Space Flight Mechanics
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Lecture – 78
Orbit Determination (Contd.)

Welcome to lecture 78 and we have been working with our with determination problem in that context I showed you how to get the correction in the initial state. Now I will return back to that again and that was particularly for one observation I have shown but we have multiple observations in that case. How do we do it?

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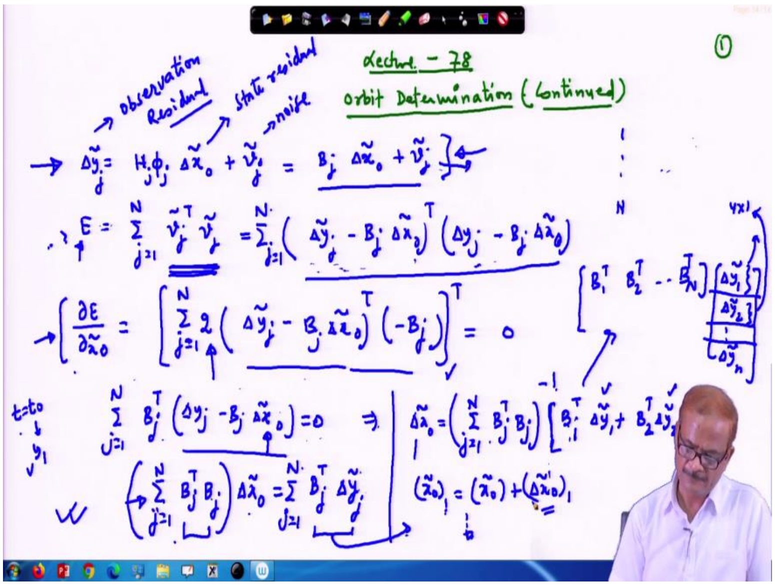

$$\Delta\tilde{y}_j = H_j\varphi_j\Delta\tilde{x}_0 + \tilde{v}_j = B_j\Delta\tilde{x}_0 + \tilde{v}_j$$

So let us go to that. So see for the same equation and linearised equation it is shown here in this place. This is Δy_j and this also be called as observation residual this is called state residual. This is your noise vector and we have casted in this format. Now if we have multiple observations this error v_j which is appearing here this must be minimised over all the observation. So we are making from 1 to total number of n observation.

So I have here some remark. So here I have taken a summation of this over the n number of observation and so I can put here it in this format till this; this is ok. Now again if we differentiate it you can check or either you can refer to some book on linear algebra. So if you differentiate it, it can be casted here in this format. This is from the previous one only. By putting it differentiating it like this and these two factors comes because there are two similar factors present here.

$$E = \sum_{j-1}^{N} \tilde{v}_j{}^T \tilde{v}_j = \sum_{j=1}^{N} \left(\Delta\tilde{y}_j - B_j \Delta\tilde{x}_0\right)^T \left(\Delta\tilde{y}_j - B_j \Delta\tilde{x}_0\right)$$

It will come here in this format. This is in the transpose format. And if you take the transpose of this so you can reduce it to this format. From here our work starts now the summation j equal 1 to N B_j^T B_j times $\Delta\tilde{x}_0$ this quantity will be equal to B_j transpose B_j transpose times Δ times B_j^T times tilde and information is equal to 1 to capital N with j here the subscript j is present in Δ y_j.

$$\frac{\partial E}{\partial \tilde{x}_0} = \left[\sum_{j=1}^{N} 2 \left(\Delta\tilde{y}_j - B_j \Delta\tilde{x}_0\right)^T \left(-B_j\right)\right]^T = 0$$

Now look back here in this equation we got it in the same format this together it is a summation. Summation is present v transpose z, B transpose v time α_0 we have got it in the this format. But for because the summation sign is here so we need to put it in a proper format so that we can work it out Δx_0 is separately. Δx_0 is common as you can see this is at the initial state only and therefore this can be taken as a common here in this place.

$$\sum_{j=1}^{N} B_j^T \left(\Delta\tilde{y}_j - B_j \Delta\tilde{x}_0\right) = 0$$

So multiplied is 2 matrices summit up over all the observation and similarly we you have the matrix here who submission is required and then this can be worked out. Therefore $\Delta\tilde{x}_0$ this will be equal to summation B_j transpose B_j equal to 1 to N this inverse and then you have to write for this matrix. And this matrix can be summarised as you can write here in this format B_j^T times summation it will be B_j^T times Δ or let us say we can expand it and write it here 1 times Δ y_1 B_2 these are all different time instant.

