Space Flight Mechanics
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Lecture No - 75
Orbit Determination

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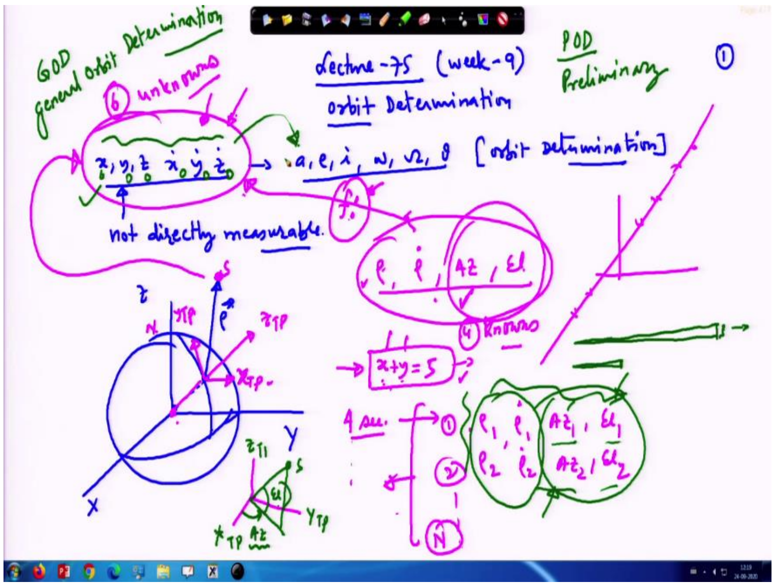

Welcome to lecturer 75 the last lecture we have discussed little bit about the time. Today we are going to discuss about the orbit determination in this Lecture. Let us first state what is the orbit determination? As has already you are learnt that given xyz, $\dot{x}$, $\dot{y}$, $\dot{z}$ find out a, e, I, ω, Ω and θ. So, you are trying to find the orbital parameters from the Cartesian coordinates.

Now here in this case xyz is not directly measurable. What do we measure from the Earth? This is our XYZ the inertial frame so somewhere on the ground, somebody sitting and making this observation. So he is making is doing the observation for, so what he is doing? That he is trying to get people let us first describe the axis reference frame this is X_{TP} topocentric and this is Y_{TP}. So, Y_{TP} points toward the north and Y_{TP} points towards the east and Z_{TP} vertically radially outward.

So, in this observation is made and the satellite is located here. So as a whole what we can do that I can measure the range, range rate, $\dot{\rho}$, azimuth and elevation these are 4 things can be measured at most. That no way xyz and $\dot{x}$, $\dot{y}$, $\dot{z}$ can be measured. That is the in the inertial frame can we have directly this Cartesian coordinates xyz and $\dot{x}$, $\dot{y}$, $\dot{z}$? No it is not possible. So therefore to get this we measure this and then try to estimate this one.

And as you know from your basic mathematics that if you have $x + y = 5$ so there are two unknowns and only one equation involved so here in this case you can have infinite number of solutions. This represents basically a line here you have any line in this place. There are many points which will be satisfying the equation of this line. This kind of problem we called as it indeterminate problem. You cannot work it out, you have infinite number of solution.

So, similarly here in this place they are connected by certain equation let us call this is the matching F which maps from your observation to the Cartesian coordinates from observed value to Cartesian co-ordinate values. But this map in is available to us and that is available in terms of equations. But has you can see that here in this case we have one equation and 2 unknowns we cannot solve. So similarly here in this case. We have four unknowns 4 known. These are the other known and these are the unknown.

There are 4 known and there are 6 unknown. So from 4 we cannot determinate it; we can get infinite number of solutions. So then what to do, so then we take many of the research observation. So we can have a series of the observation say our satellite is there every 4 second I am making one observation for everyone second there is one observation. This observation then can be queued up and let us write this has ρ_1, $\dot{\rho}_1$ and Az_1 and εl_1.

Similarly the next observation we make we write as ρ_2, $\dot{\rho}_2$ and Az_2 and εl_2 and this way we can have N number of observations. Now using this N number of observation it is possible that we can estimate xyz and $\dot{x}$, $\dot{y}$, $\dot{z}$ as closely as possible. But let say that these things are obstructed by errors in measurements you know the angles are troublesome quantity. If you are measuring angles from this place, this is your topocentric reference frame and the satellite is located here.

