Space Flight Mechanics
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Lecture No - 74
Time (Contd.,)

Welcome to latest 74. So, we have been discussing about the time. We have already done the transformation part and if required little bit more on the transformation I will discuss but today we start with the time.

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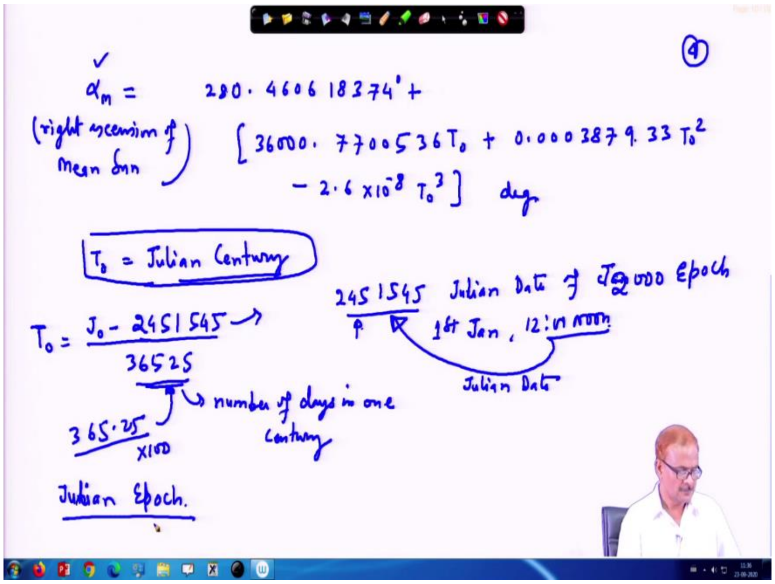

So as we discuss they are basically 4 times scales involve one is 4 times type of times. What are you stated that we have the time which is involved in the equation of motion which we have called as the dynamical time then one time is associated with Vernal Equinox that we are called as a sidereal time once time related with the movement of the sun that we are called as a solar time. So we have time and then we have this sidereal time, solar time sidereal time and then dynamical time and the last one we have discuss it was the atomic time.

So brief review I am just because it is not possible to go into the details of all these things but whatever I discussed it will be very useful and rest of the things for this part you can refer to the book by Vallado, Astrodynamics. Greenwich means sidereal time if you are measuring from the mean of date position if we are measured from the mean of date position along the true equator. So we are getting the GMST.

And if we are measuring from the true of date along the vernal equation equinox true of date from there if we do measurement along the true equator so we get the AST Greenwich mean sidereal time another was apparent time so these 2 issues we have discussed so I will return back to those things again. For GMST we have written as α is plus hour angle of sun and in which this we have written in terms of UT - 12 where hour angle of sun is UT - 12.

So this is in terms of hours when we are telling here GMST, and I am showing here the angle. It does not matter so in terms of angles or hours is the same. So, ultimately it will convert this in terms of angles because we required all the things in terms of angles here for the conversion. So, UT was the universal time and the part of this UT_1 we will be utilizing here in this place. So we have 2 parts of this available. The other party is not used for UT_1 and UT_0 these are 2 things we used. So this is associated with the celestial ephemeris pole CEP that means if we look into the transformation. As I have told that this is ITRF International Celestial Reference Frame so this is the frame fixed in the figure of the Earth.

It is rotating along with the earth but it is a fixed but it is not the spinning one and the other point which we have called as the CEP is the point of the location of the pole after we have given the Precision nutation and the rotation of the earth corrections ok so there after this pole is of the Earth is located here in this place. So this is the axis about which the earth is rotating. So this is the spin axis what is strictly speaking this is not just spin axis but this is rather the angular momentum axis.

This is not this spin axis in true sense. But here hugely we will take this has to be the spin axis and because I will have to go into the dynamic to explain all the things so I skip I choose to not to discuss all the things rather that tell that this is a spin axis and this is spin axis it will keep moving in the figure of the earth. So is this is a feature of earth sometimes spin axis is here and sometime it is here sometime it is here that time it may be here or that time it may be here.

