Space Flight Mechanics
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Lecture No - 72
Transformation from Celestial to Earth Fixed Reference Frame

Welcome to lecture 72, so we have discussed about the transformation from celestial to body reference frame celestial to the earth fixed reference frame. So, we will continue with that today will conclude that and thereafter we discuss something about time and then will come to the orbit determination. Actually only one week is allotted to for this particular talk with which is related to reference frame and Orbit determination. So we do not have much time to cover in details.

So I will slightly touch the topics and then I will give the references to which you can refer if you want further details and move to the next the trajectory transfer.

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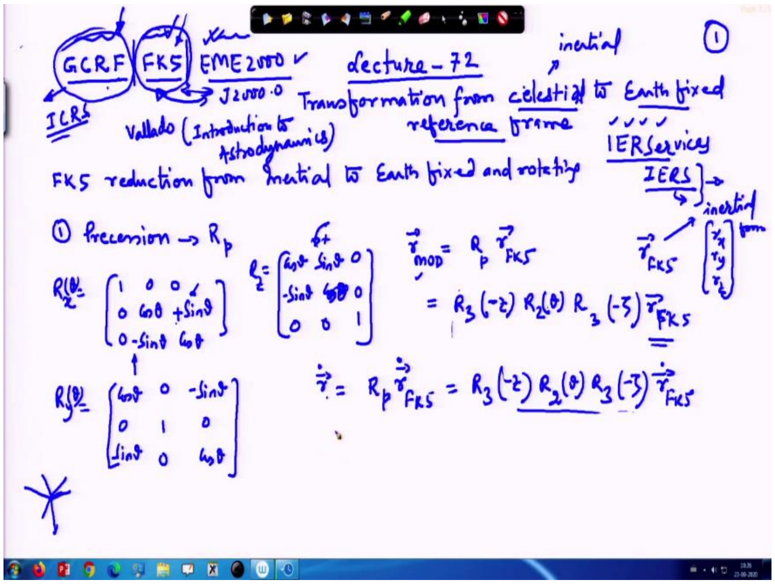

So let us start ok so till now in this transformation from celestial to earth fixed reference frame where celestial refers to your inertial reference frame. So in this there are two ways you can do as I have told you that I can have the Transformation in the geocentric celestial reference frame I can have transformation with respect to Fk5 this is another frame. And then, also, as I told you that we have also the EME earth mean equator of 2000 or what we call as the J 2000.

They are closely related to each other ok but some slight differences are there. At outer level it does not matter once we are looking for the astronomers its important but not for earth. This especially for the earth satellite it does not matter. But if you are the satellite tracking somewhere near Pluto or something then it will. So difference will come ok if you are looking for the Stellar distance for the very distant star at that time this things matter.

So for our case this is immaterial with respect to which you are working out in this especially you call this as the J 2000 sometimes people refer to this Fk5 as J 2000 what is strictly speaking this was the earlier one used the EME 2000 later on in the Fk5 we have the star catalogue included and then the geocentric celestial reference frame. But this is usually they are referred to the same J 2000 or EME 2000 and Fk5.

But GCRF it falls under the category of international celestial reference system, ok, so it is a little different as compared to this because here it is based on the sources of radiation from the Stellar background. And while Fk5 mainly involved star catalogue. So these are things we have discussed in the past. So today I am going to conclude this. So in the Fk5 reduction inertial to earth fixed. Earth fixed means it is rotating along with the earth.

So as we have discussed and then we have one concept it is called the non-rotating origins. So we do not have time to discuss all those things. I thought of discussion but time does not permit. And for the non-rotating origin slightly it is given in your The David Vallado otherwise you will have to refer to the books on astronomy. But the papers are available by giving note and based on that mainly what I will suggest you that you look into the IERS International Earth Rotation System or services International Earth Rotation this is S for services. I E R S this provides you the related software written in Fortran.

And also it provides you the documents related documents do not in very great details, but all the necessary information are; how to use those things are also available. How to put the poles in serial order so that you get the final result, so these things are available, so we conclude here. The first was the Precession we applied Precession and ok so the precession modelling. So, this we can write as the rotation due to precession.