$$\sum_{j=1}^{N} B_j^T \left(B_j\right) \Delta\tilde{x}_0 = \sum_{j=1}^{N} B_j^T \Delta\tilde{y}_j$$

$$\Delta\tilde{x}_0 = \sum_{j=1}^{N} (B_j^T \left(B_j\right) \Delta\tilde{x}_0)^{-1} [B_1^T \Delta\tilde{y}_1 + B_2^T \Delta\tilde{y}_2 + \cdots]$$

So, ok t equal to t_0 corresponds to y_1 and therefore otherwise I need put here 0 that I am not doing. So n number of observation t_0 corresponds to y_1 this you should remember. So B_2 times Δy_2 and so on and remember these are vectors. You just need to separate it out. So this quantity can be written as B_1^T, B_2^T ... B_N^T and then here it comes as $\Delta\tilde{y}_1$, $\Delta\tilde{y}_2$ and $\Delta\tilde{y}_n$.

$$(\Delta\tilde{x}_0) = (\Delta\tilde{x}_0)_1 + (\Delta\tilde{x}_0)_2$$

So from here to here this matrix is 4 into 1 vector again this is 4 into 1 vector and the same way and here to actually code it on the computer so it is very easy to do. You do not have to worry so much. In the coding in the mathematics it may appear very complex. But once you code it becomes easier. So this is the correction to be given to you are made n number of observation that you give correction to $\tilde{x}_0$.

After the first iteration I will write it like this after the first titration this gets reduced to $\Delta\tilde{x}_0 + (\Delta\tilde{x}_0)_1$ and continue with this. So if you do this estimation step of few numbers of times and you will see that your system has conserved; your system has converged. And how do we know that it has converged? You can keep estimating this error which is nothing but the observation residual.

You can keep estimating for either It is better not of your observation residual we have written at this quantity. So you can look for initially what the value of this E was and later on how it is converging. So the smaller this quantity becomes the better your estimate will be. After certain tolerance limit you can stop the iteration. Ok, so this is the process of estimation. But few of the things we are still left with that I will complete now.

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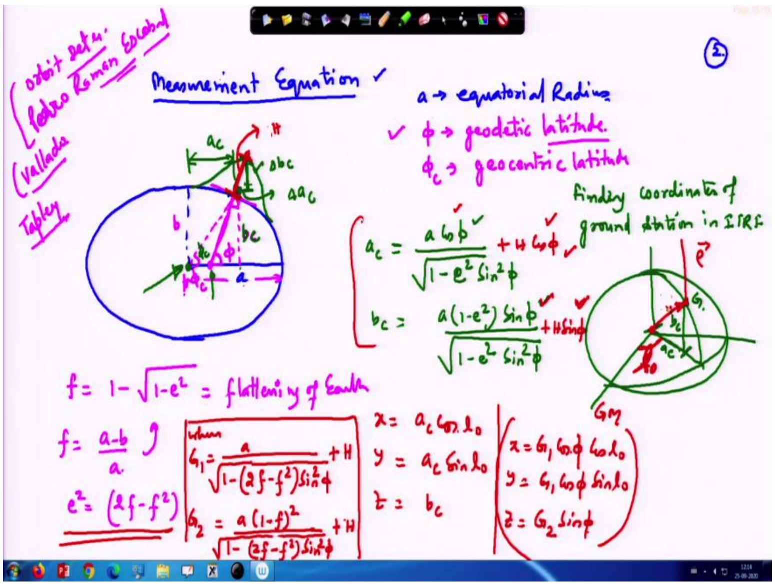

We have to get the measurement equation this we have not done measurement equation is required and it is linearization is also required. So how do we get all of them? So our Earth is little flat and from here to here let us say this distance a. And a this is called the equatorial radius equatorial radius and on the whole it is flatten. You may be situated somewhere here. So this is tangent to this and if we draw a perpendicular at this point, it will come to this point and this will angle we can write as ϕ.