So it is shifting and it is shifting in a square of 30 X 30 meter around the pole. So it is about this it will be shifting in an area of 30 X 30 and correction already we have applied in terms of x_p and y_p in which we have called as the polar rotation. Polar rotation basically as I told you it arises from the dynamics which is involved with $\mathbf{I\dot{\omega}+\Omega I\omega}$. So this is the equation the Aerial dynamics it is written like this and this is the torque on the right hand side and if we shift this torque to 0.

And if we solve this equation find that equation. It appears how the axis will move. I do not want to discuss that particular issue at this stage that is no way scope also here for discussing it. But this x_p and y_p we have already taken into account in the polar motion. So, this we have written as TRO are something the rotation matrix we have used. So we are discussing this part $\mathrm{GMST} = \alpha_s + \mathrm{UT} - 12$ and if we write the same thing in terms of angles for all the things will be in terms of angle.

And this 12 will become 180 degree and UT is universal time. So this also we need to convert to angle. So this is in times of time and here it comes in terms of angle.

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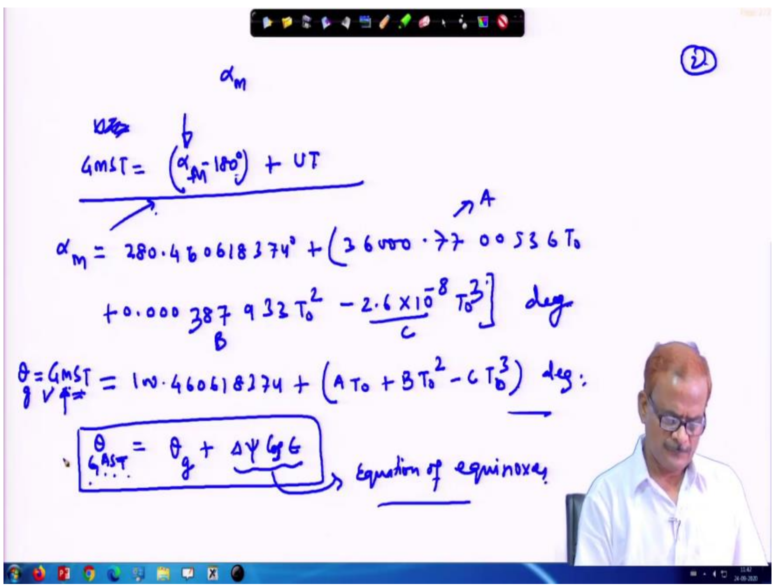

And therefore we have written UT equal to or say for the GMST equal to UT - or α_s - α right ascension of this one we go back. So this is your right ascension. So α_m – 180^o and Plus the other quantity which we have written here α_s + UT. So, this UT is added to this and this quantity α is is known to us this we have written last time. So alpha S or α_m the symbol is I have used this is the mean Sun and therefore it I might have written here is α_m.

So let us continued with α_m rather than the α_s and this α_m this quantity we have written in terms of angles raised to

$\alpha_m = 280.460618374^{\circ} + (36000.7700536T_0 + 0.000387933T_0^2 - 2.6 \text{ X } 10^{-8}T_0^3)°$.

So, this is your α_m so if you insert here in this place. So,

$$GMST = 100.460618374 + (AT_0 + BT_0^2 - CT_0^3)°$$

So, from here GMST is available and this in if you are writing the right hand side terms right hand side in terms of angle and left hand side also you can write in terms of angle. So this is θ_G. So that give the location of the Greenwich Meridian with respect to the vernal Equinox mean of date because you are this is the mean, Greenwich mean sidereal time and therefore this is with the mean of date.

True of date vernal Equinox and then we have added the equation of equinoxes to this so theta AST, GAST in which the apparent sidereal time this we have written as θ_G or GMST whatever you want to write this symbol does not matter much. If you write consistently it is;

$$\text{GAST} = \theta_g + \Delta\psi \text{Cos}\varepsilon.$$

This part I have written as the equation of equinox. So, from here the rotation angle of the Earth is available to you which will go into the spin motion of the earth.

First we have given the precision correction and then we have been given the nutation correction and then we are giving the Earth rotation correction and then we are giving the polar motion correction and then this is your $\vec{r}$ with just get vertex from the just Fk5 or the inertial frame as your choosing say accordingly the things will produce matrices will differ. So, from there it will gets converted to $\overrightarrow{r_{PEF}}$.