And all the rotation matrix is written that we have discussed earlier. At rotation about the x-axis is given by

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

if you are rotating by θ it will written by $cos\theta\ cos\theta\ -sin\theta$ and $sin\theta$. Ok so if we are rotating the reference frame.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{bmatrix}$$

So will put here plus and this is minus and if we are rotating the vectors will replace this with plus and will replace this with minus. So here in this case choose this has plus similarly Ry this we choose as we will put one here in this place and then 0 0 will put here.

$$R_y = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}$$

If you are rotating by θ this is rotation by θ using the right hand rule. Here this will be $cos\theta\ cos\theta$ and in this case this gets to minus $sin\ \theta$ and here this will come $sin\ \theta$. So this is the difference between rotation between x and y and similarly Rz θ if you are rotating by θ about the z axis the rotation is been given so this we can $cos\ \theta\ sin\ \theta$ minus will not be there in that case vector rotation is there.

$$R_z = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So that at this place the minus sign will appear just opposite of this vector rotation by θ here this minus sign is already given if it is the matrix frame rotation, then here the plus sign will appear. So this is with plus sign and then minus $sin\ \theta$ and that $cos\ \theta$. So these are 3 rotations required for modelling any general rotation in terms of Euler angles. Here in this case we have the vector given in the say r Fk5 or r ITRS whatever you want to write so r Fk5 in this case. This is your vector which you can write here also in this format.

$$\vec{r}_{Fk5} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

So it the component rx ry and rz so this is the inertial frame and then converting from the inertial frame to the earth fixed frame. So this we have already done this part. So just I am concluding today. First once we convert this then it comes to mean of date r MOD and then the corresponding the precession R P times r Fk5 where the R P we have already modelled we have written earlier those details I am skipping because this is the concluding lecture on this.

$$\vec{r}_{MOD} = R_p\, \vec{r}_{Fk5}$$
$$= R_3(-z)\, R_2(\theta)\, R_3(-\zeta)\, \vec{r}_{Fk5}$$

So R3 this was – z R 2 minus θ; R 2 + θ this is R 3 -z R 2 + θ and then R 3 $(-\zeta)$ we have written and these times r Fk5 for this portion you can refer to book by Valladu Introduction to Astrodynamics. Similarly the velocity here it can be written so from Fk5 we have come to the MOD and velocity it will be indicated by r dot and this also because it is a snapshot of the axis. This will be returned this way and the same rotation matrix will play here its role. R 3 R 2 θ.

$$\dot{\vec{r}} = R_p\, \dot{\vec{r}}_{Fk5} = R_3(-z)\, R_2(\theta)\, R_3(-\zeta)\, \dot{\vec{r}}_{Fk5}$$

Ok even in this case say because what we have done that this is my inertial frame and then we have to convert to another frame which is also taken to be a snapshot at a particular time. And therefore this rotation is not considered even if you considered. So, that the rotation rate of all of them will be negligible its, neglected.

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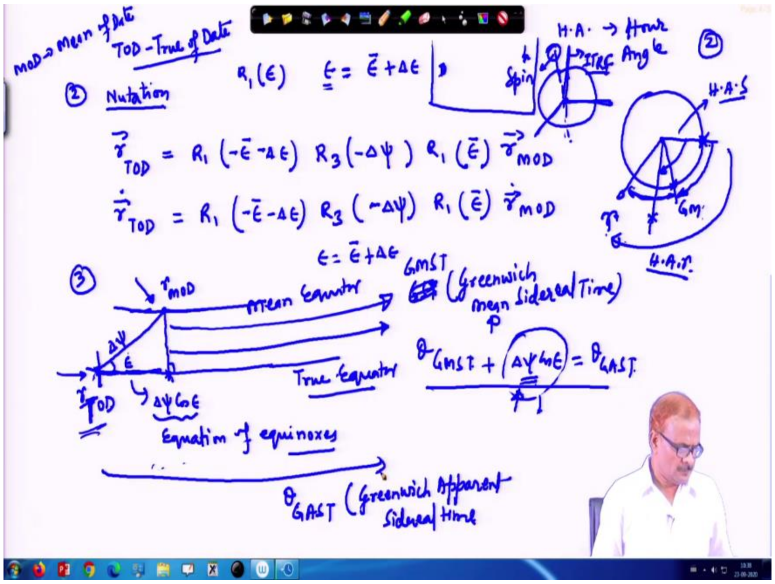

