Space Flight Mechanics
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No - 71
Orbit Determination (Contd.)

Ok Welcome to lecture 71 and if you remember in the last lecture we have worked with the Precession motion. Now, we looked into the nutation motion So already. I have told that this is the luni solar precession over that the nutation is riding this perturbation motion is modelled in a similar way and we can represent this nutation rotation. But nutation we have written as N so I will just I will indicate it by N.

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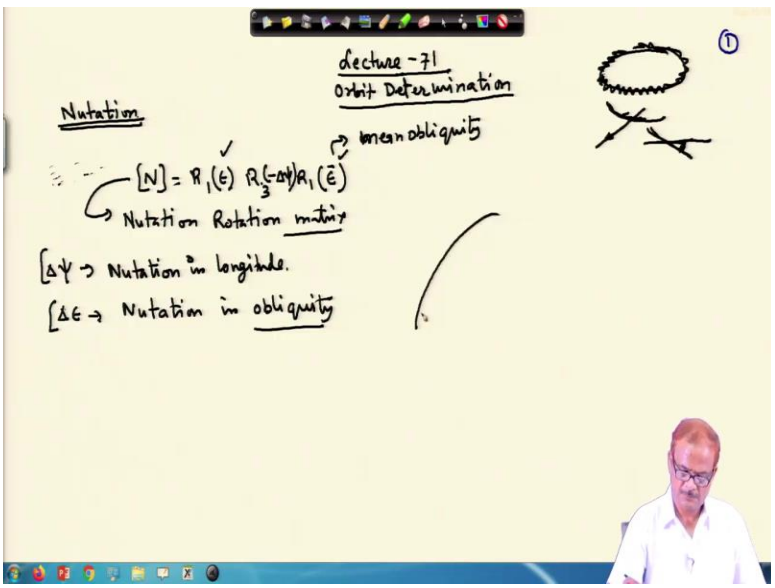

This is the rotation matrix. This motion can be written in the form first we give the rotation about the x axis by $\bar{\epsilon}$. This is called the mean obliquity and then we give the next rotation about the Z Axis by $(-\Delta\psi)$ and the last rotation we give about again about the first axis of the x-axis by ϵ. This is your nutation matrix, nutation rotation matrix. Here $\Delta\psi$ it is called nutation in longitude that means longitude for your vernal Equinox it is a moving on the equator because of this. And the $\Delta\epsilon$ here this is not ϵ.

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$$[N] = R_1(\epsilon)\, R_3(-\Delta\psi)\, R_1(\bar{\epsilon})$$

This is ok for $\Delta\epsilon$. I will later and this ϵ is ok here. And this is $\bar{\epsilon}$ both ok and what does then all they mean it will be clear later on. Nutation obliquity that means inclination with ecliptic of the equator this changes this is modelled as ΔE. Ok so to model the nutation we will go on to the next page.

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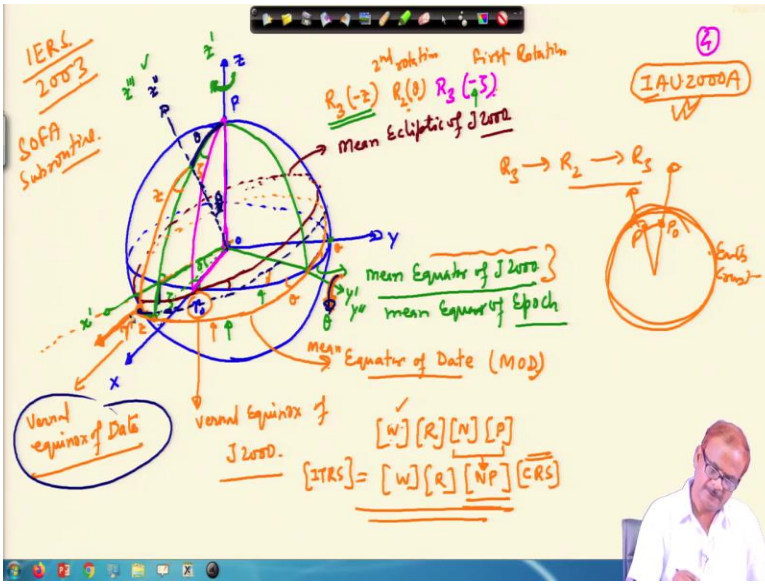

If we you go back we see that our axis has come to this point. This is your x-axis finally. X axis was first here and then we finally moved so here this will appear as x''', z axis has come here in this place z''' is here and similarly y-axis will move to this place somewhere on this equator it will move here in this place is the y'''.