PES stands for Pseudo Earth fixed here the pole the CEP shown here so the corresponding axis that I choose this x, y and z year. So this is referring to your pseudo earth fixed and this point is called as the CIO where you have the ITRF the celestial intermediate origin. And its location of CEP with respect to ITRF it is decided by x_p and y_p which we have written here. So, if you do not want to go into all the naming and other things which pole and which origin it may be difficult to remember even sometimes say it is ok. But the concept wise it should be clear that what we are trying to do.

So this way, our rotation angle is available to us for the matrix, the rotation matrix. We have written as you can see that there we had this as

$$R_s = \begin{bmatrix} Cos\theta_{AST} & Sin\theta_{AST} & 0 \\ -Sin\theta_{AST} & Cos\theta_{AST} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{So, } \theta_{AST} = \theta_{GAST} + l$$

It is referring to the location of your meridian the ground the station where you are located and observing the satellite.

So this will be written as theta GAST + λ or let us say this λ is built in terms of l and this we call the longitude of the ground station. So add this and then you get the final θ_{AST} which will enter here this place. But $\dot{\theta}_{AST}$ which we get during this rotation matrix or again the symbol I do not remember this symbol I have used but let us say this is this spin. I write this is the R_s.

So the R_s dot matrix differentiation in that your $\dot{\theta}_{AST}$ appears but in this matrix $\dot{\theta}_{AST}$.

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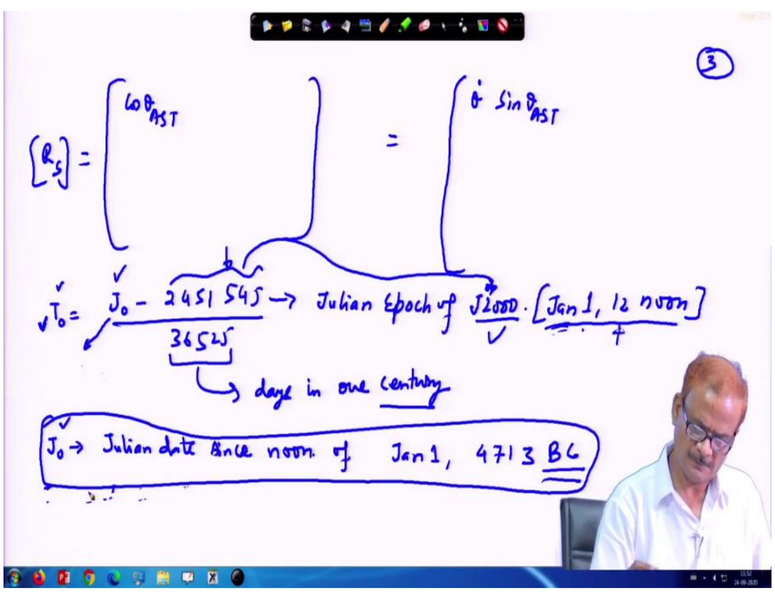

Let us go to the next page. So in the matrix ones, we are differentiating this quantity $Cos\theta_{AST}$ this is your $R_s$once we differentiate this so we have written this as $Sin\theta_{AST}$ times $\dot{\theta}$. So the $\dot{\theta}$ what is written here. This is nothing but your quantity that we have written in the previous lecture. That is the rotation rate of the earth. But the angle is available to you. From angle the total angle θ_{AST} becomes

$$\theta_{AST} = \theta_g + \Delta\psi \text{Cos}\varepsilon + 1$$

Here, $\theta_g$is given by this.

If we differentiate this with T, now this here also this appears as a function of T. So if you differentiate this quantity, you will see that it will come in terms of and where T_0 it appears in terms of the Julian date, which we are going to write later on. What the quantity is the T_0 is defined in terms of Julian date. So next let us complete this part $\dot{\theta}$ we have to write and this we have written earlier. So let me first complete the other part.

$$T_0 = \frac{J_0 - 2451545}{36525}$$

So these are days in one century. In one year 365.25 days are there; and therefore this message 365.25 into 100? This is the Julian Epoch of J 2000 that means the January 1, 12 o'clock noon. At that time what is the Julian date of that particular time and why it is taken to be the 12 noon the reason for this is that while if you work with this for in the night; while the astronomers are doing observation so date change does not take place that is why this is done like this.