Ok some matrix multiplication I am not doing here. To save our time then the second one then we have given the nutation. And nutation was similarly modelled and it convert it from mean of the true of date. This is TOD is true of date. So after the precession correction, we are coming to the mean of date and after nutation correction, We are coming to the true of date. Mean of date and true of date.

$$R_1(\epsilon); \;\; \epsilon = \bar{\epsilon} + \Delta\epsilon$$

$$\vec{r}_{TOD} = R_1(-\bar{\epsilon} - \Delta\epsilon) \; R_3(-\Delta\psi) \; R_1(\bar{\epsilon}) \, \vec{r}_{MOD}$$

So these are corrections that we are giving so here this was written as; if you remember we have written it as R 1 perhaps E prime or E i I have used there that was the last lecture I do not remember exactly the symbol what we have used. But here if I write simply E so this implies this epsilon implies epsilon bar which is the mean value of Epsilon and plus delta E. So, this we have modelled then as – E bar – delta E I might have written here in terms of epsilon R mean of date to true of date this is converted.

$$\dot{\vec{r}}_{TOD} = R_1(-\bar{\epsilon} - \Delta\epsilon) \; R_3(-\Delta\psi) \; R_1(\bar{\epsilon}) \, \dot{\vec{r}}_{MOD}$$

Similarly it will be given by the same rotation matrix there is no change it is a snapshot of the time or either even if you consider the rotation rate so these are just negligible. The third rotation its related to siderul time. Since it is rotation related to the rotation of the earth the siderul time we have not still considered and or either discussed about that. So, the rotation of the Earth it can be modelled in two ways one is with respect to the distant star which we called as the siderul date rotation with respect to siderul time.

And another rotation with respect to the sun or the sun rotations with respect to the particular Meridian of on the earth. Say here in this case and if I show earth like this here say this is your the vernal Equinox which we are showing at gamma as gamma. So from here to the Greenwich Meridian distance perhaps we have not discuss this Greenwich Meridian you might have heard about this and this may be the local Meridian.

So angle from here to here you have and the angle from here to here we have. Similarly the sun can we this is called the from here to here if we measured from the particular Meridian towards in the opposite direction of the angle from this place to this place this will call the hour angle of gamma which is the hour angle of vernal equinox, H dot A is the hour angle of gamma which is the hour angle of vernal Equinox H.dot A this is the hour angle.

So hour angle of vernal equinox and similarly if we have the sun here and if I show that so this will be the hour angle of the Sun. So, will come to that concept a little later right now it is not required; so thereafter we have to model the; where the your; the location from which on the earth you are looking at the satellite, so that location with respect to the universal frame where it is aligned. So, the first nutation and rotation correction precession and nutation rotation, corrections are given.

And thereafter then we are coming to the Earth rotation. Ok, so this is the proper rotation of the earth about its axis but as I have mentioned earlier that this axis itself, it keeps changing with respect to the figure of the Earth. Figure of the earth means you have the earth here and in this itself the Earth rotation Axis it keeps changing it is not constant. And this happens because the way the things are happening here it is directly related to Alice dynamical equation.

Alice dynamical equation again, I will have to describe ok if we skip all those things, but it happen because of the rigid body rotation of the earth and beside this there is the plate tectonics involved that means the surface of the earth moving with respect to each other for all those things are involved. So either one axis is fixed in the figure. That means your this axis is rotating this is your spin axis. But this axis is not rotating and this is called the ITRS.

The International Terrestrial Reference Frame we can say ITRF. We convert it from what we are trying to do? We are trying to convert from the inertial reference frame to ITRF which is fixed in the figure. This is not rotating like the spin axis of the earth. Spin axis of the Earth it keeps changing with time within 30 meter plus minus 30 meter or you can say the 30 meter into 30 meter square area about the pole it keep shifting.