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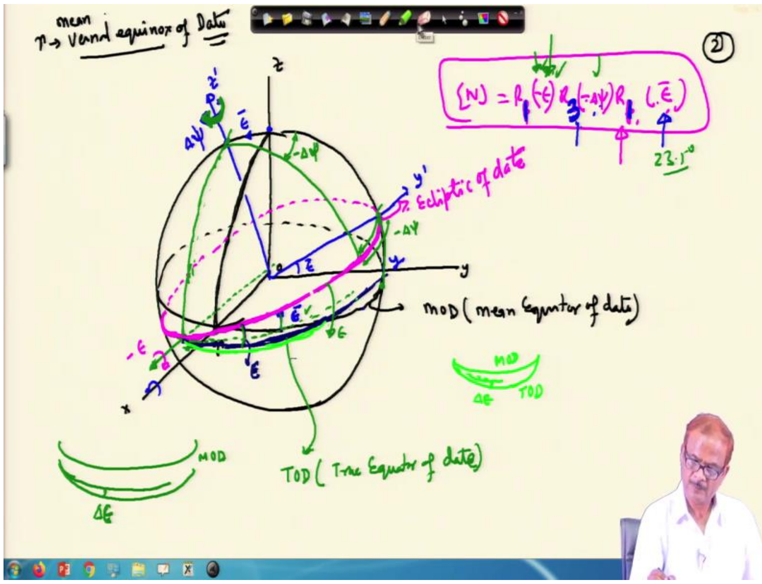

So these black axes we pick up and put here in this place and again, but still we will write this as xyz. We will not write as x''', z'''. You can write but it is not very convenient carrying so many of indices and this remained so. So as per these already I have written here. This is the mean equator of date. So here in this case this constitutes the MOD mean equator of date. So the precession rotation we have already given now we have to go to the nutation rotation.

As I told we draw the planes and here your γ is here. This γ we have written as vernal Equinox. This is the vernal Equinox of 2000. This is the vernal Equinox of mean vernal Equinox. This is basically because this is the mean equator of date so with this the mean term will come. So this is your mean vernal Equinox and once we give the nutation correction then means some will drop out and then we get only vernal Equinox of date.

So here you have the ecliptic as shown earlier. So the nutation rotation as we have written earlier. This is modelled by R_3, and we have written as $R_3(-\bar{\epsilon})$ $R_2$with $(-\Delta\psi)$ was there and then R_3 again we have taken with $(-\epsilon)$. So that means the first rotation we have to give about the Z axis.

$$[N] = R_3(-\epsilon)\, R_2(-\Delta\psi)\, R_3(\bar{\epsilon})$$

Sorry this is R 1 and this is also R 1 about the first axis we have to give rotation by$(-\bar{\epsilon})$.

$$[N] = R_1(-\epsilon)\, R_3(-\Delta\psi)\, R_1(\bar{\epsilon})$$

First rotation we give your from this point to this point and this is your $\bar{\epsilon}$ that means this axis will move from this point to this point as a result of this, this point will move from here to this place. This is your $\bar{\epsilon}$. So this becomes z'. So the motion is along Z direction. This is a positive rotation about the x axis. The next rotation is given by - $\Delta\psi$ and that is about the 2 axis ok.

So if we rotate about like this so the y axis this will go and get here in this place. So this is your y' this is your y and this is y direction. This is 3 here we have written it correctly here this is R 1, R 3, R 1. This is about the third axis this about the first axis. So the first rotation takes this to this place. This is your $\bar{\epsilon}$. The next rotation we give about the z-axis about by - $\Delta\psi$ so that means what has been shown here so you have to rotate by – $\Delta\psi$ means by $\Delta\psi$ you are giving rotation in this direction.

Negative, so this is negative direction you can see it is going over the blue line. So, as a result of this the plane from here to here this point to this point this will move and come here on this axis. So this angle is then your $\Delta\psi$ with - sign here also we can indicate this is - $\Delta\psi$ because this is in the negative direction. As a result of this, this γ point also it will move along this by $\Delta\psi$.

So it goes here somewhere say and from this point to this point we can connect it. This is your $\Delta\psi$ from this place to this place this is $\Delta\psi$ and the third rotation then given about R 1 by - ϵ this is your axis now and about this axis you have to give another rotation by ϵ. So; another rotation by ϵ about this point that means this the pink line what we have shown here this pink line this particular line. Now we have to move it. Ok by ϵ angle.

