Space Flight Mechanics
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Lecture No - 73
Time

Welcome to lecture 73. Today, in this lecture we are going to discuss about the time which is required very much for while you are working with the transformation of the axis for your coordinates you are trying to convert from the inertial frame to the earth fixed frame and finally to your topocentric frame. I am already that I have to go and see what the axis that we have described we are called this is the inertial frame and then the Terrestrial frame which is fixed to the earth and rotating along with the Earth and the topocentric frame from which is fixed at the position where the observer is located with proper orientation.

These are 3 axes I have described earlier in the reference frame section. So, today I briefly in this lecture touch the concept of the time and thereafter will go to orbit determination.

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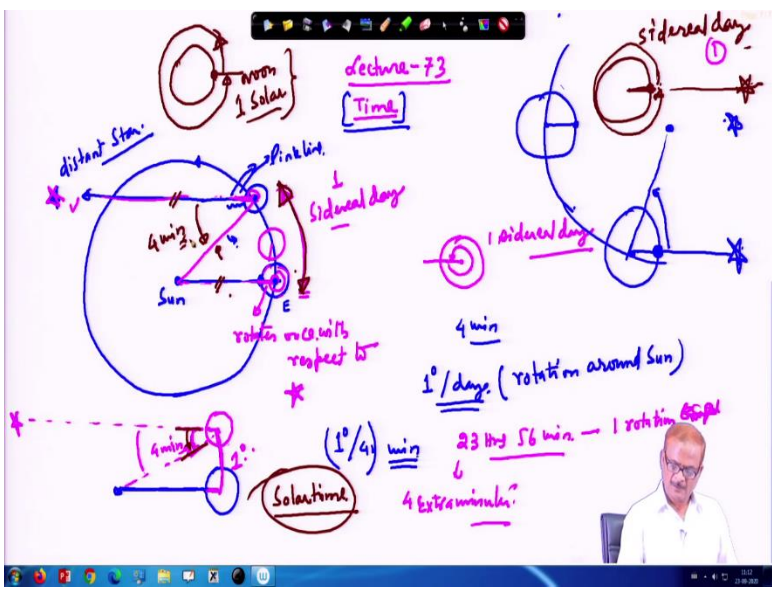

So, hopefully in the next 2, 3 lectures we will wind up the whole thing. So here in this figure I have shown you sun and the earth is moving around the; this is your earth moving around the sun in this direction and suppose somebody on the equator or at certain Meridian of longitude he is sitting then sun comes over head at certain time. You can see from this place that if I draw a line joining the centre of the earth and this point. Change the colour of this line the pink line it is pink line will organ become horizontal.

So, the impression given on earth, so it will rotate as the earth goes from this position to this position for this pink line will come here later on as it progresses further it will rotate all the way And by the time it reaches you say the earth rotates once and Earth rotates with respect to the inertial frame. So inertial frame rotation, say is the distance star is here, so it is restricted to distance star. So this pink line it rotates once rotates once with respect to the star.

So, the time taken from this place again coming to this place this pink line which is on the earth it rotates here all the way and comes again to this position and the time involved is called the sidereal day. In the meanwhile, your earth is earth is rotating on its axis in the meanwhile your; there are two things involved in 1 sidereal day your Earth is moved around the sun from this position to this position.

From here, it is started and it has reached to this position in 1 sidereal day. In addition Earth is rotating on its own axis so this is a pink line. So this line will rotate and again, it will come to this position with respect to distant star miss these horizontal lines say. So, this also constitutes one sidereal day. Once; one rotation of the earth with respect to distant star but in the meanwhile the earth also rotates around the sun.

So, centre of mass of the Sun or it moves from this position to this position. It is moving from this position I am showing by blue dot to another blue dot so how much time it will take for this pink line, the pink line which is shown here. To come and coincide with this. So, if we do; we will say that it will look from the Earth exactly what is happening say there is a distant star somewhere very distant star is there.

And you are sitting here on this place and as the earth is rotating, so what you see that after certain time Earth is rotating around the sun. So, as the earth has come to this place and from here this is your Meridian and distant stars will look like this because it is very distant. So it will look again like this. So, right now you are here and sun is here in this place. But you have your position has not come under the sun.

So, this position to go to this place your position this is your position and you have to go up to this place. So, this takes another some extra time and that extra time is around 4 minutes and how it is related? It is related to the fact that by the time earth rotates on its own axis on an average Earth is rotating around the sun 1^{o} /day. This is rotation around the sun rotation around the sun. This is one degree per day and on its axis it rotates 1^{o} per 4 minute or 1 by 4^{o} per minute.

So, this is the sun and your pink line is here right now. By, the time earth as rotated this pink line is coming to the pink position again here in this place. So, the one rotation of the Earth is complete with respect to the distant star this one rotation is complete. But with respect to the sun it is not yet complete. So this extra angle which has covered this is around 1^{o} and to cover this extra angle it requires 4 minutes more.