If date changes then they have trouble so they continue with the observation in the same date and therefore this measurement is done from the well known. Zero is the quantity this is the Julian date since noon since noon of January 1, 4713 BC before Christ. And for finding the Julian date for any time. So there is a particular equation, many equations are available and we can use one of them for sorting out the problem. So Julian date at any point of time this can be written as J_0 what we have written here.

So the J_0 is the Epoch of J2000 + UT say what will do here on that particular you are looking for Julian date on a particular date. Let me go to the next page and write this part properly. So here this part is; J_0 is the Julian date noon of January 1, 4713 BC. So from there this is counted. So suppose as of today we are counting this. And this is the Julian Epoch of J2000 so from this we are subtracting and this then we have written this T_0.

And this T_0 is playing role in the GMST it is playing its role here. So date for today I will write it on the next page.

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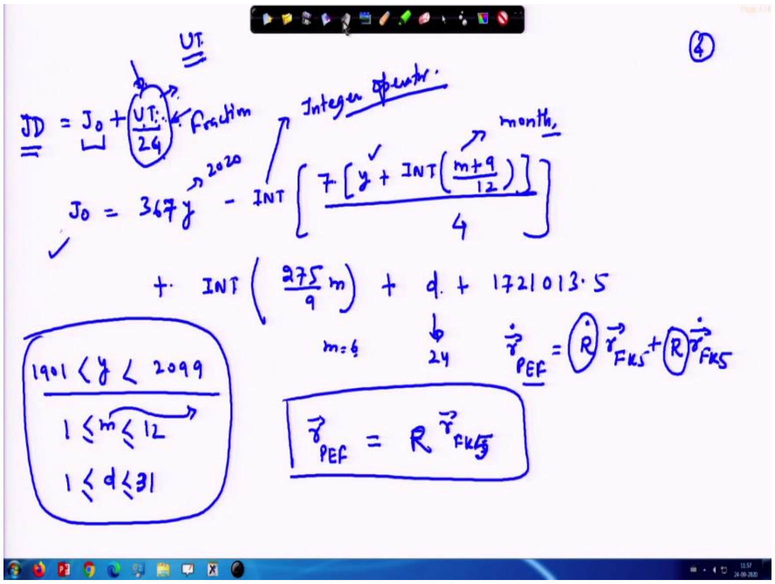

This J_0 can be already we have equation 310 so J_0 + UT divided by 24 this will convert this universal time into the Julian date. If I consider at this instant if the UT is given the universal time for this instant is given to me divided by 24 converted to day. So how many days already past it will be obviously fraction and add to this J_0. So this will complete the job and where J_0 will be given by 367 this is the year y stands for year here - INT.

$$J_0 = 367\,y - INT\left[\frac{7\left[y + INT\,\left(\frac{m+9}{12}\right)\right]}{4}\right] + INT\,\left(\frac{275}{9}m\right) + d + 1721013.5$$

INT this is the integer operator means any number having fraction it will convert to the integer. This is not the only equation which is applicable other equations are also available for converting. This year M stands for month + number of days ok because so why this kind of very peculiar equations is coming in is because every month is not of the same length that is the simple reason.

So day + 1721013.5 where y lies between 1901 this equation is valid for this 2099. If date goes beyond this, this equation will not be valid, m lies between 1 to 12, so depending on the month you will put here m say if it is June so I will put m equal to 6 here in this place if it is today 24, so I will put here 24. This is the year 2020 so 2020 will be placed here in this place and they all fall within this range. And to this if this UT the time will be given to you at what time you are looking for.

If you add this then this gets converted into Julian date and this Julian date is important for us to work with. So once we have done this part. Then we have done the rotation part. So this part is over the polar motions are already we have covered. So, the complete rotation for converting $\overrightarrow{r_{FK5}}$, $\overrightarrow{r_{PEF}}$, pseudo earth fix given write in short like this and this we have written earlier. So $\dot{\overrightarrow{r_{PEF}}}$ velocity of the point of the satellite you are considering in the pseudo earth fix term this will be given by

$$\dot{\overrightarrow{r_{PEF}}} = \dot{R}\,\overrightarrow{r_{FK5}} + \mathrm{R}\,\dot{\overrightarrow{r_{FK5}}}$$

And you know of course, what is this matrix are this already we have written last time so no need of mentioning it again here.