And this we call as geodetic latitude ϕ this we call as geodetic latitude. From here to here if we connect and let us this we can write as ϕ_c this is called the geocentric latitude. Thereafter it may happen that your; this is a ground station it is not just located on the surface of the earth but rather it is located somewhere at certain height. If it is located at certain height so for that also the correction has to be given.

$$f = 1 - \sqrt{1 - e^2}$$

$$f = \frac{a-b}{a}$$

So flattening of the earth if it is defined as let us say that this quantity we write b this is a polar radius. So this can be written as 1- a^2 under root where f is defined as a - b divided a you can reduce it. Already you know that what is the minor axis and this is ellipsoidal shape and this is the ground station in that plane. This is basically appearing as an ellipse and this is the centre of the ellipse.

$$e^2 = 2f - f^2$$

This is not the focus is it perpendicular dropped from perpendicular at this point on the ground station if you draw it, but somewhere the horizontal axis, and this point the angle that it makes these are calling geodetic latitude. The e^2this is related to this by $2f - f^2$. There is a book on orbit determination by Pedro Raman Escobal, it is a old book but it gives you all the description also you can look into the Valladao. This is the second book where you can look for some of the description, but this is very nicely described.

And the third book on this may be Tapley statistical orbit determination by Tapley. So these three authors Pedro Raman Escobal Valladao and Tapley they were referred to this book so he will get some of these things listed here. And this is called flatting of earth and it is important to take into account all these things because if you take a wrong value of the radius then your result will get affected seriously.

So let us say this quantity that already been chosen as a the distance from this place to this place and upto this place, this distance without bc this as b_c and the distance from this place to this place this horizontal distance with writers a_c. Therefore if you look here in this place a_c and b_c it can be written here and for that you are the equation a_c is equal to the sum of the details which I am not able to tell you here because too many details cannot be accommodated in so much of small lecture.

$$a_c = \frac{a\cos\phi}{\sqrt{1-e^2Sin^2\phi}}$$

$$b_c = \frac{a(1-e^2)Sin\phi}{\sqrt{1-e^2Sin^2\phi}}$$

So this is your ϕ_c this is acos ϕ divided by 1 - e^2 $\sin^2\phi$ under root and b_c is equal to a times 1 - $a^2\sin\phi$. So a_c and b_c is known. So, what will be the co-ordinate of this ground station in the shape in orbit system in Terrestrial reference frame what will be the co-ordinate of ground station this is what we are looking for. So, finding coordinates of ground station in ITRF International Terrestrial Reference Frame.

So a_c, b_c is known now if you look into the figure like this. So here this is the ground station. So this quantity is your b_c and these quantities your a_c where is a_c and b_c are measured from the centre of the ellipse not from this point not from here, but rather from this point as I have shown here a_c and b_c. So therefore now it become c_g so this distance from the centre. This distance is a_c and this distance is $b_{c,}$ immediately you can see what will be the coordinates of your ground station.

Ok and thereafter if above this if continued to a height h. Ok if you continue to height h and this ϕ and ϕ_c they are connected together this also we have to remember they are not independent of each other. So if this is the height above this plane and this is a perpendicular we need another; this extra part this is Δb_c to be added and this is Δa_c to be added to get the coordinates of your orbit.

$$a_c = \frac{a\cos\phi}{\sqrt{1-e^2Sin^2\phi}} + H\cos\phi$$

$$b_c = \frac{a(1-e^2)Sin\phi}{\sqrt{1-e^2Sin^2\phi}} + H\sin\phi$$

You may be on a hill like this you are sitting on a hill you are in this position and the quantity here this is; this quantity is ΔH which is being measured from this point to this point. This is your ΔH or simply H or you say simply H, this is the height. So to get the final value what we need to do just add this H Cos ϕ to get a_c and here H sin ϕ to get b_c. So this is the; then coordinate of the ground station at this point.

$$x = a_c \, cos\phi$$

$$y = a_c \sin\phi$$

So this way then if you are measuring and say this is the Greenwich Meridian and from there this you writers as l_0 or rather symbols of perhaps I have used earlier Lambda or I do not remember. So maybe we can keep this as the latitude, longitude as l_0. So in the Greenwich Meridian or in the ITRF, with respect to the ITRF what we see that the co-ordinate then becomes a_c cos l_0 and this is the x-coordinate y-coordinate becomes a_c sin l_0 and z coordinates equal to b_c which comes after adding this altitude.

$$x = G_1 \cos\phi \cos l_0$$

$$y = G_1 \, cos\phi \sin l_0$$

If H is equal to$_0$ this gets reduced to the previous value. Ok so once these things are known we will not complicated and combine all the equations so that it looks very clumsy. We will leave it like this. This is enough at this stage. However, once you combine. Ok. So after combining what you will look at that H can be written as G_1 cos ϕ so that means ϕ we can take it outside, so, G_1 cos ϕ times Cos l_0.

$$z = G_2 \sin\phi$$

$$G_1 = \frac{a}{\sqrt{1-(2f-f^2)\sin^2\phi}} + H$$

$$G_2 = \frac{a(1-f)^2}{\sqrt{1-(2f-f^2)\sin^2\phi}} + H$$

Similarly ϕ equal to G_1 cos ϕ times sin l_0 and z equal to here also the sin ϕ can be taken outside. So this becomes and we can write here G_2 sin ϕ times simply G_2 Sin ϕ where G_1 equal to a divided by 1 minus this e is converted into in terms of this. So once you convert in terms of that so it looks like this $\sin^2$ ϕ under root 1 + H and G_2 is a$(1-f)^2$ $\sin^2$ϕ under root and + H this is casted in a simple format.