So here you have to give rotation by -ϵ because this is a negative rotation. Once we give this, this line then it will come to this place. From there it will appear like this dark blue line. So, this total angle from this point to this point this ϵ. And this we call as mean equator of date and then it went here and then it came down to this place, this called true equator of date. If you see here; if this is very distorted figure actually is highly distorted figure because these angles are very small.

Ok but here ϵ is ok around 23.5 degrees let us say this is ok. It is not much distorted but other angles are very small and once we isolate like this gets distorted. So here somewhere above this line what I will show that if this is the MOD mean equator of date. And true equator of date lies somewhere it is like this and if we draw a parallel to the mean equator here this angle is approximately $\Delta\epsilon$.

So, we have gone from this place to the ecliptic from the mean equator of date we have gone to the ecliptic and from ecliptic, we have taken a turn by coming from this place to this place by $\Delta\psi$ and thereafter once we are given $\Delta\psi$ and again, we are dropping the ecliptic back by ϵ angle. And this is going down by ϵ angle. This is shown here by this dark line. Actually, this line is very much distorted and therefore it appears to go like this.

Otherwise in actual figure if you look into some of the references so, actual figure should look this way. This is the mean equator of date MOD and this is the true equator of date. There is a small angle between this which is $\Delta\psi$.

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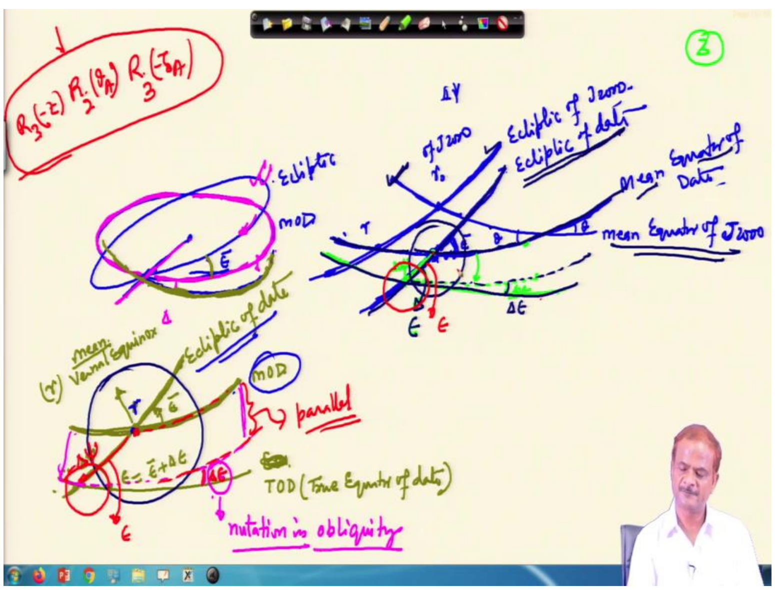

This is MOD and then this part is going to the ecliptic and then from here this is after the giving the correction $\Delta\psi$ and suppose this is the part and this is the centre. So about this centre then you are rotating by this angle is $\bar{\epsilon}$ and you are rotating here by ϵ and so this will get back and almost it will come like this. It will be going like this and if the thickness is very large. First let us show this in the pink.

So this is your MOD and then this is the ecliptic you are taken to by giving ϵ rotation. And finally $\Delta\psi$ was given which we have taken along the ecliptic like this. Ok. So we are assuming that equator has came to this place. This x axis has come to this place and thereafter about this axis we are giving the rotation. So that rotation again, it will bring back this from the ecliptic to below the equator by small angle and that angle we referred to as;

So, giving this rotation so in the figure what we can draw using the graphics tools and here it is not very clear. So this will go like this some small rotation. So, this angle was ϵ and then you are taking it down and I will show it on some other figure where this will become clear. From here itself I am not able to make it using this thick. This thick line so I will show it through some other figures it will be much more better.

See exactly what we have done that this is the mean equator of date and this show as equator of date of the TOD, true equator of date. This line is showing ecliptic of date so mean equator of date what we have done we have gone by $\bar{\epsilon}$ ok and there after we came down by this is the ecliptic. Ecliptic of date so along this we have came down by $\Delta\psi$. This was your vernal Equinox γ we have shown, we are using mean vernal Equinox with the people are not using this term.