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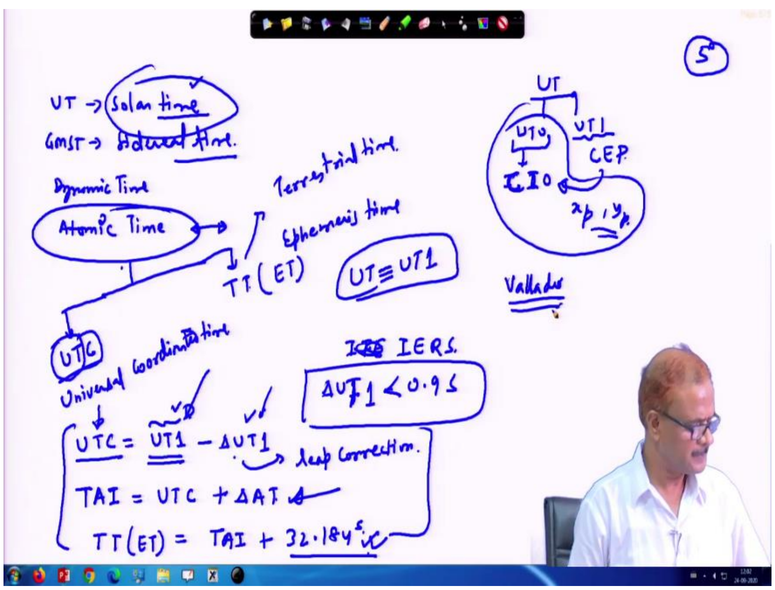

So as we mentioned earlier that UT this is connected with the solar time. And while GMST this is related to your Sidereal time. And then we have the dynamic time and atomic time. So atomic time this is the general time keeping method for; by different agencies and this is the time you get also on your mobile you will see that on your mobile automatically gets updated. This is maintained precisely and I have written about this because this is due to the oscillation in the energy states of the CGM atom.

So atomic time event it can be divided into many parts and we are not concerned with all of them. But one time is written as UTC is again, you do not get confused with this it is related to the Solar time. This is UTC and the universal coordinated time. Another one is written as TT and also is written as the ET ephemeris time. This is the Terrestrial time both are same thing. And they are connected with each other. So UTC-Universal coordinator time is connected with the UT_1 universal time.

Now this remember the UTI told that I will divide it into two parts one is UT_0 and other is UT_1. So this is related to celestial ephemeris pole while this is related to the your what we have written as intermediate UTI celestial intermediate origin. So to this we will give go after the giving the correction for the x_p and the y_p. These issues after I am not taking here into account because we do not have Time to get into all those things, but remember this UTC also its related to UT which is nothing but your solar time.

And solar time also we have connected it with the sidereal time that how they are connected through equation that we have written. If we subtract certain amount of quantity delta UT_1 this quantity is always less than 0.9 seconds and these quantities are supplied by it International Earth rotation services IERS supplying all these quantities. So if you give this correction to this and UT at a particular time when you were nothing told UT implies that this is UT_1 if nothing is written.

We will always assume that UT equal to UT_1 and these are called leap corrections and there is a jump and you will give certain correction you come to the universal coordinated time and then the atomic time TAI. This is connected with

$$TAI = UTC + \Delta AT$$

again this quantity supplied to you. And the terrestrial time of the ephemeris time this is

$$TT\ (ET) = TAI + 32.184\ \text{sec.}$$

Terrestrial or ephemeris time so; these are the properties of relations which are used.

Till now what we have looked at for modelling the rotation of the earth we require the time also that the time is measured in the Julian date as the UT or the UT_1 you know that this is connected with the UTC these are the corrections which are available from the IERS it is a given this quantity is always fixed 342.184 and for this details you read Vallado. So we have done this part then what we are left with.

Last part what we are left with was $\dot{\theta}$. So the theta dot appearing here $\dot{\theta}_{AST}$ what we have written here.