This exercise I have not done otherwise it will take a lot of time to combine all these things. Now we go to this point again. This is your ground station and this is the $\vec{\rho}$. And from the centre of the earth to this place, this is the radius of the earth. Let us say this is the $\vec{r}$. We will go on the next page.

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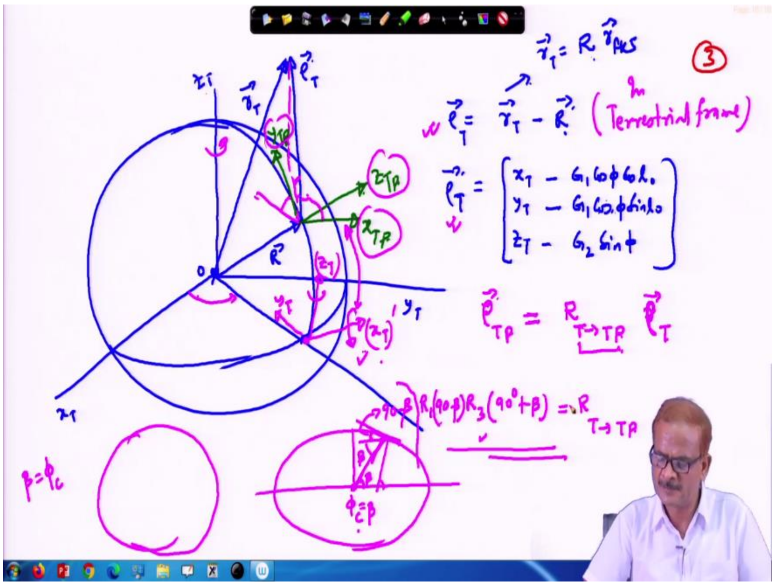

For simplicity I am not drawing the ellipsoid rather showing it in a straightforward manner. Let us say this is R and from here you have this $\vec{\rho}$ and this is the $\vec{r}$. So this is in the topocentric frame this is in the topocentric means this is x_T white and y_T and z_T and this is O for rotating along with the earth. Then the ρ in the top 10 entries from this can be written as r T – R and from where this is coming?

$$\vec{r}_T = R\,\vec{r}_{FK5}$$

This is coming from R topocentric is equal to the full rotation matrix, the Precision nutation the rotation and the polar motion that we have included a distance $\vec{r}_{FK5}$. From here this is appearing that the topocentric coordinates we can write as y_T and z_T and -R so your coordinate on the ground this is your R this is forming R, if you just look on the surface of the earth. Ok, so this will get simplified it will get eliminated. But if you are on a hill here as in these positions place it as shown.

$$\vec{\rho}_T = \vec{r}_T - \vec{R}$$

$$\vec{\rho}_T = \begin{bmatrix} x_T - G_1 \cos\phi \cos l_0 \\ y_T - G_1\, cos\phi \sin l_0 \\ z_T - G_2 \sin\phi \end{bmatrix}$$

So that time we have to take into account this particular part. So this can be written as G_1 cos ϕ times we have written cos l_0 – G_1 sin ϕ times G_1cos ϕ times G_1Cos ϕ times sin l_0 and z_T we have written as G_2 times sin ϕ. But the observation that we are doing we are not doing in this same rather we are doing the observations in a frame which we had written as x_{TP} y_{TP} and here z_{TP}. And here azimuth is being measured along this direction.

So particularly don here. This is your azimuth and then, then this will be the elevation angle. Now what we need to do that we need to transfer this so, ρ_{TP} this will be given by rotation from topocentric to the terrestrial to the topocentric frame. Ok this is in the terrestrial frame. So this is in the terrestrial frame if I use topocentric then correct it this is the terrestrial; in terrestrial. So this is transfer from rotation from terrestrial to topocentric.