So, rotation of this pink line from this place to this place this requires 4 more minutes. So, you say the rotation you may be aware of that Earth rotates on its axis and 23 hours 56 minutes in one rotation complete one rotation complete on its axis. But the 4 extra minutes which required? That is required for covering this extra angle which is involved here. And this is because of the movement of the earth around the sun.

So to cover this angle it requires 4 extra minute and thereby this drawn line will come to this position over the brown position. So it the time to switch from this place to this place and this extra time will be add we call this is the solar time. You can understand the difference solar time is matter with respect to the sun. For a person on a particular Meridian two consecutive one moment of the complete movement of the sun you are here over it and say this is moon time and sun started over your meridian from here.

And again, it returned back. So this constitutes one solar day. So the same thing is being shown whatever we have discussed all this concept is based on here in this place. And if a distant star you are here and distant star is there. Many light years away and you are sitting here on the earth and distant star once you rotate and distant star is visible to you. We can see that this we have written as sidereal day. One sidereal day with respect to a star on complete movement is there and in one solar day with respect to the sun there is one complete rotation.

So, this is the difference and the difference comes because the earth has moved from this place to this place and because you can see that here in this case this line and this line of parallel. Here in this, this is getting inclined to this line. This angle has to be covered and this requires your 4 extra minutes. So this concept now we can move with this concept can move onward.

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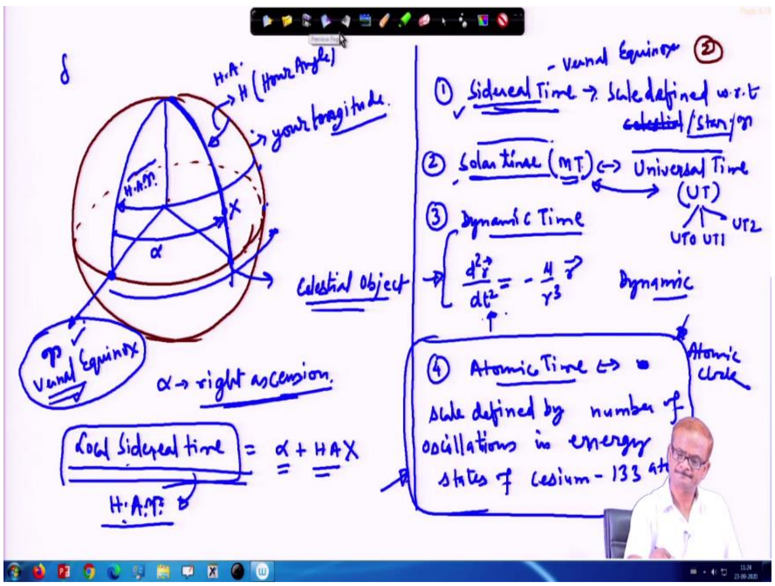

Then we have the and for the definitions you can look into the; see in this chapter if I keep writing the definitions and other things we would not be able to cover the concept. So you can look into the books because anyway this that just details which you need to remember and if you do not understand will again forget it. So, for other things you can look into the books that I have referred too. And this is vernal Equinox and this is this maybe this say we will take as the Greenwich Meridian for some this we can say let us write this is a celestial object.

The first let us discuss one concept and then go to another one celestial object and this is your meridian your longitude. So, the angle from this place to this place this is called the hour angle. From your Meridian if you go in the backward direction in the west direction this we called as hour angle. But you can write this is as H.A.. So, the hour angle of vernal Equinox from your Meridian which is located here.

This will be measured from this point up to this. So, hour angle of gamma (γ), it is written like this hour angle of gamma some celestial object is here. Once you are looking from the Earth so you will see celestial object on the celestial sphere. So, this is what is being described with respect to the vernal Equinox the location this is written as alpha, and Alpha this is called the right ascension. So this is the right ascension of the celestial object.

So, local sidereal time sidereal time is measured with respect to the star. So here in this case now, we state that this will be measured with respect to the vernal Equinox because your vernal Equinox this is the line pointing towards the; right now it is in the constellation of Pisces, but earlier as it was in the constellation of Aries and therefore this the on-off Aries this sign is used. So, the local sidereal time instead of star now we will say that because this is a fixed thing. The vernal equinox quite frequently we are using with respect to this all the angles we are measuring and making it very convenient instead of using another star.

Vernal Equinox definition is very convenient to work with so therefore this we call so as the hour angle of the Vernal Equinox and we use this. And from vernal equinox of any celestial object. This is a celestial object longitude of position so this we called as the right ascension. This is the particular notation used and if we measure distance along this the Meridian then we call this is the declination.