So it is small value but still those connections are given if you are doing very precise work at that time those corrections are required and those are measured and it is applied. So International Earth Rotation Services, they keep all these records and keep uploading on the website of IRS. So, the rotation of the Earth it can be modelled in two ways. We have already done this rotation of the spin axis. Now we are concerned with the rotation of the Earth.

So, rotation of the earth as we have discussed that we have the; here say the vernal Equinox gamma mean of date (γ_{MOD}) and then we got the gamma true of date (γ_{TOD}) after nutation correction gamma true of date. So, the angle which is measured from this place now let us join this and this was your 2 ecliptic and this we have written as $\Delta\psi$. If we check the projection on this, this is 90 degree so this projection then this and this angle is epsilon.

Epsilon (ϵ) means epsilon bar + delta E ($\bar{\epsilon} + \Delta\epsilon$) these things already I have described. So these part becomes delta Psi cos epsilon. So $\Delta\psi \cos\epsilon$ this is called the equation of equinox with certain correction which we are not going to discuss again. Equation of equinox so that the angle which is measured from this place from the true of date. Angle which will measure true equator and this is the true equator and this is the mean equator they are not parallel to each other.

So that angle measure around the true equator this you will write as θ_{GAST} and this we refer as our θ AST the Greenwich say Greenwich Apparent Siderul Time and here we will write this as and if we measure angle from this place so this is called GST or GMST we can write this as Greenwich mean siderul time. We can see why the mean value is coming because it is being measured from gamma mean of date and this is being measured from the vernal equinox gamma true of date this is T from the true of date and along the true equator.

And if we measure from this place the distance along this direction from here then we call as this as the GMST, so GMST and plus delta Psi cos epsilon. This is equal to θ_{GAST} and here are some more correction are added.

$$\theta_{GMST} + \Delta\psi \cos\epsilon = \theta_{GAST}$$

So we are skipping those terms say if you want to go into the details so you can look into the book by Vallado. So, will be measuring distances along the true equator. And along the true equator distance measured from here and distance measured from the gamma TOD.

So, this is the difference so here the other part as I am telling you that the other part the corrections I have not added here and simplify calling this as the equation of equinox because we do not have time and there are stellar angle another definition is there which is referred to the non-rotating origin. Again that concept; because I have not discussed so I will not go into that we do not have time to discuss all this issues.

But it is a very big development which started in 1976 around and later on it has been adopted now that ITRS all those things they are basically working with this non-rotating origin, and it is much more comprehensive because they are the all the rotations are combined together rather than the Precession nutation breaking them separately. So they have been combined. Ok so this rotation is model by your; the angle measured from the true of date is as I have written this is the θ GAST and θ GMST and what this ST stands for. This is siderul time.

Ok again I am writing here this is siderul time. And there is one solar time and sideruld time these are different so I will come to that little later.

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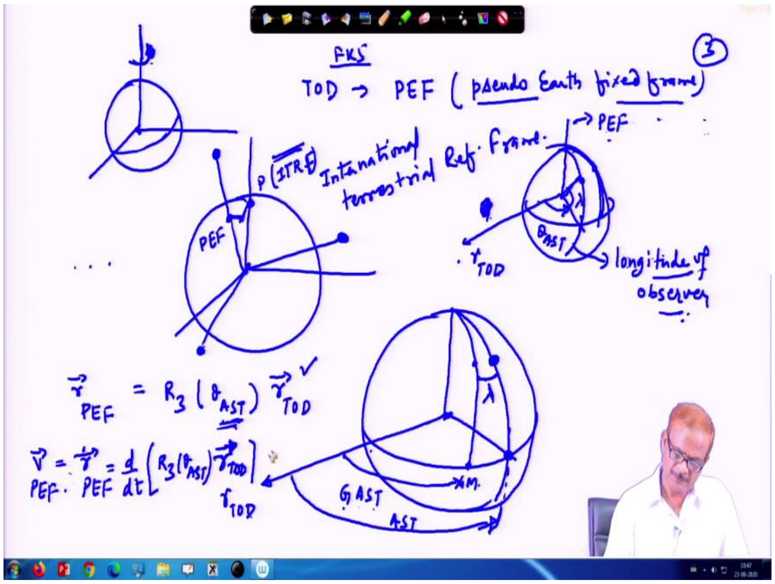

First let us wind up this so I am trying to recall those things and then we will go into the next lecture. So once this Rotation is given that means this is the earth and say this is the spin axis. And once you have given the corrections for precession nutation and then along the true equator your rotation will take place. So you are rotating along the true equator. This is the thing you are modelling your giving the rotation and that rotation will be related to the spin of the earth.