And then we can write here ρ terrestrial and this Matrix how this is decided? This is decided by your whatever the angles is a given to you. So you have to utilise all those angles. So for getting this you need the geocentric angle that means say your; if I take this and this is the geocentric latitude and this is your geocentric this you are writing as ϕ_c. So we have to write here in terms of the ϕ_c.

This angle is ϕ c or maybe we can write this for simplicity is not occurring in this c we can write in terms of beta. So, go to this green shown in green to this coordinate in this system what is required you need to rotate from here to here and come to this point. So such that your x_T comes to this point. So will write this is the $(x_T)'$ this gets parallel to this vector and x_{TP} and x_T then becomes parallel so at the time you will see that z_{TP} z_T comes in the vertical direction and y will go on the opposite side here in this direction so if you do this rotation.

So how much this rotation will be given by? That will be given by R about the third access by $90° + \beta$ R ϕ_c where β equal to ϕ c. So this is the first rotation to be given. Here this angle we have to decide ok so here if you are looking at this place this is the tangent if this angle is β so this angle will be β and this angle will be $90 - \beta$. So to concise y_T direction with y_{TP} then you need a rotation about x axis and that rotation is R_1.

So R_1 we can write here. So R_1 here about the x axis to be rotated by this angle or has its shown here 90 - β then your; this reference frame it concise with this reference frame otherwise you do normal taking the components and then breaking it up. This is the total rotation which maps from T to T_p Terrestrial to topocentric frame. And after multiplication this is rotation about the third axis. And this is rotation about the first Axis first axis and one rotation we have given about the third axis.

So this parallel think this TP is not located at this point but it is located here, but I am just rotating it so that the components can be converted. And for this part maybe I have worked in the satellite attitude dynamic control; their you can go and look how the matrix or rotation is done or either you can refer to any book on the Valladao or etcetera some matrix rotation is given you can look into that also.

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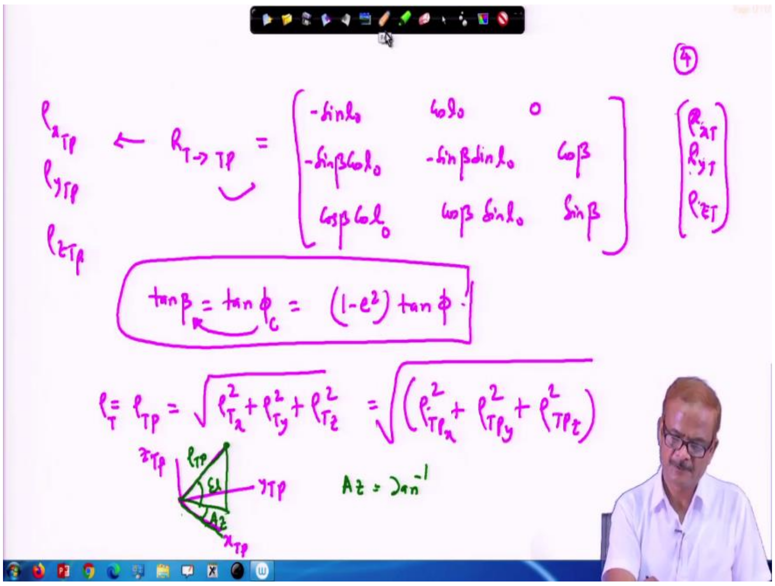

$$R_{TP} = \begin{bmatrix} -\sin l_0 & \cos l_0 & 0 \\ -\sin\beta\cos l_0 & -siin\,\beta\sin l_0 & \cos\beta \\ \cos\beta\cos l_0 & \cos\beta\sin l_0 & \sin\beta \end{bmatrix} \begin{bmatrix} \rho_{xT} \\ \rho_{yT} \\ \rho_{zT} \end{bmatrix}$$

So, from this place your R_T to R_{TP} this will available and this Matrix can be written as minus sin l_0 Cos l_0 minus sin β this is a very long topic which I am trying to compress and tell you in short is a difficult task for me also. And this is the transformation Matrix and this way once you operate on this by; if you operate on this here write here x_T, y_T and z_T or we have written ρ_T we will operate on this ρ_T.

$$tan\beta = \tan\phi_c = (1 - e^2)\tan\phi$$

And this will be ρ_T and ρ_{yT} that we have to write ρ_{xT} ρ_{yT} and ρ_{zT}. So if you operate on this so this will get converted into $\rho_{x_{TP}}$, $\rho_{y_{TP}}$ and $\rho_{z_{TP}}$ so this will get converted into the topocentric frame. Where tan β equal to tan ϕ_c this is connected with ϕ. So ϕ_c I have replaced in terms of B just so that I do not have to carry the subscript.