So, the right ascension and declination is shown by delta so that concept again we are not going to take up here your local sidereal time is the time say earlier you will just overhead ok over the vernal Equinox your Meridian was just over this. Now it has moved from this place to this place. So how much time has gone so that time we call as the sidereal time instead of star we are measuring with respect to this. So, this become so convenient if we take vernal Equinox and this is the reason instead of a star we are taking the vernal Equinox.

Because all the angles for the conversion we require with respect to vernal equinox. So local sidereal time just becomes right ascension and plus the hour angle of that particular or any object this we also we can write. Hour angle of any particular object the celestial object here in this case or both the things can be also explained in terms of hour angle that is the hour angle of the local sidereal time is nothing but you are hour angle of this. This is the way we can write.

Here also hour angle is appearing but this is our angle of your particular celestial object let us name this as the x and this is the right ascension of the particular celestial object. So the solar time we have discussed and the sidereal time also we have discussed. Now we have four broad category for the time system for measuring system we are having. This we can classified into the sidereal time already we have discussed.

This is sidereal time it is the scale is defined with respect to celestial objects celestial or star define with respect to star or vernal Equinox. And then the solar time this is written as MT or MST is called the mean solar time. Why mean solar time? because the rotation movement of the sun as appearing from the Earth it is not uniform sometimes it moves fast sometimes slow and moreover the location of the; neither it moves over the equator not over the earth apparent motion it keeps changing.

It passes the equator to the north in summer and comes to the South in the winter so this apparent movement with respect to the equator this is also involved and moreover this motion is not any form and therefore the mean solar time is defined. And to this connected is also universal time which we write as UT. There are 3 definition for the UT, UT_0, UT_1 and UT_2. So, again it is just a matter of details and other things which we; it is not possible to go at this stage because of the lack of time.

But MT and UT they are connected together so we have sidereal time the Solar time of the universal time. Instead of solar time you can write universal time and then we have dynamic time dynamic time. So, if we describe the motion of the celestial objects, ok in the dynamic itself you will see that the $\frac{d^2r}{dt^2}$ as we have written the motion of the any object or the motion of the sun and earth system. So there you will naturally see that the t is appearing.

So, the t is appears, so this we call as dynamic time because it is involved in the equation of motion. And fourth one this we can call as the atomic time and this is connected with the electron movement. So this is the scale you write this here scale defined by number of oscillations this you may be aware of from your basic 12th class. The question of oscillations in the energy states of atom or atoms.

So this time it is maintained by various agencies in the world and India also has its own time keeping systems of the atomic time is maintained by this atomic clock. Dynamic time directly appears in the equation of motion. Solar time as I told you this is with respect to the motion of the sun around the earth. And this is basically the mean motion of the sun around the earth because the actual motion it keeps wearing with time in the sidereal time which we are now relating with the vernal Equinox instead of star. I will relate this with the vernal Equinox.

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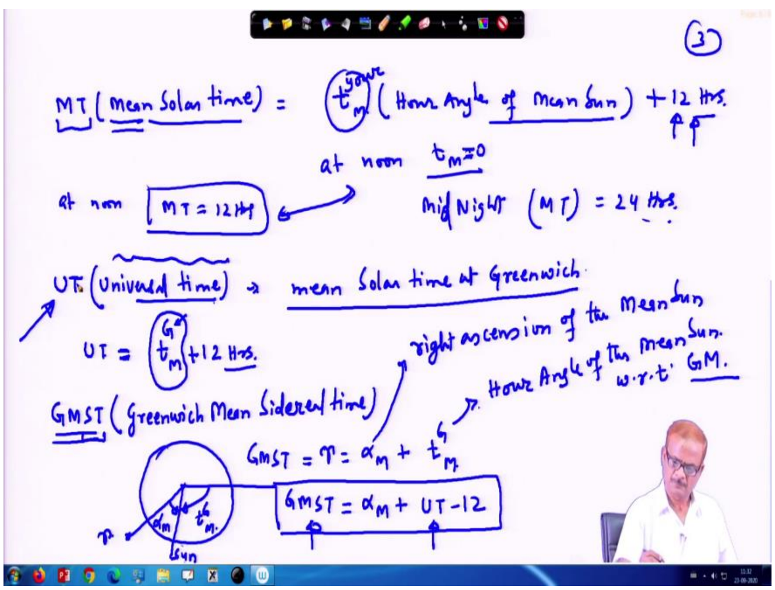

There are many, many definitions involved and unfortunately we do not have that much of time to discuss all these issues. Let us first check the mean solar time mean solar time. Hour angle of mean time plus 12 hours it is defined such that 12 hours is why it is been added. It is because at the noon. At noon t_m is taken to be 0 so that once sun is over head at that time. It is just 12 o'clock of noon. So that is for this 12 hours is added.