So this is X_{TP}, Y_{TP} and Z_{TP} and we measure angle here this azimuth (Az) and this is elevation (εl). So these quantities are not accurately measured if you do little bit of error here and say you satellite is you are looking for the satellite at moon for Chandrayaan 1 and Chandrayaan 2 so in that case happened? You will be creating a lot of error with the azimuth and elevation. If you use this and because it is small error in this it will create;

If you have a line if you have a shorter line say and you rotate it by small angle here say 1^{o} so you will be creating this much of error. On the other hand if you rotate the same thing by 1^{o} so you will be creating this much of error. You know that the distance increases these angles will create more trouble so in the actual estimation once we do the general orbit determination GOD orbit determination so at that time. We do not account for the as the angles rather we work with the difference range and range rate.

And you may have multiple ground stations from where you are measuring the this doing these observations and then they can be merged together to find out the value of xyz, $\dot{x}$, $\dot{y}$, $\dot{z}$ at different instant of time or maybe the inertial point of time when we can write this as x_0, y_0, z_0, $\dot{x_0}$, $\dot{y_0}$, $\dot{z_0}$. So our objective here is to look how this can be done that if these quantities are given here.

Here in this case will take all this 4 quantity because if we do here for this earth satellite so in that case and that too for the preliminary Orbit determination P stands for preliminary. So for preliminary orbit determination we can take this just at the instant. The satellite is injected into the orbit and after that you need to know the in which Orbit the satellite going that can be calculated using if you know the Cartesian co-ordinate correctly, then only will be able to evaluate this quantity a, e, I, ω, Ω and θ.

So for that reason; if we do here for all of them and what I will be doing in a brief because again it is not possible to do all the; in the great details.

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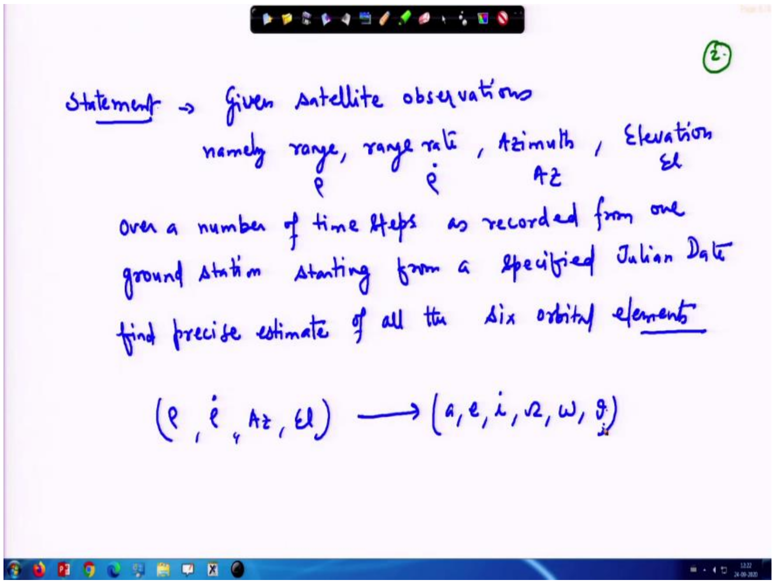

The statement for the orbit determination we can write like this. Given satellite observations namely range which we are indicating by ρ, range rate by $\dot{\rho}$, Azimuth (Az) and elevation (εl) over a number of times steps as recorded from one ground station, multiple ground stations may be involved, possible. So from one ground stations starting from a specified Julian date find precise estimate range rate may not be 100% accurate.

But as closely as possible find precise estimate of all the six orbital elements. The problem gets reduced to $(\rho, \dot{\rho}, Az, \varepsilon l)$ is given from there we need to find a major axis eccentricity, inclination, nodal angle, argument of perigee and theta, which is the true anomaly at that instant of time.