So once we convert to the true of date. So in the Fk5 reduction this comes to the pseudo earth fixed what we called as the pseudo earth fixed. While this is being called as the pseudo earth fixed because your spin axis this is not fixed with respect to the figure of the earth. This is your pole of the ITRF International Terrestrial Reference Frame ITRF International Terrestrial Reference frame and with respect to this your axis are shifted somewhere.

So here this is your shifted axis which is the spin axis. So this part we are calling as the PEF. So currently your orientation of the axis so after giving the precession correction nutation correction and then rotating it comes to this position. So, your rotated axis it is here in this position. To this once we give the correction then we come to this ITRF. So those connections are also modelled. So we have r pseudo Earth fixed this will be modelled as R 3 θ AST times r true of date.

$$\vec{r}_{PEF} = R_3\,(\theta_{AST})\vec{r}_{TOD}$$

So apparent siderul time this is θ AST because the siderul time is measured with respect to the vernal Equinox and therefore here ST stands for the siderul time. And if it is up to Greenwich line, so this will be GAST and if we are doing it on the location for the location of the Meridian of the this observer. So the observer may be located at certain Meridian with respect to the; so this is the centre of the earth.

And so this is the Meridian of the longitude of the longitude line of this will be a little better longitude of observer and this is vertically the figure axis figure z-axis. This is particularly named so I will come to those things later on. So, your vernal Equinox may be directed along this direction of the true of date and with respect to this then if it is the observer longitude then this will be written as θ GAST.

Now, one thing I would like to correct here. This is the; say first we confine ourselves to PEF. Let us say this is PEF pseudo earth fixed frame. So it has rotated from this place to this place by θ AST which is the longitude of the observer. Let us not complicate at this current time step and this is your true of date vernal Equinox and from there your location of the longitude of the observer perhaps we require another figure to make it more clear.

This is gamma true of date and here you are located. So this is your longitude and longitude is being measured from the; on the earth we measure it from the Greenwich Meridian. So this may be your Greenwich Meridian and from there this lambda or capital lambda whatever the value you want to use the symbol this is the angle between your Greenwich Meridian and the observer Meridian. And your location is somewhere here in this place.

So, from this place to this place remember this is with respect to the true of date. So this becomes AST and we apply before this to Greenwich apparent siderul time and if we go to this place up to here so this we write as simply as AST apparent siderul time because this is for your location and this is for the Greenwich location. So for this correction is given so that with respect to the inertial frame where you are located that is known and there after because the earth spin axis it is shifting with respect to the axis of figure.

So for that another correction is required that becomes the final correction so V EF the velocity in the V EF is equal to r dot this is PEF r dot PEF this quantity will be R 3 and we have to convert it. Ok so we need to take the derivative of this because this is a rotating frame. So we need to take a derivative of this. So d by dt and R 3 θ dot AST gamma dot true of date. This is r true of date.

$$\vec{v}_{PEF} = \vec{r}_{PEF} = \frac{d}{dt}[\, R_3\, (\theta_{AST})\vec{r}_{TOD}\,]$$

We can convert from one Frame to another frame.