In hour angle already we have define this is you are measuring west from the observer meridian. You are sitting at any meridian and from there then you are measuring. So at noon mean solar time becomes 12 hour according to the definition we are using. Now, how the universal time is defined so at midnight we can also note it at midnight. At midnight the solar times MT will be 24 hours means 12 + 12.

That means sun is just opposite to your meridian. Now the UT the universal time which I told that it is related to the MT. And this is defined as mean solar time at Greenwich. So instead of your Meridian it is Greenwich Meridian. So you are sitting at the Greenwich Meridian and then defining this time. And therefore it can be written as; remember this is the mean solar time at Greenwich Meridian. So, t instead of just t_m now this becomes this is your G this was related to your location this is related to Greenwich location plus 12 hours.

Greenwich Mean sidereal time now they are getting connected the Solar time and the sidereal time. Greenwich Mean Sidereal STA stands for sidereal time and MTS stand for mean solar time instead of writing solar time. It is mean solar time as I told you because of the non uniform motion of the sun has appeared appearing from the Earth. So therefore GMST this becomes equal to this is your vernal equinox.

This is the Greenwich line and here this is your Alpha Greenwich Meridian α_G or α_m whatever you want to write this is the t_m^G, so GM is the Greenwich sidereal time now we are going to define. Greenwich Meridian let us say one thing here we will locate the sun not the Greenwich because we are writing here t_m^G. So we will t_m^G is here. We are writing as the sun. This is your sun location so t_m^G + α_m this is the right ascension of the Sun.

So this gamma angle this is α_m + t_m^G and this is nothing but your GMST Greenwich Mean Sidereal Time. So, α_m is the Right ascension of the of the mean sun this is the right ascension of the mean sun. And as for this hour angle of the mean sun and so this is hour angle of the mean sun with respect to Greenwich Meridian this completion we have to mention. And therefore GMST this gets reduced to; now we can connect together.

Here this is $t_m^G = \text{UT} - 12$. So, $\alpha_m + \text{UT} - 12$ so this is how your Greenwich Mean sidereal time and the universal time they are connected together. And this universal time this plays a lot of role in the modelling of the rotation of the earth. And then this times are connected to the atomic time also, it was finally for the measurement. We are also maintaining the atomic time.

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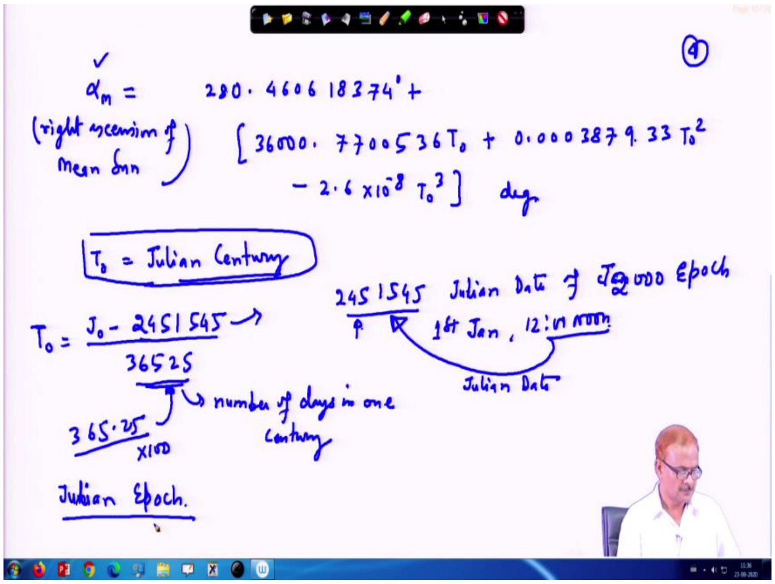

So, α_m mean sun right ascension of mean sun this is written as 280.460674º plus these are really very precise values this all in degree. The right ascension of the mean sun and it is available from this equation and this has been modelled. By observation and other things this has been modelled. So once this is available so you can see that α_m is available and once α_m is available here then GMST and UT they get connected together.

Because this was the remaining unknown so this known is described here. And what is this T_0? This T_0 is measured in Julian century. So this definitions we will write it now so T_0 is the quantity here $J_0 - 2451545$ the whole thing divided by 36525. In one year we have 365.25 days. So in one century we will have 365 this into 100 this gives you this many days. So these are days in since number of days in one century.

This is number of days in one century and this is precisely the Julian date and what is Julian date? Again I will have to write here. This is actually the Julian date of J2000 Epoch. That means on 1st of January 12 o'clock noon. The Julian date is precisely this value. And from where this number is coming; this number is coming from the Julian Epoch and Julian Epoch it is referred to some date before Christ. So, we will continue in the next lecture. So, thank you very much.