But for convenience let us write this is the point γ . Then we come to this place and thereafter what we do. About this then we are giving deflection by ϵ. So this angle is your ϵ that means this ϵ is $\epsilon = \bar{\epsilon} + \Delta\epsilon$. You can see that if this is the line here is the mean equator of date and parallel to this if I draw parallel to this if I draw this line like this ok this to work parallel ok then this angle is your $\Delta\epsilon$.

So this should be clear from this place which I am not able to make here because the angles we are taking large here in this place. So it is not very much visible here also see once we are gone hear from here we are going down by ϵ we went up from here to hereby $\bar{\epsilon}$ and we are coming down by ϵ. So the net location is; as shown here this is $\Delta\epsilon$ so this is called nutation in obliquity.

Ok on this figure we can also make the actual precession also can be shown on this line. Say this is the mean equator of J 2000 and like this and this is an ecliptic of date. So here this is ecliptic of

J 2000. This constitutes your γ_0 the vernal equinox of J 2000. And from there, then we are given the precession correction and by giving the precession correction, we got to the point where we got the MOD.

So MOD we have shown here in this place like this. So this is your mean equator of date, this is in mean equator of date and this angle we shown by θ we can see the conversion. So the mean equator of date is here and this is the line we are taking up. So again go back so the mean equator of date which is shown here in orange. So this one is picked up and the corresponding γ is there. So we have picked up and this is the corresponding γ .

And then the next part we have moved it from to the ecliptic of z. So we are taking of the ecliptic of date and ok so this part first we had the mean equator and the ecliptic of J 2000 from there we got this line the mean equator of date and also we have shown earlier that this part this line which is the ecliptic of date. This is the point here which is referring to γ . So this is the point here which is γ .

And with this point, then we start with and from here then what we have done. Then we are going to the equator of date. First what we did that we have given here the rotation R 1 by $\bar{\epsilon}$ ok which is about the x-axis. So about this axis then we are giving rotation by $\bar{\epsilon}$. So giving this rotation by $\bar{\epsilon}$ and then moving by $\Delta\psi$ MOD and ecliptic of date and then this is your; this part we pick up for this is given by$\bar{\epsilon}$.

And then move by $\Delta\psi$ along this line. So we move from here to here by $\Delta\psi$ along this line. And so this is your $\Delta\psi$ and both are combined together now the nutation and precession. And then again convert it once we have given this. Ok from here if we show little bit $\Delta\epsilon$ upward, this is $\Delta\epsilon$. So $\bar{\epsilon}$ and this line and this line they are parallel. Rotating about first giving bringing this point to this point and then rotating by ϵ this angle is your of ϵ and this angle is $\Delta\epsilon$.

This angle is ϵ and this angle is $\Delta\epsilon$ and I will make it little darker. All these three curves are all the two nutation and precession are combined together. So we have started with J 2000 ecliptic of J 2000 and equator of J 2000 and it is written here in this place. Then we came to the ecliptic of date and mean equator of date which is shown by this line. And this is a ecliptic of date which; so we come to this point.

This point and this point they are the same. And this point and this point they are the same. So, taking this point here in this place ok and then giving Δψ here this rotation along this line and then again giving rotation by ϵ we get this final result. Ok so from here to here we have given rotation by ϵ. This rotation is given by ϵ and with respect to this line then this line makes it these 2 are parallel.

Ok and therefore this appears as a Δϵ and I hope that this makes it clear to you and I will supply you the graphically drawn figure as I can show if the camera can be focused on this.

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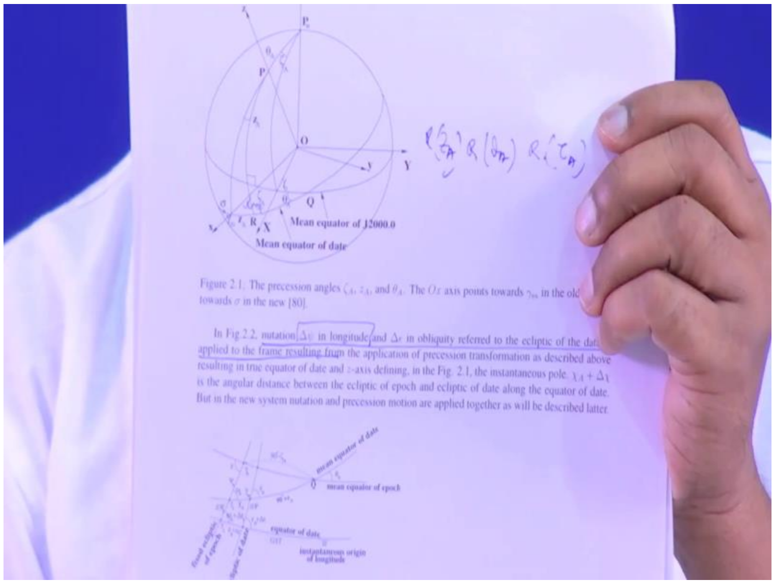