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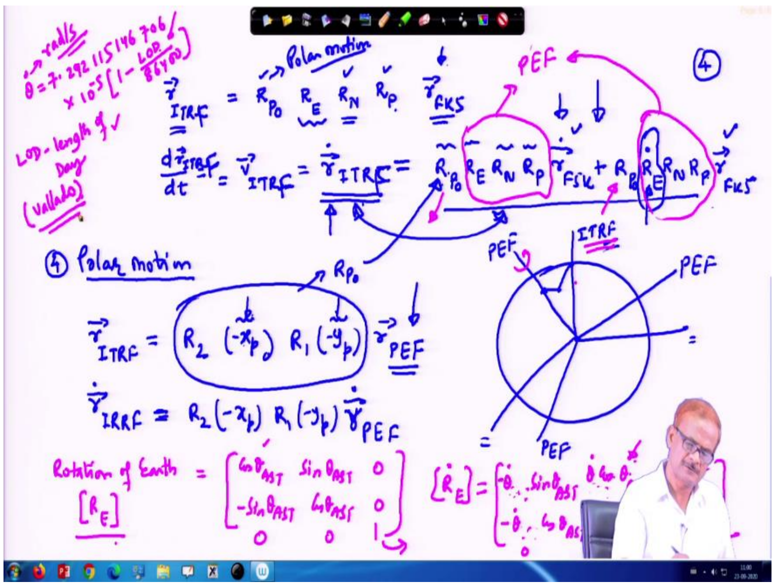

And final one, this is the; what we have the total thing we are using that we are writing here r in the Fk5 and then we are operating on this first by the rotation due to the precession then we are giving the rotation due to the nutation. Then we are giving rotation due to the rotation of earth and then the rotation due to the polar motion correction which still we have not done. Ok, and this brings you to the frame r ITRS.

$$\vec{r}_{ITRS} = R_{Po}\, R_E\, R_N\, R_P\, \vec{r}_{Fk5}$$

And if you differentiate this gives velocity, in the ITRS this is r dot ITRS and here all this rotations are small. These are not counted only these rotations are small and only the Earth rotation is bigger and this is counted and this is tangible. So here this gives you R P times RE RN times R P r Fk5 dot this is the velocity in the inertial frame. And plus R P times R E dot R N R P times r Fk5. So this is the complete rotations.

$$\frac{d\vec{r}_{ITRS}}{dt} = \vec{v}_{ITRS} = \dot{\vec{r}}_{ITRS} = R_{Po}\, R_E\, R_N\, R_P\, \dot{\vec{r}}_{Fk5} + R_{Po}\, \dot{R}_E\, R_N\, R_P\, \vec{r}_{Fk5}$$

We are interested here if you are looking for the conversion from Fk5 to ITRS. So this is the complete rotation sequence that we you need to give. Ok this part what we will do rather than writing here it in this way we remove this part, not to confuse with the our things that we are developing its better to just show. It is better to just show this part what we have shown here in this place. This is your velocity in the International Terrestrial Reference Frame these are all frames.

$$\vec{r}_{ITRF} = R_{Po}\, R_E\, R_N\, R_P\, \vec{r}_{Fk5}$$

$$\frac{d\vec{r}_{ITRF}}{dt} = \vec{v}_{ITRF} = \dot{\vec{r}}_{ITRF} = R_{Po}\, R_E\, R_N\, R_P\, \dot{\vec{r}}_{Fk5} + R_{Po}\, \dot{R}_E\, R_N\, R_P\, \vec{r}_{Fk5}$$

What is the difference between the frame and the system I already discussed in the beginning. So, once your velocity in the inertial frame and velocity in the position in the inertial frame with the known so you can find out velocity in the rotating frame this is what it implies and the corresponding relationship is given by this. And vice versa if you are to go in the opposite direction so accordingly, you have to follow the rules.

And this part is especially other things here just angle angular and angles involved while here the derivative of this matrix is involved. This is a rotation matrix, whose derivative is involved. So we will have to write for that part but before this we write for the polar. Polar here the precession and polar we have not differentiated. So will write this as r this is only precession nutation Earth rotation and then let us say this is the polar.

$$\vec{r}_{ITRF} = R_{Po}\, R_E\, R_N\, R_P\, \vec{r}_{Fk5}$$

$$\frac{d\vec{r}_{ITRF}}{dt} = \vec{v}_{ITRF} = \dot{\vec{r}}_{ITRF} = R_{Po}\, \overbrace{R_E\, R_N\, R_P}^{PEF}\, \dot{\vec{r}}_{Fk5} + R_{Po}\, \overbrace{\dot{R}_E\, R_N\, R_P}^{PEF}\, \vec{r}_{Fk5}$$

We will make it different from the precession. So this is your polar motion, polar motion and Precession nutation they are different. So the; fourth one that we are considering this is the polar motion. So, now polar motions it is model by r ITRF this is the pseudo earth fix. Here this spin axis is located with respect to the figure of axis this is your figure of axis ITRF reference frame and this is your PEF. This is referring to PEF X Y and Z axis and this is for ITRF.