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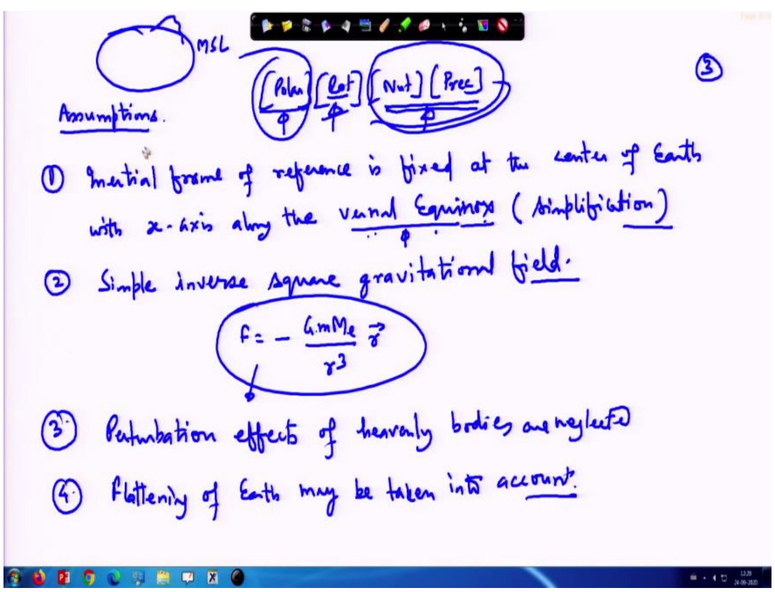

And what are the assumptions that we need to make? Inertial frame of reference is fixed at the centre of earth with x axis along the vernal Equinox. This may be a simplified measurement see if in the previous thing the precision and correction we are given and then the nutation correction we have given and thereafter the rotation and then finally the polar motion correction we are given. So, quite often if you are doing for a short period of time so this can be neglected nutation and Precision.

The polar rotation also you can neglect only based on this rotation the mean the spinning of the earth you can work with. But as we have decided described that the inertial frame Fk5 or either the International Celestial Reference Frame or it may be the EME2000. So we need to choose any one of them and with respect to this how your Earth is currently oriented that you need to work out if you want to go in a precise way.

If you do not want to go in a precise way then just you need with respect to the vernal Equinox what is the current orientation of the earth that is all. So in that we neglect the nutation and Precision correction and also the polar correction we can delete and simply we can keep the rotation part here. So this is simplification. So inertial reference frame which vernal Equinox is not mentioned here but we know that what is the direction of the x?

So in the case of EME2000 which is pointing toward the vernal Equinox J2000 from there we are coming to the mean of date and then from there we are coming the true of date then giving rotation and then giving the polar motion. But in this way, this is a simplified way of stating it. Simple inverse square gravitational field that means that as you will already have covered the general perturbation theory there are many infinite number of terms appears in the gravitation modelling.

So, especially due to the earth those part we are just neglecting and writing here that F gravitation maybe just retain as GmMe earth divided by y cube r this the simplest gravitational model to be used because this does not complicate the whole problem still we understand what we are trying to do. Perturbation effects of heavenly bodies are elected. So this is obvious from this point, perturbation effects of heavenly bodies are neglected.

Flattening of earth may be taken into account may or may not be taken into account. But see here if you do not take flattening of the earth into account that it will introduce a larger error. So flattening of earth will be taken into what is the altitude at which you are situated? This is the Earth model you are seeing, so this is the mean sea level and you may be sitting on somewhere on a hill from the top of the hill then you are observing you have to do the correction for all those things.

We have to without that it should not possible just taking the radius of the earth and then working. So your measurement whatever you are doing and trying to estimate, your estimate will not be correct. It will introduce large amount of error.

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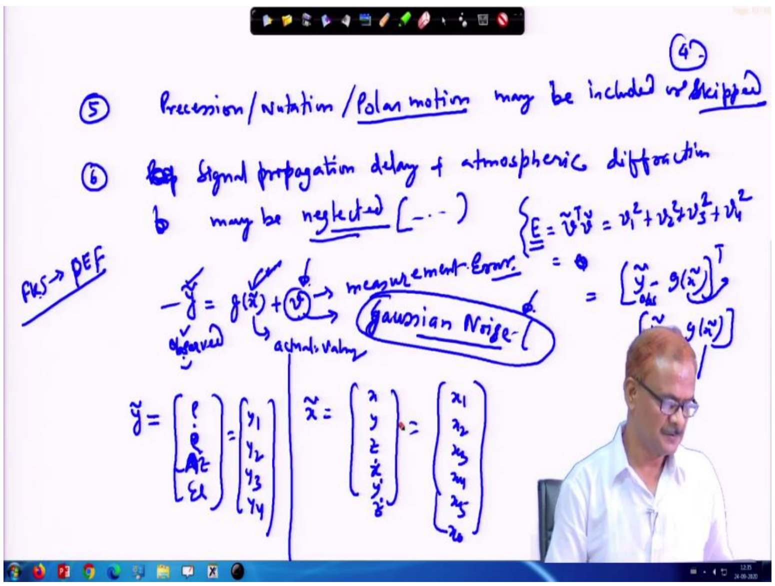

So precision, nutation and polar motion this may be included may not be included depending on your move. Polar motion this is not the same polar motion as we seen included or skipped. In actual process once the measurements have been done. So in the signal delay is a deflection in the signal once it is passing through the atmosphere it may get reflected by many layers of the earth atmosphere. Somewhere it is cold or somewhere it is hot. The density of the atmosphere is varying.

