then this angle is also theta angle. Of course, I am not trying to show all these things otherwise that figure gets very complicated. And thereafter on this equator on this line you move backward by z.

So that means you are rotating again about the z-axis means resulting z double prime. z triple prime will be here in this direction. We rotated from y to y prime and then we give another rotation. So, y double prime is here. And if we now rotate by z that means we go from this place to. So I use some other colour. I will choose orange colour it will come and this cuts here in this point. So the next rotation this is z. your axis then moves from this place to this place. This is z. So it comes here.

So now we can turn all the things. This blue line already we have indicated. This is the mean equator of J 2000 and this is called the Mean Equator of Date (MOD). Once we give procession correction then we get mean equator of date. So this was the equator of Epoch and now this earth equator of J 2000. This is the mean equator of date which we get by giving these three rotations in sequence. Means first we are giving the R 3 then we are giving R 2 and then again, we are giving the R 3 rotation and these are all called the orthogonal rotation. We need to go and because all the axis about which we are moving they are mutually perpendicular to each other.

$$R_3 \rightarrow R_2 \rightarrow R_3$$

And therefore they are orthogonal rotation and also orthogonal rotation has certain property. So you can look for all those things in the Satellite Attitude Dynamics and Control course. Ok, so what we observed from this finally; again, I am not going to match this line with the point O because matching this it will create problem. So gamma 0 is here and then we write this point as gamma this is vernal. This is called vernal Equinox of date.

And gamma zero this is called as vernal Equinox of J 2000. So this is what the; your precision correction to the earth Motion. Going from the inertial reference frame to the terrestrial reference time it involves certain motion out of that; first is we have to correct for the precession and then for the nutation. So as I earlier referred in this International astronomical unit this 2000A here all this motions, precision and nutation has been combined together.

In fact if I write here mathematically so say if I write here the CRS this is the coordinates in the celestial reference system. So from there if I want to convert to the ITRS International celestial reference system, so I have to go through certain motions. In the modern one this IAU the nutation and precision is combined together, but in the older one, the first the precision correction is given and then the nutation correction is given.

And then the rotation of the Earth is taken into account which we call as the spin and then the polar motion of the Earth. The polar motion of the earth is nothing but the shift in the rotation axis of the earth with respect to the crust of the earth. That means if this is the crust of the earth and this is the rotation axis right now. Ok. This is the point P 0 later on this axis will not remain the same after sometime this may come to P 0.

$$[w][R]\underbrace{[N][P]}_{[NP]}$$

$$[ICRS] = [w]\,[R]\,[NP]\,[CRS]$$

With respect to this is with respect to the crust. This is not with respect to the inertial frame so with respect to the earths crust. This is the earth crust shown here so this will come to this point and this point is called polar motion. So this correction is then given and also this correction is indicated by w. While here this new system which is referring IAU 2002. These 2 are combined together and represented like this and then r and finally this correction is given.

And there are various ways of doing this of course they are correlated with you can convert from one form to another form, but I will suggest for this that you read the IRS convention manual of 2003 and also you can look into the SOFA subroutine. Ok finally we have to wind it up. If I am saying that this is a vernal Equinox of the date that means ecliptic must pass through this point. So, ecliptic must pass through this point. So we stop for this particular lecture here and will continue in the next lecture.