So with respect to this the rotation is written in terms of X and Y but these are small values given in the angular terms. Ok so how much is the displaced from this position its done. These things already I have discussed. So here again if we are looking for velocity because this rotations are almost; rotation rates are almost negligible. So therefore its rates are not taken into account and just it is written as r dot PEF.

$$\vec{r}_{ITRF} = \overbrace{R_2(-x_p)\, R_1(-y_p)}^{R_{Po}}\, \vec{r}_{PEF}$$
$$\dot{\vec{r}}_{ITRF} = R_2(-x_p)\, R_1(-y_p)\, \dot{\vec{r}}_{PEF}$$

So once we give the polar correction. So this is how it follows. This part is referring to R p0 this is your R P0. So these together with bring set in the PEF. Here also this together itself bringing it into the PEF and from there. Then you are converting it into the figure fixed axis which is ITRF ok and modelling of all these things so finally we conclude here in this place. Rotation of earth; about the spin axis so that we are modelling as $cos\theta_{AST}$ as I have told you this is the rotation about the corresponding z axis.

So after nutation you are giving this rotation by θ_{AST} as I have shown on the previous page. About this, this is your PEF; if you give rotation about this so you come to the PEF frame otherwise it was in the gamma true of date. So, here this rotation this is the AST what we have written in terms of angle, we can write this as θ_{AST}. So $cos\theta_{AST}$ this is $sin\theta_{AST}$ $-sin\theta_{AST}$ and $cos\theta_{AST}$ 00 001.

$$Rotation\ of\ Earth\ [R_E] = \begin{bmatrix} cos\theta_{AST} & sin\theta_{AST} & 0 \\ -sin\theta_{AST} & cos\theta_{AST} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So this is your rotation matrix R E and if you differentiate this with respect to time so you get R E dot. So, R E dot this quantity will be given by just differentiation of this Matrix so this will be

$$[\dot{R_E}] = \begin{bmatrix} -\dot{\theta_{AST}}\ sin\theta_{AST} & \dot{\theta_{AST}}\ cos\theta_{AST} & 0 \\ -\dot{\theta_{AST}}\ cos\theta_{AST} & -\dot{\theta_{AST}}\ sin\theta_{AST} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\dot{\theta_{AST}}$ and cos then becomes $sin\theta_{AST}$ with minus sign here and sin theta dot becomes cos θ AST multiplied by $\dot{\theta_{AST}}$. All of them this is the way of matrix differentiation and here $-\dot{\theta_{AST}}$ cos this term we can remove and just took it $\dot{\theta}$.

And then here in this place $-\dot{\theta_{AST}}\ sin\theta_{AST}$ 000 differentiation of one this becomes 0 here in this place rest of the things they remain as 0. So, these are the things and you just need to insert these values, of course if I going the details, it will take many lectures to discuss all the involved issues here in this. And I am trying to hurry that is why I am not able to put forward all the related issues, time is not there.

So by now we have completed this week, but still we have to go further to complete this week lecture. Or Maybe we will just write one equation and then there after we will stop. In this equation what the $\dot{\theta}$ is appearing here. This is written as 7.292115 so if these are written so accurately because if you are propagating for a longer time, at that time this angle still matter. LOD is this called the length of day and you can refer to Vallado for all these details.

$$\dot{\theta} = 7.292115146706 \times 10^{-5}\ [1 - \frac{LOD}{86400}]$$

I am not going to cover and this $\dot{\theta}$ is written in Radians per second. Why I am telling that I am not going to cover because this will not be of immediate interest to you. Until and unless you get into the orbit determination area or go to the astronomy area this may not be very useful or either once you are working in the mission planning area for the orbit determination so at that time these things will be important to you.

Because we do not have that much of time to discuss what exactly all these things are. Why I am skipping few of the things. Ok so we stop here at this point. And for reference you can go to Vallado and look into this. Thank you very much.