So for that we have to introduce the corrections which we call as a signal propagation delay and atmospheric diffraction may be neglected which we want to account. Normally once the ISRO is working or NASA is working or other agencies are working they take into account all those things.

So the data is measured in the computer all these corrections are given and after that what we get it is not purely raw data, but it is having measurement errors, errors are still present.

If I say that what does mean by measurement error? If I say that y = g(x) is the relation between the observed value this is observed value and this is the actual value. For relation between not observed value and this is observed value just like this is a Cartesian coordinates. The coordinates are there we can write it and here this is a ρ, $\dot{\rho}$, Az and εl as such. Once you observe this naturally they go with this the error, this call the measurement error.

And the estimation Orbit determination, it involves estimation theory, filtering theory. They list as for as estimation and Kalman filter that so many varieties available. So those are used for estimating this x from this y in the presence of this noise and this noise is assumed to the Gaussian noise. Gaussian noise because its distribution will be Gaussian and some assumption can be made but here those things become very important once you go into the filtering area like the Kalman filter and other things.

We have more assumption in that involved, but we are not going to tackle all those things. I will I am trying to show you the concept behind this to actually implement you need more rigorous work on this and a more rigorous lecture also. So the ultimately the objective then becomes and if I put a tilde means these are vectors. $\tilde{y}$ in this case? This consists of

$$\tilde{y} = \begin{bmatrix} \rho \\ \dot{\rho} \\ \mathrm{Az} \\ \varepsilon\mathrm{l} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix},$$

This is one observation and this consists of

$$\tilde{x} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix},$$

So this dot we will keep it here and we can write this in terms of x_1, x_2, x_3, x_4, x_5, x_6 and similarly this can be written here as y_1, y_2, y_3 and this becomes easy to handle. So in the estimation list as far as estimation what is the objective that you reduce this error. Error function defined like this,

$$E = \vartheta_1^2 + \vartheta_2^2 + \vartheta_3^2 + \vartheta_4^2$$
$$= [\widetilde{y_{obs}} - g(\tilde{x})]^T$$

This implies this is nothing but a nu 1 square nu 2 square nu 3 square + nu 4 square and this quantity can be written as $[\tilde{y} - g(\tilde{x})]^T$ times are $[\tilde{y} - g(\tilde{x})]$. So objective is to minimize this error.

If you minimize this error then using this observed values this unknowns x can be estimated. This is the whole orbit determination process involved in nutshell. And I need to elaborate already I have taken the conversion from the inertial frame to the Terrestrial frame we have done the conversion from the Fk5 to PEF pseudo earth fixed PEF we have written we have converted from Fk5 to PEF and this is one of the method.

And as I told you that this is Fk5 reduction. We can go for the order reduction you have the ITRS either non rotating origin concept. But obviously that requires the discussing all the things it require to shortlist 20, 30 lectures to cover in details. We are not working with all those things here and simply we will look into the basic concepts what we are what is all about the orbit determination.

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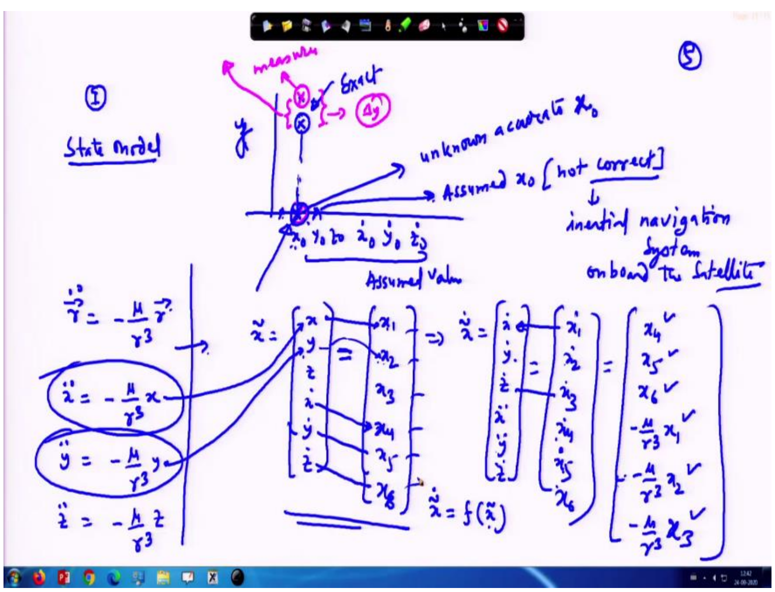