$$\rho_T = \rho_{TP} = \sqrt{\rho_{T_x}^2 + \rho_{T_y}^2 + \rho_{T_z}^2}$$

Also ρ magnitude in the terrestrial from this will be equal to ρ magnitude in the topocentric frame and this quantity $\rho_{T_x}^2$ where $\rho_{T_y}^2 + \rho_{T_z}^2$ under root ρ_{TP_x} and now azimuth and elevation in the topocentric fram that we have to decide. So once we have done this so we can draw the frame here in this is x_{TP}, y_{TP} and z_{TP} direction. And here you have the ρ_{TP} or whatever ρ_{TP} they are same in the magnitude this is azimuth angle and this is the elevation angle.

So we can immediately write azimuth equal to tan inverse y_{TP} azimuth angle will be; we are measuring the azimuth from x direction. So we will have y_{TP} divided by ϕ ok tan inverse we will write it clearly on the next page.

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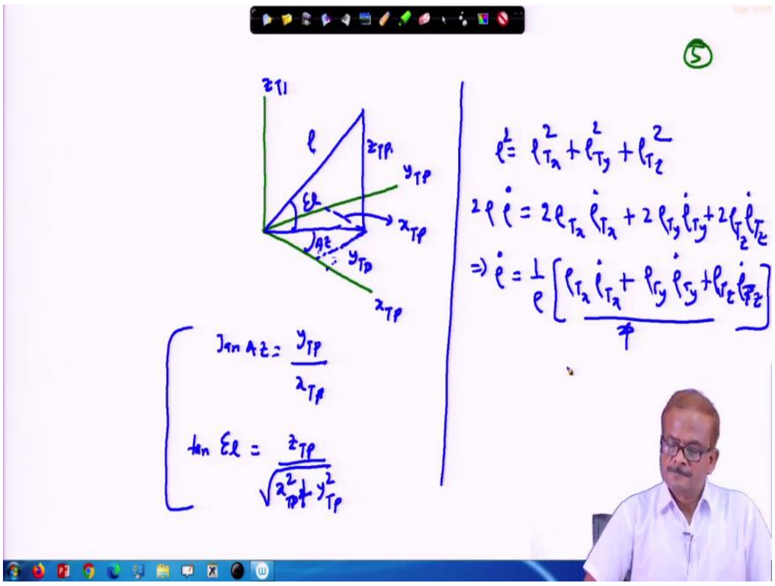

$$\tan Az = \frac{y_{TP}}{x_{TP}}$$

$$\tan \mathcal{E}l = \frac{z_{TP}}{\sqrt{x_{TP}^2 + y_{TP}^2}}$$

This is x_{TP}, y_{TP} and z_{TP} this is ρ this quantity from here to here this angle is azimuth. This angle is elevation and this is the corresponding co-ordinate which will be shown here. This coordinate will be y_{TP}, this coordinator will be x_{TP}, and this coordinate will be y_{TP}. If I write azimuth here this azimuth angle will be y_{TP} divided by x_{TP}. Similarly the tan elevation this will be z_{TP} divided by $x_{TP}^2 + y_{TP}^2$ under root.

Few more things are required here it is a simple differentiation and this implies ρ dot and the same thing can also be written in terms of topocentric components exchange range rate it is available here. So what we are trying to do that if we go and look back this is ρ topocentric and this R topocentric this vector from here this in the terrestrial frame then we get aware of this length the range.

$$\rho^2 = {\rho_T}_x^2 + {\rho_T}_y^2 + {\rho_T}_z^2$$

$$2\rho\dot{\rho} = 2\,{\rho_T}_x{\dot{\rho}_T}_x + 2\,{\rho_T}_y{\dot{\rho}_T}_y + 2\,{\rho_T}_z{\dot{\rho}_T}_z$$

$$\dot{\rho} = \frac{1}{\rho}\left[{\rho_T}_x{\dot{\rho}_T}_x + {\rho_T}_y{\dot{\rho}_T}_y + {\rho_T}_z{\dot{\rho}_T}_z\right]$$

So exactly what we are trying to do here in the inertial frame the radius vector what this R_T this here it is shown to be in the terrestrial frame. If it is given in the inertial frame, so I have to first convert this into the terrestrial frame and so from terrestrial I have to get to the ρ, $\dot{\rho}$, azimuth and elevation.