So, whatever the figure I have shown you the colour the same figure is shown here in this place. If you look at this place, this is the same figure. By giving the proper; in this is screen I am not able to draw a very neatly but here if you show graphically it is drawn very neatly and all the angles are also shown. So, upper figure that refers to the precession and lower figure here both the nutation and precessions are combined together which I have shown on the screen.

Ok and see we can wind it up on there if you saw it on the paper, so there it is just written as $R(\zeta_A)$. so do not go by that because we are given first the negative rotation. And that was about the third axis, and then we give the rotation about the 2 axis by (θ_A) this what we have done. This by θ about the two and then again, we are giving the third rotation by about the Z axis by - Z. Ok, so this is by - Z. So this is the actual thing on the paper whatever was written in the pen that is not corresponding to this.

This was just symbolic but here the correct sequence of the; sequence with signs are given here in this place. So this material I will provide you later on while the course runs, but this is the right figure.

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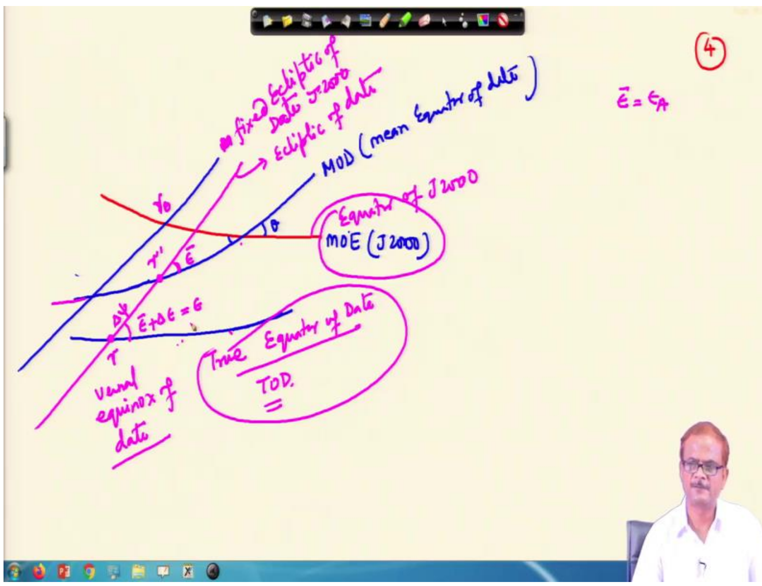

Let me draw a phrase so that it remains in record here so quickly I will draw the figure. So mean equator of date and this is mean equator of Epoch we should write in MOE J 2000 and this is mean equator of date. This angle is θ this angle will also be θ. And then we have drawn a line like this. And this is γ_0 this is the ecliptic fixed this called the fixed I have used the word mean, but this also this is a fixed ecliptic of date not mean it is a fixed ecliptic of date.

Here we separately have written ecliptic of J 2000. So this is point is γ_0. There after this is converted here and then again we have ecliptic of date. Once we have got the ecliptic of date so we here the point γ ; where we are getting. And the angle between this 2 this we are writing as $\bar{\epsilon}$. Here on the paper it is written as $\bar{\epsilon}$ is written as ϵ_A . And finally once the nutation correction is given so this angle becomes $\bar{\epsilon} + \Delta\epsilon = \epsilon$ and therefore this is the equator of date which also we write as the true equator of date (TOD).

And this is your final γ point. So we will write this is prime (γ'). So this is your final vernal Equinox of the date. The distance between these and to angular distance this we have shown by $\Delta\psi$ and there are various other representations; the angular representation we can write here but those are not necessary this suffices for our need that means coming from the mean equator of Epoch or equator of J 2000.This is the equator of J 2000. So the true equator of date(TOD) it involves the precession and the nutation correction.

Once this is done then we are ready to go into the Terrestrial frame. Today we wind up here and then next time we will take up the Terrestrial reference frame how to go up to that and thereafter we will come to the orbit determination part. Thank you very much.