The state model that means what we will do that the we assume that we have the initial Cartesian coordinates available x_0, y_0, z_0, $\dot{x}_0$, $\dot{y}_0$, and $\dot{z}_0$ these are assumed values this is not correct value. In one dimension I can show it like this that; if I show here the ρ , $\dot{\rho}$, Az and εl something like this and say only I show here y ok this may be the exact value of y and your measured value may be somewhere here.

This may be your measured value. So therefore this is the amount of error is already introduced in the measurement. Moreover this constitutes your Δy error in measurement and here this is the corresponding to this x_0 this I will do in blue. This is accurate x_0 and unknown accurate let us say x_0. So corresponding to this, this is the exact value of the x_0, but this is again not known to me. I may be assuming it to be somewhere along this line say I differ from this point and I come to either on the left hand side either I go on the right hand side.

So these are wrongly measured value. So this is the assumed x_0 not correct and these are generally available from the inertial navigation system onboard the satellite and that gives you wrong value. If it is giving wrong value to using that wrong value, then you have to get to the correct value you have to estimate this. And once you estimate this correct value then you will be able to determine the orbit.

So first we have to assume the, for this purpose we have to assume the state model. So state model is our basic as per assumption this is $-\frac{\mu}{r^3}$ and then this can be written as $\ddot{x}$ in the scalar form, $-\frac{\mu}{r^3}$x and $-\frac{\mu}{r^3}\ddot{y}$ and this we need to put in the exactas format.

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r}$$

$$\ddot{x} = -\frac{\mu}{r^3}x$$

$$\ddot{y} = -\frac{\mu}{r^3}y$$

$$\ddot{z} = -\frac{\mu}{r^3}z$$

So, let us assume that it is $\tilde{x}$ we represent as xyz, $\dot{x}$, $\dot{y}$, $\dot{z}$ in this we write as x_1, y_1, z_1 sorry it is x_1, x_2,x_3, x_4, x_5, x_6 this is the notation used those who have done already controls course so they may be aware of this transformation.

So, this implies $\dot{\tilde{x}}$ equal to $\dot{x}$, $\dot{y}$, $\dot{z}$ and $\ddot{x}$, $\ddot{y}$, $\ddot{z}$. So $\dot{x}_1$,from here what we will observe $\dot{x}_1$ is nothing but $\dot{x}$ if you look here in this place this is x_4. So, this write as x_4, similarly the $\dot{x}_2$ is $\dot{x}_5$ so $\dot{y}$ is here is x_5 so it goes as x_5 here in this place and $\dot{x}_3$ is $\dot{z}$ is here x_6 so this goes as x_6 and there after we have $\dot{x}_4$ which is $\ddot{x}$.

So $\ddot{x}$ is available to me from this place I will write this as - μ/r^3 and x then becomes from this place x_1 so this is x_1. Similarly $\dot{y}$, $\dot{x}_5$ is nothing but your $\ddot{y}$ and $\ddot{y}$ form this place is - μ/r^3 ,y and then y the quantity which is present here so this y is directly related to x_2. This x is related to this point and this related to x_1 only y is related is given here which is nothing x_2.

So this will replace as x_2 and the last one similarly the z will be x_3. In short notation we can write $\dot{\tilde{x}}$ is equal to call to f($\tilde{x}$) because y, because it is a function of x_1, x_2, x_3, x_4, x_5 and x_6 there $\tilde{x}$ is nothing but the quantity we have shown here in this place x_1, x_2, x_3, x_4, x_5, x_6 and

$$r = \sqrt{x^2 + y^2 + z^2}.$$

So with this I conclude today and in the next election will continue with this.

So this is the state model which is available to us and we will try to put in a proper format and let us see how the things evolve. And the process of orbit determination other things you can refer to the Hobbit Curtis book on space dynamics for engineers. There the theoretical methods of orbit determination. There are many other great books written which discusses completely the how the theoretical way of doing and theoretically done.

But whatever we are doing here it so the computational method and this is the method applied in reality in the orbit determination problem if we have to find the orbit of the satellite. So this is the method that we go. Thank you very much.