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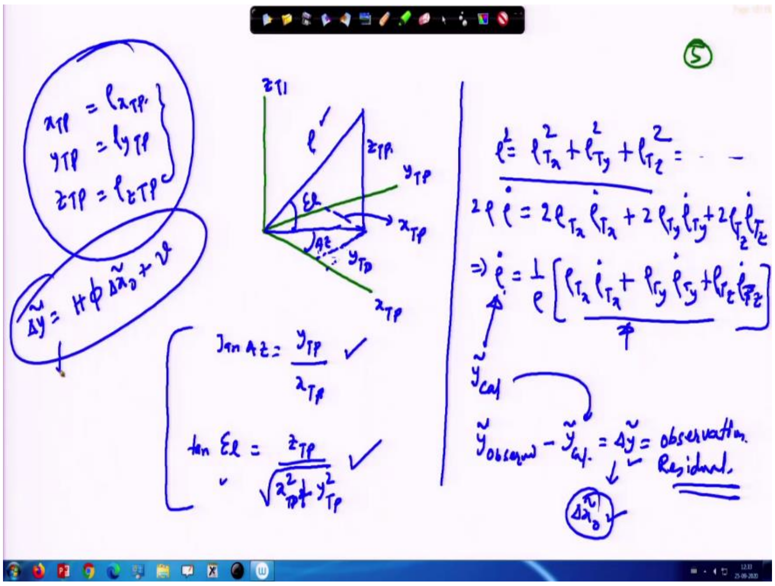

So the steps, we are following. So first what we do assume $\tilde{x}_0$ and then propagate this state to n number of times. So you will get here $\tilde{x}_1$, $\tilde{x}_2$ up to $\tilde{x}_N$ and these are in the propagated in the inertial frame and this comes from the inertial navigation system. So you are propagated. Now each of them needs to be converted into the terrestrial frame. So this is in inertial and then comes to Terrestrial reference frame.

So you know the rotation matrix the complete precision, nutation, rotation and the polar motion though this R_{FK5} reduction and then you are coming to the Terrestrial reference frame. Until early the velocity part also can be reduced. The equation we have written where the R dot also appears. So once this reduced to this then this has to be reduced to the topocentric frame. And in topocentric what we are measuring?

We are measuring ρ, $\dot{\rho}$, azimuth and elevation. So already I also knew that once we get the quantities in terrestrial frame so by multiplying with this Matrix you can Terrestrial frame values can be converted into the; that is topocentric frame. And ok so once you have converted your into topocentric frame and then the corresponding here x_{TP}, y_{TP} and z_{TP} they are nothing but $\rho_{x_{TP}}$, $\rho_{y_{TP}}$ and $\rho_{z_{TP}}$.

Frequency wise a series of mathematical reduction we are able to come to this point and once this three are known immediately the azimuth angle and from here the elevation angle can be estimated and also together with this the $\dot{\rho}$ that can be estimated. What will be the rate of change of this range either you write it like this are topocentric one both will come to the; he will come to the same result because they are connected by the same equation.

This way the observations you are able to get. One is y calculated another one you are directly observing your instruments so those are y observed. And then this is y calculated so it gives you $\Delta\tilde{y}$ and this I have written as the observation residual. Rest other things you need to work out. Now this observation residual is directly related with $\Delta\tilde{x}_0$. If this is known to this can be estimated using the equations we have developed earlier.

$$\Delta\tilde{y} = H\phi\Delta\tilde{x}_0 + \nu$$

The Jacobian Matrix and then the observation matrix so Δy was connected like this ΔH times ϕ times $\Delta\tilde{x}_0$ times ν it was connected like this and this we have tried to minimise and get to Δ x_0 and this is the process to be followed. But if this is the theoretical process and I will supply you with the written material on this because this is the last lecture and after this I am not going to cover any more on this.

And you can refer to the lecture notes that I will be supplying you. But in the actual practice what is done that I will give you.