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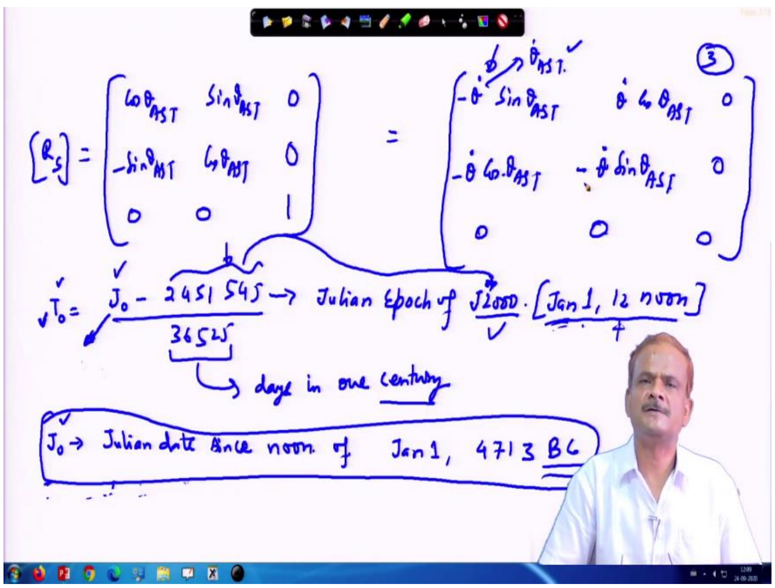

This we have written as

$$\dot{\theta}_{AST} = 7.292113\ 146706\ X\ 10^{-5}(1 - \frac{LOD}{86400})\ \text{rad/s}$$

The whole thing will come in Radian per second and this is 1 - LOD divided by 86400 a number of day number of seconds in one day. LOD is called as the length of day the basic symbol but the here it stands for LOD is the instantaneous.

This is the instantaneous rate of change of UT_1 that means it is the rate of change of UT in seconds with respect to a Uniform Time Scale which is the uniform time scale this is your UTC or TAI stands for the international atomic time. And this is uniform because it is done precisely using the atomic clock and its where as in the case of the UT this is measured with; this is described with respect to the motion of the sun.

So this is the mean motion of the sun and that is not uniform. So therefore it is written here the rate of change of UT_1 in seconds with respect to the uniform time. Which is your in this case the UTC or the TAI so this is available from IERS International Earth Rotation Services, which is based in France you can download all the software and the files from that place. One particular file this is the sofa the name full name.

I do not remember but here are you get all the subroutines how to apply them for finding bring the conversion. And all the programs are written in Fortran and besides this you will find many documents which are there conference documents which are published for the convention they are adopting over a period of time. They are up grading different convention making it more robust more precise more rational.

So those inside for business documents also you get and also the subroutine are available. So with this I hope that the things are clear now that the angle which is appearing here this angle we differentiate. So the $\dot{\theta}$ is available from this place. $\dot{\theta}$ is nothing but $\dot{\theta}_{AST}$this is available to earth. There is another concept as earlier told you in the last lesson. The Stellar angle is not the same as the θ_g we have written earlier. It is a different.

So cos once we differentiate then there will be a minus sign a here and then we had the $Sin\theta_{AST}$with Plus sign similarly we get $\dot{\theta}$ $Cos\theta_{AST}$. But here this plus sign then there is this quantity gets to 0. The other quantity we have designed is -$Sin\theta_{AST}$ again we got your $\dot{\theta}$ times sin, sin becomes cos this is $-Sin\theta_{AST}$and here in this place this was $Cos\theta_{AST}$.

So this gets converted to $\dot{\theta}$ $Sin\theta_{AST}$ and this again computed – T. With all these information this is process is complete. Now only thing that the numerical can we set it but those in numerical see if you use the matrix method it will be long multiplying and the so for the exam is not possible. But the conceptually we will deal with this. So, in the exam will not have the numerical if you want to do look into the numerical you look into the book by valado.

Though there he has fixed some of the problems about the time also about converting someone from one Frame to another friend. And it is a nicely describe there any time we pick up your calculator in work with this will be able to do this. But for example will not do this part, only theoretical aspects or the conceptual things will look into the exam and neither in the tutorial. We will have that kind of thing because this is just one week a lecture and we are accepting that by much larger amount ok so this we stop this here and then we will go to the orbit determination problem. So thank you very much.