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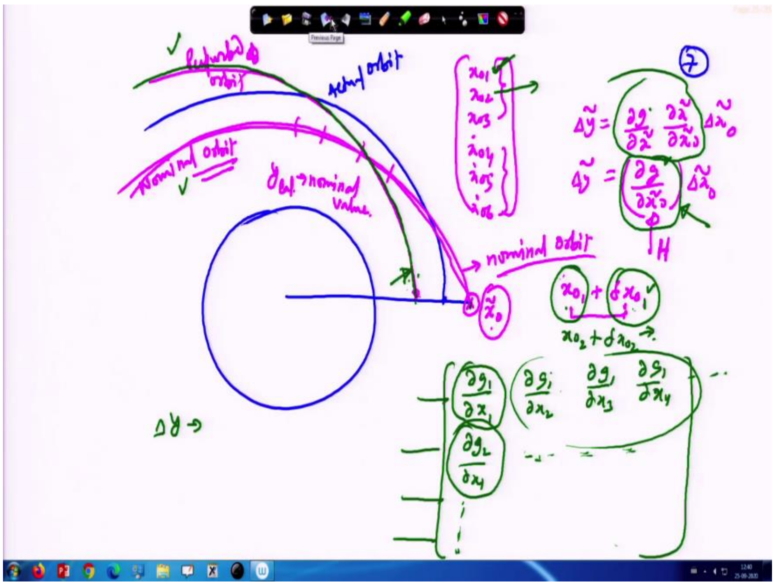

So instead of calculating so much of matrices this is the Earth ok and this is your say the actual orbit which is unknown actual orbit. Then you have one; this is the assumed value $\tilde{x}$ is your assumed value. And using this then you get to the orbits so this is your we called them as a nominal orbit. This we call it as the nominal orbit. So in calculating this Matrix $\Delta\tilde{y}$ is equal to ∂g by $\partial\tilde{x}$ times ∂x tilde by $\partial\tilde{x}_0$times $\Delta\tilde{x}_0$ we are writing like this.

$$\Delta\tilde{y} = \frac{\partial g}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{x}_0} \Delta\tilde{x}_0$$

$$\Delta\tilde{y} = \frac{\partial g}{\partial \tilde{x}_0} \Delta\tilde{x}_0$$

So instead of writing expanding like this which simply write as 0 times $\Delta\tilde{x}_0$ and we are interested in finding this matrix which we have written as H. How do we find it? Then this in the nominal value we apply perturbation one by one. Let us say this is x_{0_1} there are no number of components are there. So x_{0_1} you have and perturbation by some δx_{0_1} only in the; there are six components of this $\dot{x}_{0_1}, \dot{x}_{0_2}, \dot{x}_{0_3}, \dot{x}_{0_4}, \dot{x}_{0_5}, \dot{x}_{0_6}$ and these are corresponding to position and the velocity.

So do the perturbation in this so as a result shape you are your orbit is appearing here in this place. So as a result of this perturbation what will be the corresponding; so propagate this orbit and after propagating you can calculate the different instant of time what will be the value of ϕ calculated. This constitutes your nominal or this is called as the nominal orbit. Next once it is perturbed and as it is position is here and this orbit one you propagate using the state equation so this goes like this.

And then this we call as the perturbed orbit. So I will show you by another colour. This is your perturbed orbit. So perturbing by Δx_{0_1} in x_0 only in the first element you are doing a perturbation you can calculate this quantity here so ∂g by you need the first element here. There are series of size $\partial g_1/\partial x_1$ $\partial g_1/\partial x_2$ $\partial g_1/\partial x_3$ $\partial g_1/\partial x_4$ and so on you have to do up to 6.

Similarly you have $\partial g_2/\partial x_1$ and so on. So if your perturbing in the first element only you will be able to get this you will be able to get this will be to get this so up to here the this four elements of the matrices will be determined. Next then you restore this back to x_{0_1} position. So again we come to the nominal value then next time we perturbed the second element this one by Δx_{0_2}. And then again, we do the same propagation of the orbit calculate this quantities.

Ok. These are the partial derivatives. So how much the y value will change Δy, how much it going to change for you just have to calculate this quantity by calculating the difference. How much difference it makes with this perturbed value and the nominal orbit. If you subtract them at that point and divide by this Δx_0 so you directly get this quantity. So instead of all the mathematics it can be done numerically in a very lucid manner.

Only thing that you need to propagate 6 times nominal orbit will you propagate only once but the perturbed orbit you have to propagate 6 times because every time you are giving perturbation in one of the elements here in one of the state variables. This way you will be able to complete this Matrix once this is done to apply the method of least squares that we have already discussed.

In the previous lecture this particular part and then your correction in Δx this will be known to you. And therefore your first iteration, after doing a number of iteration, you will be able to determine the orbit. So I think this is enough at this step because only one week was allotted to this and already we have perhaps we have covered more than 10 lectures. So we stop here and then next time onwards will start with the trajectory transfer and whatever the hand written material I am having I will supply you later on thank you very much.